Prof. Dr. Christoph Bohle Tübingen SS 2019

Geometry of Manifolds II : Exercise Sheet 5

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Diese Aufgaben sind schriftlich auszuarbeiten und am 23. Mai vor der Vorlesung abzugeben. Für jede Aufgabe gibt es 4 Punkte.

Zweierabgaben sind erlaubt. Bitte bei der ersten Abgabe Matrikelnummer(n) angeben.

Aufgabe 1. Show that the vector field that in spherical coordinates

$$(\varphi, \theta) \mapsto \big(\sin(\theta)\cos(\varphi), \sin(\theta)\sin(\varphi), \cos(\theta)\big)$$

is given by $X = \frac{\partial}{\partial \varphi}$ extends smoothly to the entire S^2. Compute the flow of the extended vector field.

Aufgabe 2. Let X, Y be vector fields on a manifold M and denote by Φ_t the local flow of X. Then $Y(t) = (\Phi_{-t})_* Y$ satisfies

$$\frac{\partial}{\partial t} Y(t) = (\Phi_{-t})_* [X, Y].$$

Aufgabe 3. Show that the 1–form

$$\omega = y\,dx - x\,dy + dz$$

on $\mathbb{R}^3$ satisfies

$$\omega \wedge d\omega \neq 0.$$

Conclude that $E_{|p} = \ker(\omega_p)$ defines a distribution that is non–integrable.

Aufgabe 4. The *Lie derivative* of $\omega \in \Omega^k(M)$ with respect to $X \in \mathfrak{X}(M)$ is defined as

$$\mathcal{L}_X \omega := \frac{d}{dt}_{|t=0} \Phi_t^* \omega,$$

where Φ_t denotes the local flow of X. Show that

$$\mathcal{L}_X \omega = d(\iota_X \omega) + \iota_X (d\omega),$$

where $\iota_X \omega$ denotes the so called *inner product* obtained by plugging X in the first slot of ω. (Hint: show that both sides of the equation give rise to a linear operator on $\Omega^*(M)$ that preserves degree and commutes with $\wedge$ and d. Prove the claim using that both operators thus obtained coincide on 0–forms/functions.)