Korrelation (Pearson)

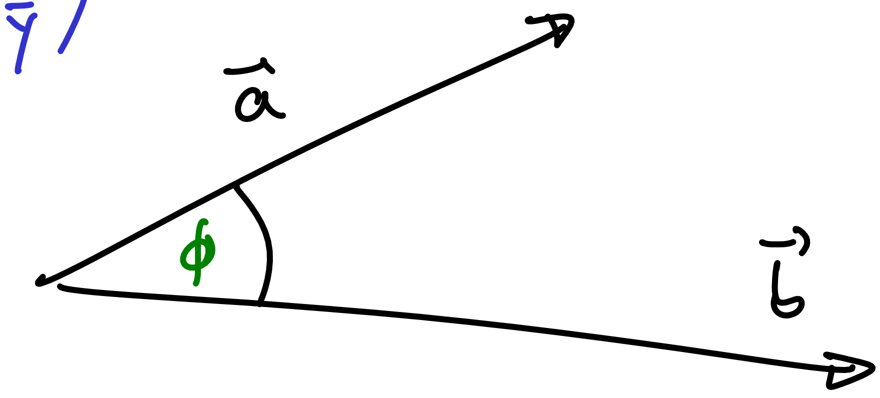
$$r_{xy} = \frac{s_{xy}}{s_x \cdot s_y}$$

$$s_{xy} = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})$$

$$s_x = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2}$$

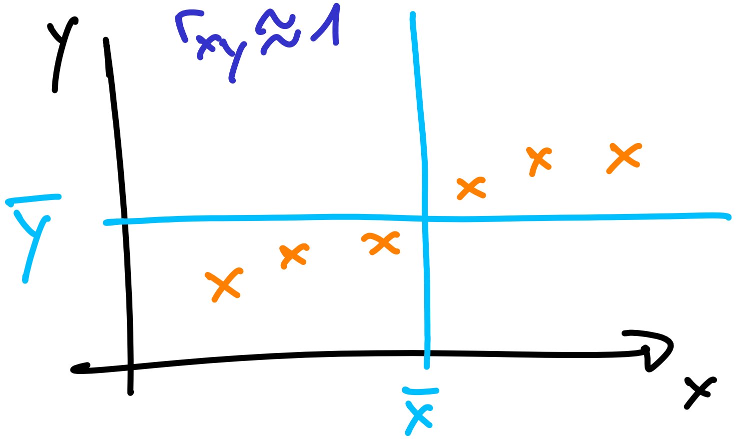
$$r_{xy} = \frac{\cancel{\frac{1}{n-1}} \sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\cancel{\frac{1}{n-1}} \sum_{i=1}^{n} (x_i - \bar{x})^2} \sqrt{\cancel{\frac{1}{n-1}} \sum_{i=1}^{n} (y_i - \bar{y})^2}}$$

$$\vec{a} = \begin{pmatrix} x_1 - \bar{x} \\ \vdots \\ x_n - \bar{x} \end{pmatrix}, \quad \vec{b} = \begin{pmatrix} y_1 - \bar{y} \\ \vdots \\ y_n - \bar{y} \end{pmatrix}$$

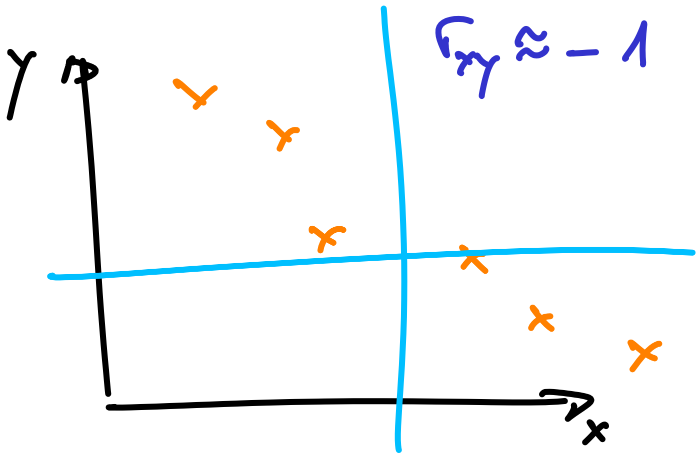


$$r_{xy} = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| \cdot |\vec{b}|}$$

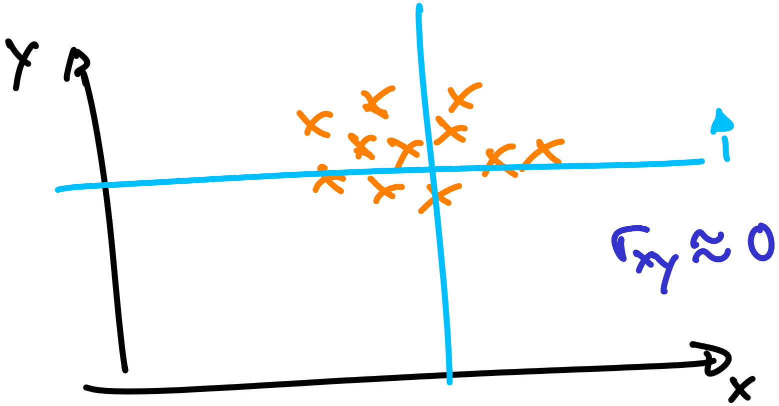
$$\vec{a} \cdot \vec{b} = |\vec{a}| \cdot |\vec{b}| \cos\phi$$



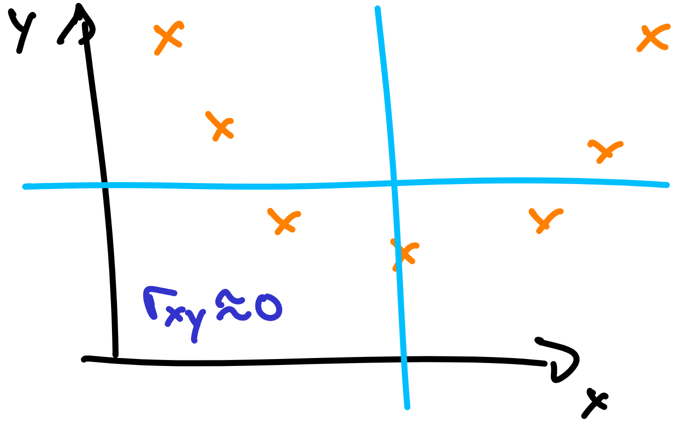
Y
$r_{xy} \approx 1$
$\bar{Y}$
$\bar{X}$
X



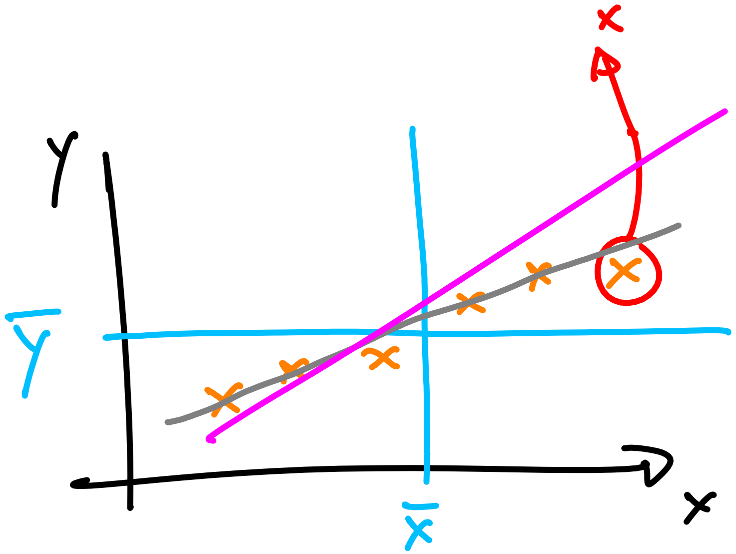
Y
$r_{xy} \approx -1$
X



Y
$r_{xy} \approx 0$
X

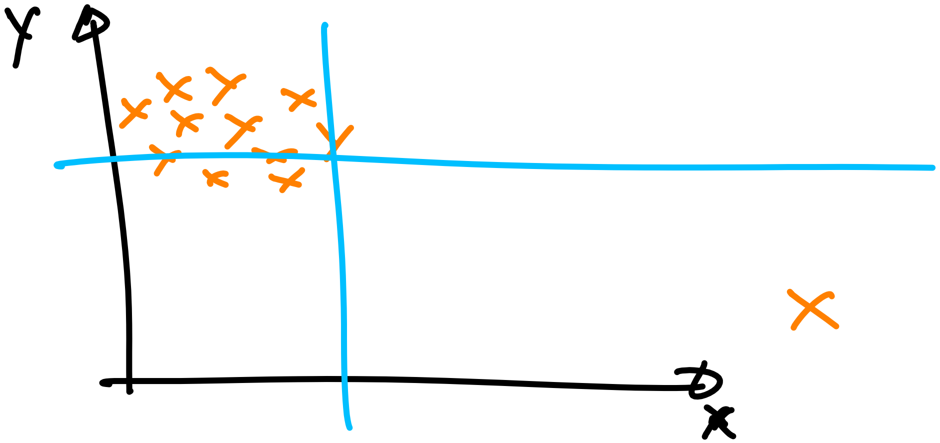


Y
$r_{xy} \approx 0$
X



$r_{xy} \approx 1$ $\xrightarrow{\text{Fehler}}$ Korrelation
sinkt

Rangkorr. unverändert

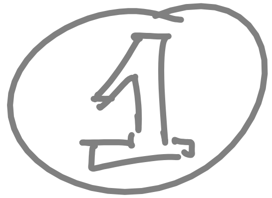


$r_{xy} < 0$ und deutlich
von Null verschieden
(aber "keine Korrelation")

Rangkorrelation bleibt klein

Etwas ganz anderes

faire Münze

Wahrscheinlichkeit $\frac{1}{2}$ $\frac{1}{2}$

werfe 3x $\rightsquigarrow$ 3x K (Kopf)

Wahrscheinlichkeit dafür, falls die Münze fair?

$$\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2} = \left(\frac{1}{2}\right)^3 = \frac{1}{8} = 12{,}5\%$$

werfe 10x $\rightsquigarrow$ 9x K

$$\frac{1}{512} \qquad \frac{10}{1024}$$

$$\left(\frac{1}{2}\right)^9 \cdot \frac{1}{2} \cdot 10 \approx 1\%$$