Prof. Dr. Christoph Bohle Tübingen WS 2019/20

Geometry of Manifolds/Geometry in Physics
Exercise Sheet 4

Jonas Ziefle 5. November 2019

Diese Aufgaben sind schriftlich auszuarbeiten und am 14. November vor der Vorlesung abzugeben. Für jede Aufgabe gibt es 4 Punkte.

Zweierabgaben sind erlaubt. Bitte bei der ersten Abgabe Matrikelnummer(n) angeben.

Aufgabe 1. Compute the basis change matrix between the Gauss basis for spherical (cylindrical) coordinates on $\mathbb{R}^3$ and the standard basis.

Aufgabe 2. Show that the vectorfield on S^2 that in spherical coordinates is given by $X = \frac{\partial}{\partial \varphi}$ can be smoothly extended through the poles.

Aufgabe 3. Show that

$$M = \{(x, y, z) \in \mathbb{R}^3 \mid x^2 + y^4 + z^4 = 1\}$$

is a submanifold. Find a chart around $p = (0, 0, 1) \in M$. Determine a linear equation for T_pM.

(Optional) Show that M is diffeomorphic to S^2. (The inverse function theorem in a version for maps between manifolds is useful here.)

Aufgabe 4. Denote by $TM = \dot{\cup}_{p\in M} T_pM$ the disjoint union of all tangent spaces of a manifold M. Show that

i) every atlas $\mathcal{A}$ of M defines an atlas of TM as follow: for $(U, \varphi = (x_1, ..., x_n)) \in \mathcal{A}$ define

$$\Phi\colon TM_{|U} \to \varphi(U) \times \mathbb{R}^n \qquad \sum_i v_i \frac{\partial}{\partial x_i}(p) \mapsto (\varphi(p), v_1, ..., v_n),$$

where $TM_{|U} = \dot{\cup}_{p\in U} T_pM$ and $\frac{\partial}{\partial x_1}(p)$, ..., $\frac{\partial}{\partial x_n}(p)$ is the Gauss basis of T_pM defined by (U, φ),

ii) TM with this atlas is a manifold,

iii) the projection $\pi\colon TM \to M$ that maps a tangent vector $X \in T_pM$ to its base point p is a submersion, and

iv) vectorfields are precisely the smooth maps $X\colon M \to TM$ satisfying

$$\pi \circ X = \mathrm{Id}_M\,.$$