Prof. Dr. Christoph Bohle Tübingen WS 2019/20

Geometry of Manifolds/Geometry in Physics
Exercise Sheet 3

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Diese Aufgaben sind schriftlich auszuarbeiten und am 7. November vor der Vorlesung abzugeben. Für jede Aufgabe gibt es 4 Punkte.

Zweierabgaben sind erlaubt. Bitte bei der ersten Abgabe Matrikelnummer(n) angeben.

Aufgabe 1. Explain why

$$M = \{(x_1, x_2, x_3) \in \mathbb{R}^3 \mid |x_1|, |x_2|, |x_3| \leq 1 \text{ and } |x_i| = 1 \text{ for at least one i}\}$$

is not a submanifold of $\mathbb{R}^3$. Describe the minimal subset $M' \subset M$ such that $M \backslash M'$ is a submanifold of $\mathbb{R}^3$. Is it possible to equip M with a differentiable structure compatible with the subspace topology induced from $\mathbb{R}^3$?

Aufgabe 2.

i) Given a submanifold $N \subset M$, prove that φ is an admissible chart for N iff $g = \varphi^{-1}$ is a parametrization as in iii) of the submanifold theorem.

ii) Show that the differential structure on S^n induced by the atlas $\{\varphi_N, \varphi_S\}$ coincides with the submanifold differential structure.

Aufgabe 3. Let $M \subset \mathbb{R}^N$ be a submanifold and $q \in \mathbb{R}^N \backslash M$. If the function $x \mapsto \|x - q\|$ takes its minimum in $p \in M$, then $p - q$ is perpendicular to T_pM.

Aufgabe 4. Prove that

i) $G = SL(n, \mathbb{R})$ and $G = O(n)$ are submanifolds of $Gl(n, \mathbb{R})$.

ii) Left– and right–multiplication by an element $g \in G$ acts as diffeomorphisms $L_g, R_g \colon G \to G$.

iii) The tangent space T_eG in the point $e \in G$ is equal to the set of trace free matrices (for $G = SL(n, \mathbb{R})$) and to the set of skew symmetric matrices (for $G = O(n)$).

iv) We have

$$T_gG = g \cdot T_eG = T_eG \cdot g.$$