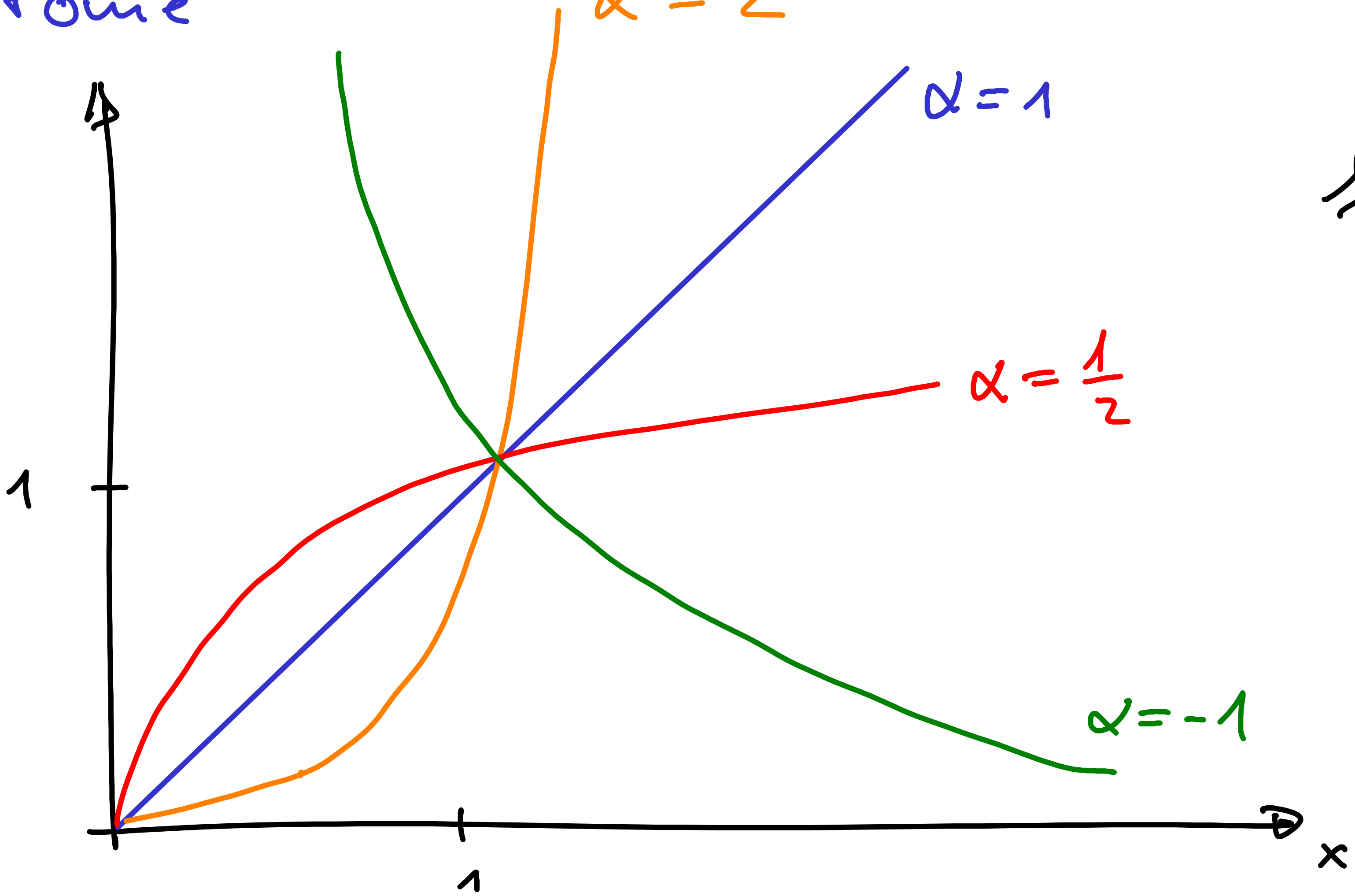
$$\sqrt[3]{9^{-2} \cdot 3} = \left[(3^2)^{-2} \cdot 3\right]^{\frac{1}{3}}$$

$$= \left(3^{-4} \cdot 3\right)^{\frac{1}{3}} = \left(3^{-3}\right)^{\frac{1}{3}} = 3^{-1} = \frac{1}{3}$$

Monotonie

$f(x) = x^{\alpha}$



$t = \frac{1}{2}$ (halbes Jahr)

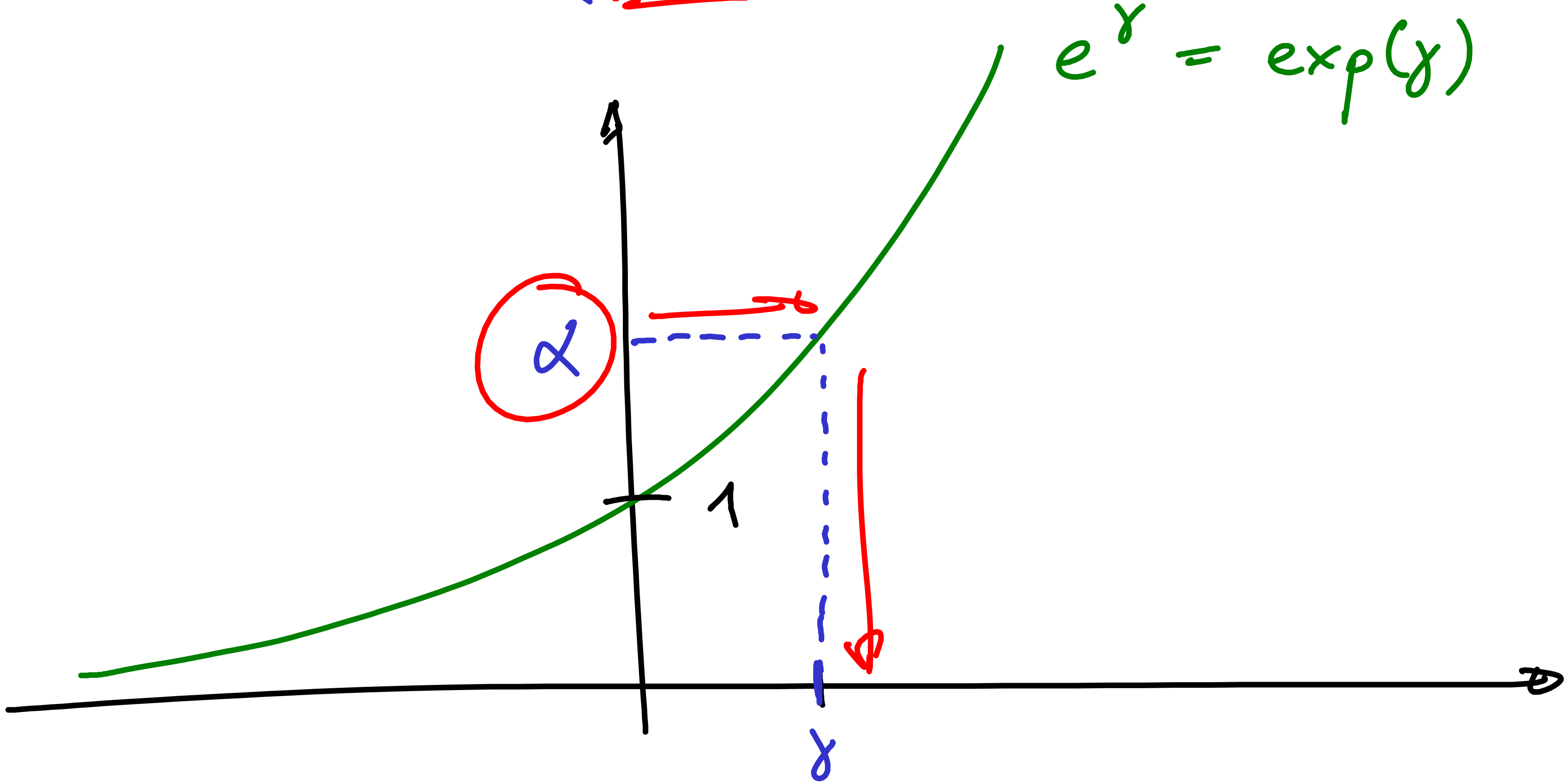
$\alpha = 1{,}06$ (6% Zinsen)

$G(0) = 100\,€$

Rückzahlung nach halbem Jahr

$$G\left(\tfrac{1}{2}\right) = (1{,}06)^{1/2} \cdot 100\,€ \approx 102{,}96\,€$$

$$\alpha^t = e^{\gamma t} = (e^\gamma)^t$$



$(\gamma \in \mathbb{R}, \alpha > 0)$

$$\alpha^{t/\tau} \underset{\alpha = e^{\gamma}}{=} \left(e^{\gamma}\right)^{t/\tau} = e^{\frac{\gamma t}{\tau}}$$

$$\underset{\frac{\gamma}{\tau} = \lambda}{=} e^{\lambda t}$$

$$G(t) = e^{\lambda t}\, G(0)$$

$$G\left(\frac{1}{\lambda}\right) = e^{\lambda \frac{1}{\lambda}}\, G(0) = e \cdot G(0)$$

$$G\left(-\frac{1}{\lambda}\right) = e^{\lambda \cdot \left(-\frac{1}{\lambda}\right)}\, G(0) = e^{-1} \cdot G(0) = \frac{G(0)}{e}$$

radioaktiver Zerfall

$G(t)$ Menge zu Beginn des Intervalls $[t, t+T]$ vorhanden ist

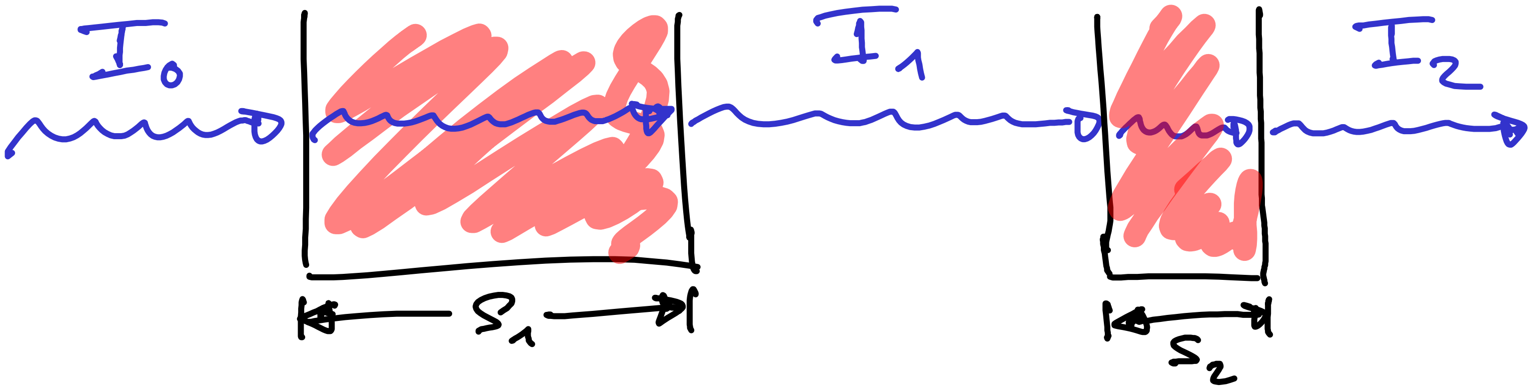
$G(t+T)$ Menge am Ende des Intervalls

Verhältnis

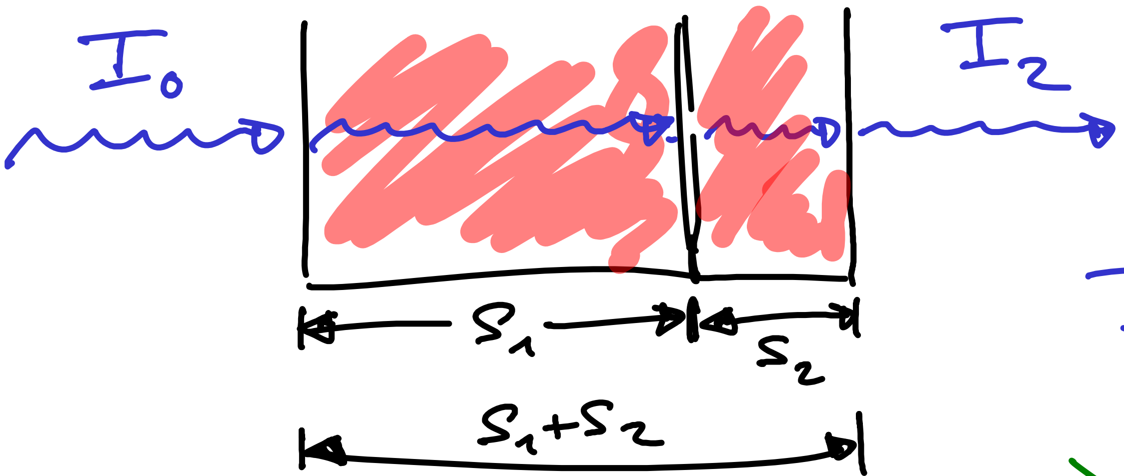
$$\frac{G(t+T)}{G(t)} = \frac{e^{-\lambda(t+T)}G(0)}{e^{-\lambda t}G(0)} = e^{-\lambda T}$$

hängt <u>nicht</u> von t ab!

zum Lambert-Beer-Gesetz

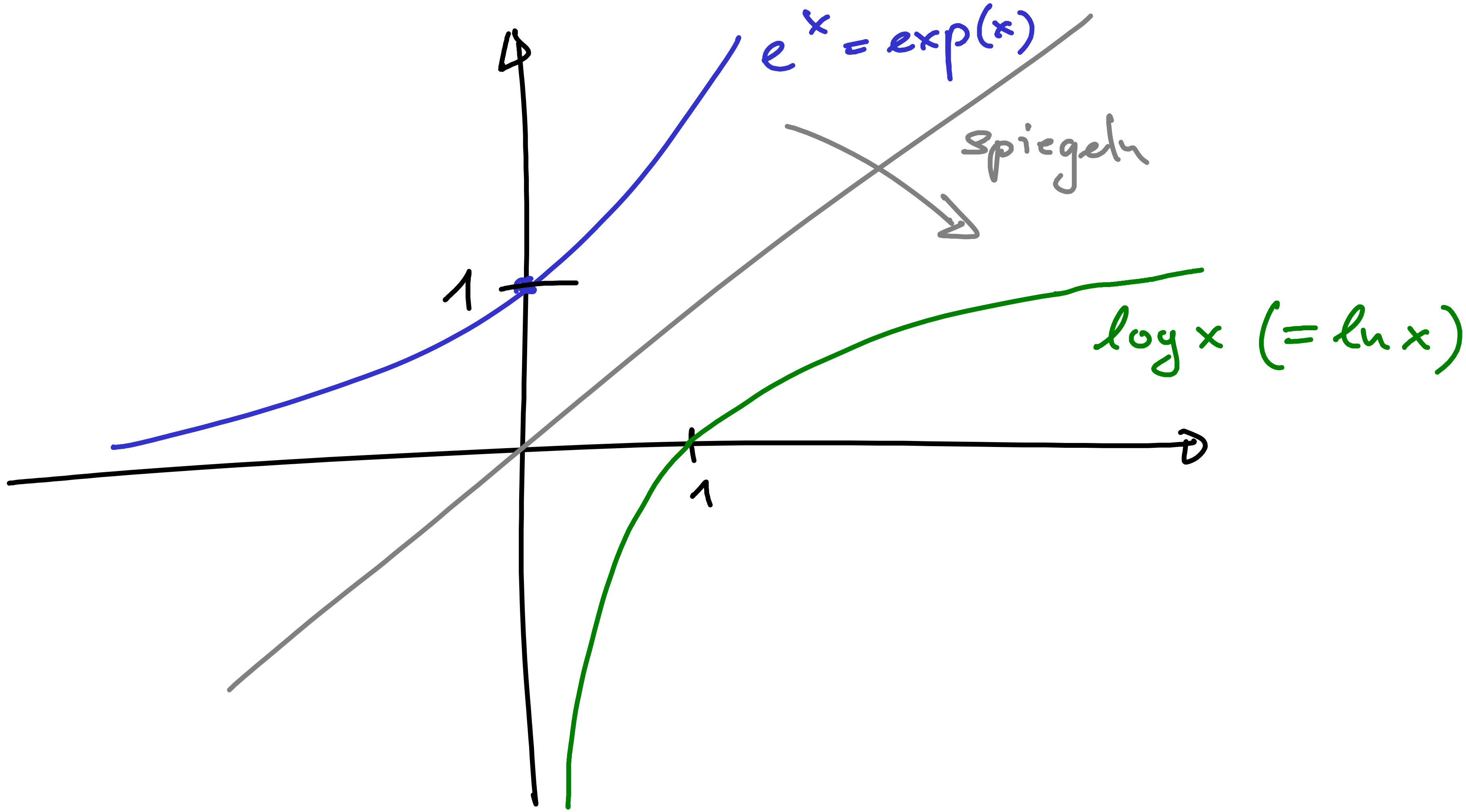


$$\underline{I_1 = \alpha_{s_1} \cdot I_0} \quad , \quad I_2 = \alpha_{s_2} \cdot I_1 = \alpha_{s_2} \cdot \alpha_{s_1} \cdot I_0$$



$$I_2 = \alpha_{s_1+s_2} \, I_0$$

$$\Rightarrow \quad \alpha_{s_1+s_2} = \alpha_{s_2} \cdot \alpha_{s_1}$$



e^x = exp(x)
spiegeln
log x (= ln x)
1
1

log - Rechenregeln

① $\log(xy) = \log x + \log y$

$x = e^a \;,\; y = e^b \iff \log x = a \;,\; \log y = b$

Potenzrechenregel

$\log(xy) = \log(e^a \cdot e^b) = \log(e^{a+b})$

$= a + b = \log x + \log y$

log und $e^{\cdots}$ sind Umkehrfkt.

② $\log(x^\alpha) = \alpha \log x \qquad (x > 0, \alpha \in \mathbb{R})$

$$x = e^\gamma \iff \log x = \gamma$$

$$\log(x^\alpha) = \log((e^\gamma)^\alpha) \overset{\text{P.R.}}{=} \log(e^{\gamma\cdot\alpha})$$

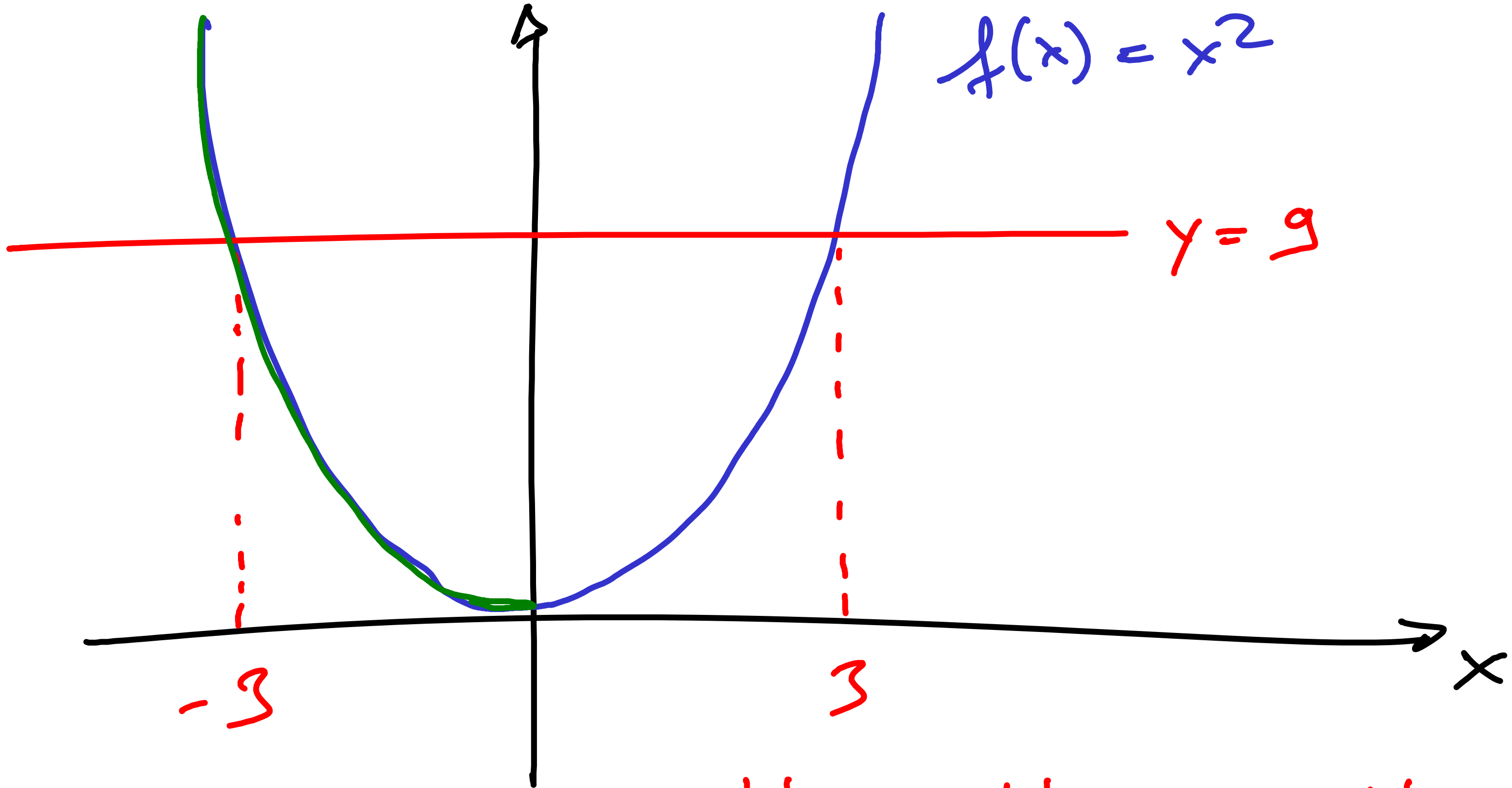
$$= \gamma \cdot \alpha = \alpha \cdot \log x$$

log ⇔ exp
Umkehrfkt.

③ $\log\left(\frac{1}{x}\right) = -\log x \qquad$ (② für $\alpha = -1$)

④ $\log(1) = \log(e^0) = 0$

Umkehrfunktionen



nicht surjektiv $\Rightarrow$ nicht umkehrbar

$$\left.\begin{array}{l} f: [0,\infty) \longrightarrow [0,\infty) \\ \qquad x \longmapsto x^2 \end{array}\right\} \text{ ist umkehrbar!}$$

$$f^{-1}: y \longmapsto \sqrt{y}$$

ebenso

$$\tilde{f} : (-\infty, 0] \longrightarrow [0, \infty)$$
$$x \longmapsto x^2$$

(linke Teil des Graphen)

$$\tilde{f}^{-1} : [0, \infty) \longrightarrow (-\infty, 0]$$
$$y \longmapsto -\sqrt{y}$$

f streng monoton wachsend/fallend

$x \neq y$

entweder

(i) $x > y \Rightarrow f(x) \underset{<}{>} f(y)$

oder

(ii) $x < y \Rightarrow f(x) \underset{>}{<} f(y)$

$\Rightarrow f(x) \neq f(y)$

injektiv