Prof. Dr. Christoph Bohle Tübingen WS 2019/20

Geometry of Manifolds/Geometry in Physics
Exercise Sheet 8

Jonas Ziefle 3. Dezember 2019

Diese Aufgaben sind schriftlich auszuarbeiten und am 12. Dezember vor der Vorlesung abzugeben. Für jede Aufgabe gibt es 4 Punkte.

Zweierabgaben sind erlaubt. Bitte bei der ersten Abgabe Matrikelnummer(n) angeben.

Aufgabe 1. Denote by $(e_1, ..., e_n)$ and $(f_1, ..., f_n)$ two bases of a vector space V.

a) Determie the matrix of basis change between the dual bases $(e_1^*, ..., e_n^*)$ and $(f_1^*, ..., f_n^*)$ from that between $(e_1, ..., e_n)$ and $(f_1, ..., f_n)$.
b) Determine the relation between the determinants $e_1^* \wedge ... \wedge e_n^*$ and $f_1^* \wedge ... \wedge f_n^*$ on V that are normalized with respect to $(e_1, ..., e_n)$ and $(f_1, ..., f_n)$ respectively.

Aufgabe 2. A *bundel metric* on a real vector bundle E is a section

$$h \in \Gamma(E^* \otimes E^*)$$

(often denoted by $\langle . \rangle$) that fiberwise is a positive definite symmetric bilinearform. Show that an $O(r)$–valued cocycle defines a bundle with bundle metric. Conversely, show that a bundle with bundle metric admits a bundle atlas such that all cocyles take values in $O(r)$. Show that every bundle admits a bundle metric.

Aufgabe 3. Determine the isomorphy classes of real line bundles over S^1. (Choose a bundle metric. Then a unit length section is either globally defined or not...)

Aufgabe 4. Show that

$$\Sigma_{\mathbb{RP}^n} = \{(p, v) \in \mathbb{RP}^n \times \mathbb{R}^{n+1} \mid v \in p\}$$

with $\pi \colon \Sigma_{\mathbb{RP}^n} \to \mathbb{RP}^n$, $(p, v) \mapsto p$ is a line bundle on $\mathbb{RP}^n$ (the so called *tautological bundle*). Is $\Sigma_{\mathbb{RP}^1}$ trivial?