Aufgabe 37

fishy.dat aus Aufgabe 32

$$E[X_j] = \mu \quad , \quad Var(X_j) = \sigma^2$$

$$Y = X_1 + X_2 + \dots + X_n \sim N(n\mu, n\sigma^2)$$

$$E[Y] = E[X_1 + \dots + X_n] = E[X_1] + \dots + E[X_n] = n\mu$$

$$Var[Y] = Var(X_1 + \dots + X_n) \underset{X_j \text{ unabhängig}}{=} Var(X_1) + \dots + Var(X_n) = n\sigma^2$$

$$\overline{X} = \frac{X_1 + \dots + X_n}{n} = \frac{1}{n} Y$$

$$E[\overline{X}] = E[\tfrac{1}{n} Y] = \frac{1}{n} E[Y] = \mu$$

$$Var(\overline{X}) = Var(\tfrac{1}{n} Y) = \frac{1}{n^2} Var(Y) = \frac{\sigma^2}{n}$$

d. h. $\overline{X} \sim N(\mu, \frac{\sigma^2}{n})$

Z - Test

$$Z = \frac{\overline{X} - \mu_0}{\sigma/\sqrt{n}} \quad , \quad \overline{X} = \frac{X_1 + \dots + X_n}{n}$$

wir wissen $E[\overline{X}] = \mu_0$, $\mathrm{Var}(\overline{X}) = \frac{\sigma^2}{\sqrt{n}}$

laut Voraussetz. $Z \sim N(0, 1)$

$$E[Z] = E\left[\frac{\overline{X} - \mu_0}{\sigma/\sqrt{n}}\right] = \frac{\sqrt{n}}{\sigma} E[\overline{X} - \mu_0]$$

$$= \frac{\sqrt{n}}{\sigma}\left(E[\overline{X}] - \mu_0\right) = 0$$

$$\mathrm{Var}(Z) = \mathrm{Var}\left(\frac{\overline{X} - \mu_0}{\sigma/\sqrt{n}}\right) = \frac{n}{\sigma^2} \mathrm{Var}(\overline{X} - \mu_0)$$

$$= \frac{n}{\sigma^2} \mathrm{Var}(\overline{X}) = 1$$

VI beim t-Test

$H_0 : \mu = \mu_0$ (gegen $H_A : \mu \neq \mu_0$)

H_0 wird angenommen falls

$t_{df, 1-\frac{\alpha}{2}}$

$$-t < \underbrace{\frac{\bar{x} - \mu_0}{s/\sqrt{n}}}_{= T} < t \quad \Big| \cdot \frac{s}{\sqrt{n}}$$

$$\iff -\frac{ts}{\sqrt{n}} < \bar{x} - \mu_0 < \frac{ts}{\sqrt{n}} \quad \Big| -\bar{x} \quad \Big| \cdot (-1)$$

$$\iff \bar{x} + \frac{ts}{\sqrt{n}} > \mu_0 > \bar{x} - \frac{ts}{\sqrt{n}}$$

Wilcoxon-Test

Summe aller Ränge:

$$1+2+3+\ldots+n = \frac{(n+1)n}{2}$$

Hälfte $\oplus$, Hälfte $\ominus$, unabhängig

also erwarte $U^+ \approx U^- = \frac{(n+1)n}{4}$