Prof. Dr. Christoph Bohle Tübingen WS 2019/20

Geometry of Manifolds/Geometry in Physics
Exercise Sheet 11

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Diese Aufgaben sind schriftlich auszuarbeiten und am 16. Januar vor der Vorlesung abzugeben. Für jede Aufgabe gibt es 4 Punkte.

Zweierabgaben sind erlaubt. Bitte bei der ersten Abgabe Matrikelnummer(n) angeben.

Aufgabe 1. Let E and F be vector bundles with connections on M.

a) Prove that there are unique connections on E^* and $E \otimes F$ satisfying

$$< \nabla\alpha, \varphi >= d < \alpha, \varphi > - < \alpha, \nabla\varphi >$$

and

$$\nabla(\varphi \otimes \psi) = (\nabla\varphi \otimes \psi) + (\varphi \otimes \nabla\psi)$$

for all $\varphi \in \Gamma(E)$, $\psi \in \Gamma(F)$ and $\alpha \in \Gamma(E^*)$.

b) What is the curvature of the connection on E^* and $E \otimes F$?

c) What is the curvature of the connection on $\mathrm{End}(E) = E^* \otimes E$?

Aufgabe 2. Let $\omega \in \Omega^1(U, Mat_{r\times r}(\mathbb{K}))$ be a matrix valued 1–form on an open set $U = \prod(\alpha_i, \beta_i) \subset \mathbb{R}^n$. Show that the existence of a smooth function

$$f \colon U \to GL_r(\mathbb{K}) \qquad \text{with} \qquad f^{-1}df = \omega$$

is equivalent to

$$d\omega + \omega \wedge \omega = 0.$$

Aufgabe 3. Prove that a connection is flat if and only if parallel transport with respect to the connection is locally independent of the path between points.

Aufgabe 4. Prove that:

a) Every isometry between two Riemannian manifolds is an affine diffeomorphism.

b) Affine diffeomorphisms preserve
   - curvature,
   - parallel transport, and
   - geodesics.

   (Prove one of the three fact.)