On self gravitating solutions of the Einstein-Scalar field equations

We will discuss the existence of geodesically complete solutions of the Einstein-Scalar field equations in arbitrary dimensions depending on the form of the scalar field potential $V(\phi)$. As a main special case it will be shown that when $V(\phi)$ is the Klein-Gordon potential, i.e. $V(\phi) = m^2|\phi|^2$, geodesically complete solutions are necessarily Ricci-flat, have constant lapse and are vacuum, (that is $\phi = \phi_0$ with $\phi_0 = 0$ if $m \neq 0$). Therefore, if the spatial dimension is three, the only such solutions are either Minkowski or a quotient thereof. For $V(\phi) = m^2|\phi|^2 + 2\Lambda$, that is, including a vacuum energy or a cosmological constant, it will be shown that no geodesically complete solution exists when $\Lambda > 0$, whereas when $\Lambda < 0$ it is proved that no non-vacuum geodesically complete solution exists unless $m^2 < -2\Lambda/(n-1)$, ($n$ is the spatial dimension) and the manifold is non-compact. The proofs are based on techniques in comparison geometry à la Backry-Emery.

Hierzu wird herzlich eingeladen.

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