Prof. Dr. Christoph Bohle Tübingen SS 2019

Geometry of Manifolds II : Exercise Sheet 7

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Diese Aufgaben sind schriftlich auszuarbeiten und am 6. Juni vor der Vorlesung abzugeben. Für jede Aufgabe gibt es 4 Punkte.

Zweierabgaben sind erlaubt. Bitte bei der ersten Abgabe Matrikelnummer(n) angeben.

Aufgabe 1. Let $E \to M$ be a vector bundle with connection ∇. Show that

i) $\mathcal{R}^\nabla \varphi = d^\nabla(\nabla\varphi)$ for $\varphi \in \Gamma(E)$,
ii) $d^\nabla \circ d^\nabla \omega = \mathcal{R}^\nabla \wedge \omega$ for $\omega \in \Omega^k(M;E)$, and
iii) $d^\nabla \mathcal{R}^\nabla = 0$.

Aufgabe 2. Given $\tilde{\nabla} = \nabla + \omega$ on $E \to M$, show that

i) $d^{\tilde{\nabla}}\eta = d^\nabla \eta + \omega \wedge \eta$ for $\eta \in \Omega^k(M;E)$ and
ii) $\mathcal{R}^{\tilde{\nabla}} = \mathcal{R}^\nabla + d^\nabla\omega + \omega\wedge\omega$.

Aufgabe 3. Let $\gamma\colon I \to \mathbb{R}^3$ be a curve parametrized by arclength. Show that

i) the derivative N' of a normal vector field $N\colon I \to \mathbb{R}^3$ is pointwise a multiple of T if and only if
$$N' = - < N, T' > T,$$
ii) if $N\colon I \to \mathbb{R}^3$ satisfies the linear differential equation in i) and if
$$N(t) \perp T(t) \qquad \text{and} \qquad \|N(t)\| = 1,$$
holds for one $t \in I$, it holds for all $t \in I$, and
iii) if N satisfies the differential equation in i) with initial value as in ii), so does
$$\tilde{N} = e^{\alpha J} N = \cos(\alpha)N + \sin(\alpha) T \times N$$
with $\alpha \in \mathbb{R}$ and all solutions to i) und ii) are of that form.

Aufgabe 4. Let $\gamma\colon I \to \mathbb{R}^3$ be a curve parametrized by arclength. Every unit normal vector field $N\colon I \to \mathbb{R}^3$ defines a frame $F = (T, N, T \times N)$, where $T = \gamma'(t)$. Show that changing the normal field to
$$\tilde{N} = e^{\theta J} N = \cos(\theta)N + \sin(\theta) T \times N$$
for some function $\theta\colon I \to \mathbb{R}$ changes of the complex curvature $\kappa = \kappa_1 + i\kappa_2$ and torsion τ of F to
$$\tilde{\kappa} = e^{-i\theta}\kappa \quad \text{and} \quad \tilde{\tau} = \tau + \theta'.$$