I will describe joint work with Costante Bellettini (Cambridge) in which we develop a regularity theory for a class of hypersurfaces of a smooth Riemannian manifold that are stationary and stable for area with respect to volume preserving ambient deformations. The hypersurfaces (codimension 1 integral varifolds) in this class are required to satisfy two structural conditions: (1) they have no classical singularities. A classical singularity is a point about which there is a neighborhood in which the hypersurface is supported on three or more embedded sheets coming smoothly and transversely together along a common boundary. (2) if $y$ is a touching singularity—i.e. a point where the hypersurface locally is supported on two distinct $C^{1,\alpha}$ graphs touching at that point—then there is a neighborhood of $y$ in which the set of points with density equal to the density at $y$ has zero $n$-dimensional Hausdorff measure, where $n$ is the dimension of the hypersurface. We show that such a hypersurface, away from a closed set of codimension at least 7, locally is supported on a smooth graph or two smooth graphs touching. Easy examples show that (1) and (2) are necessary. If the hypersurface is the boundary of a Caccioppoli set, then (2) is automatically satisfied. We also show that a collection of such hypersurfaces satisfying uniform volume and mean curvature bounds is compact in the varifold topology.

Hierzu wird herzlich eingeladen.

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