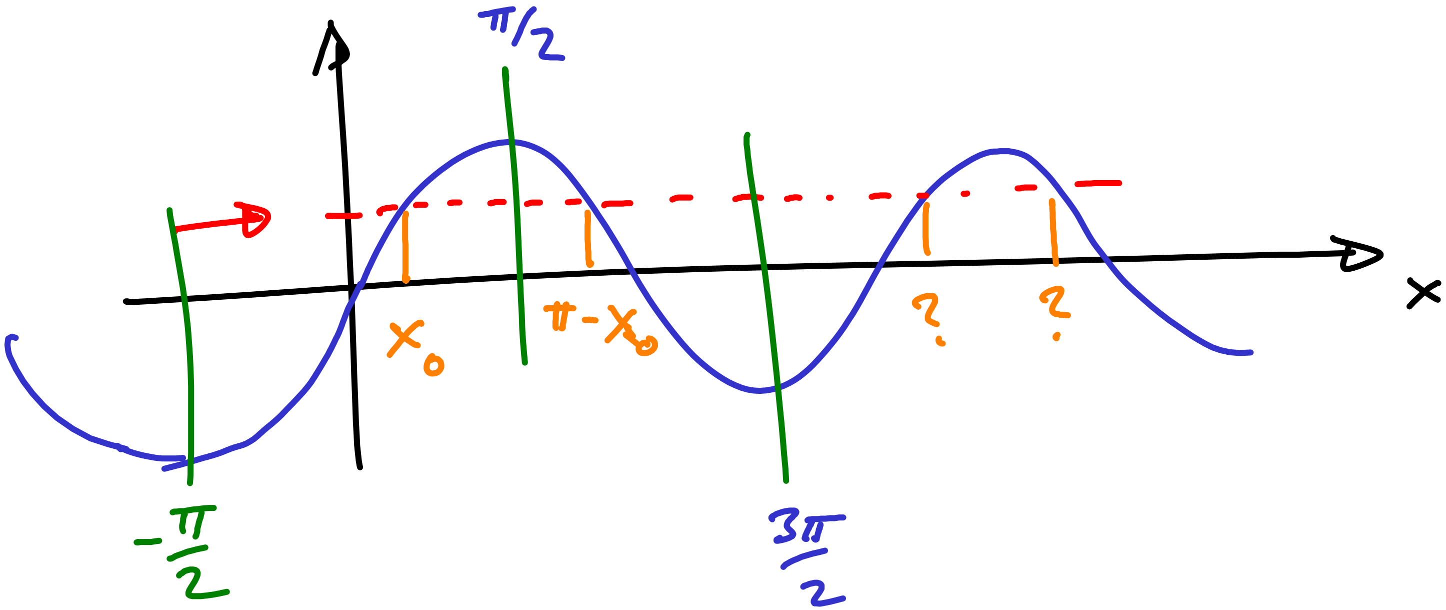
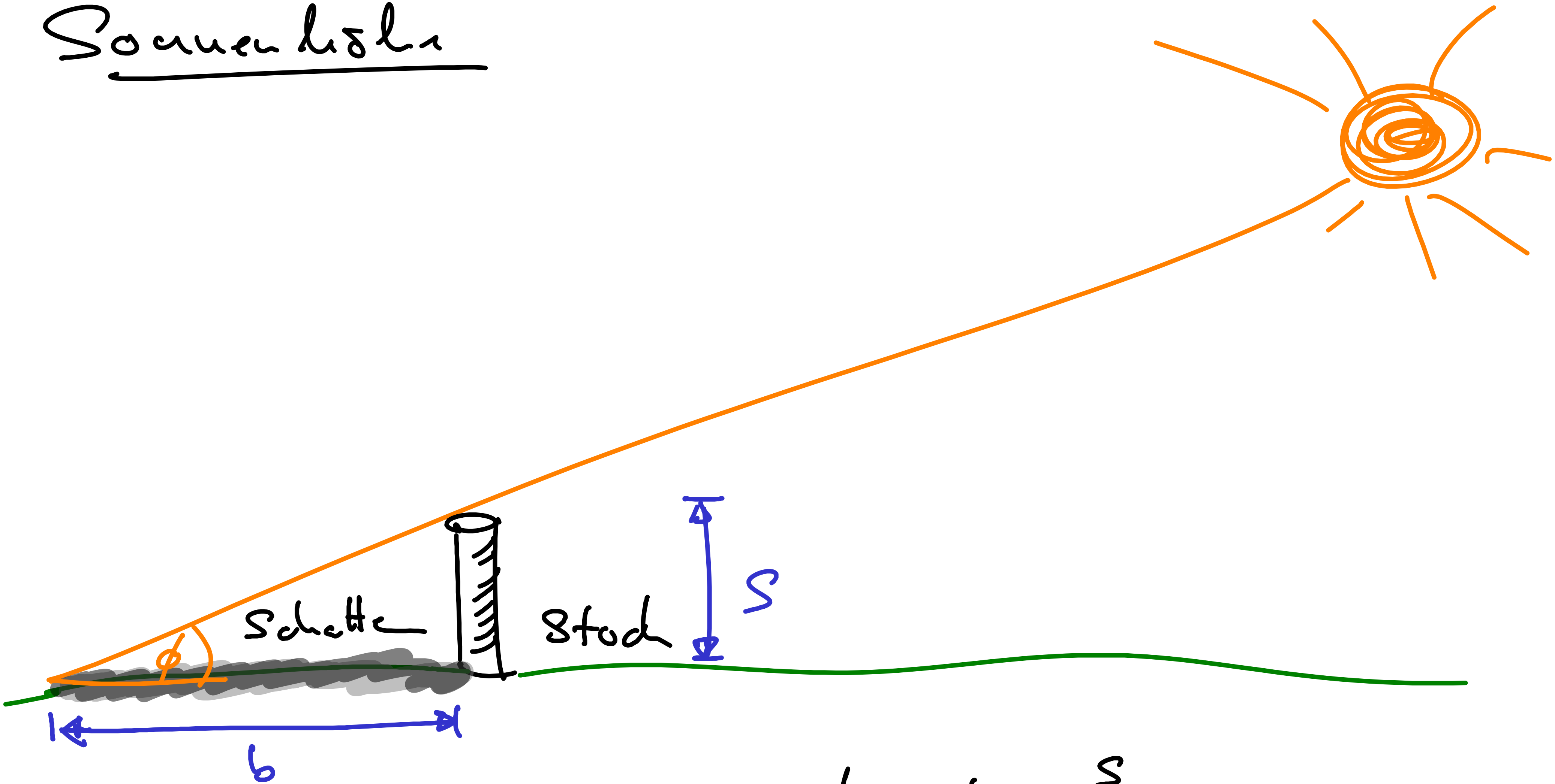
Umkehrfkt. von sin, cos, tan, ...

Bsp.: sin x



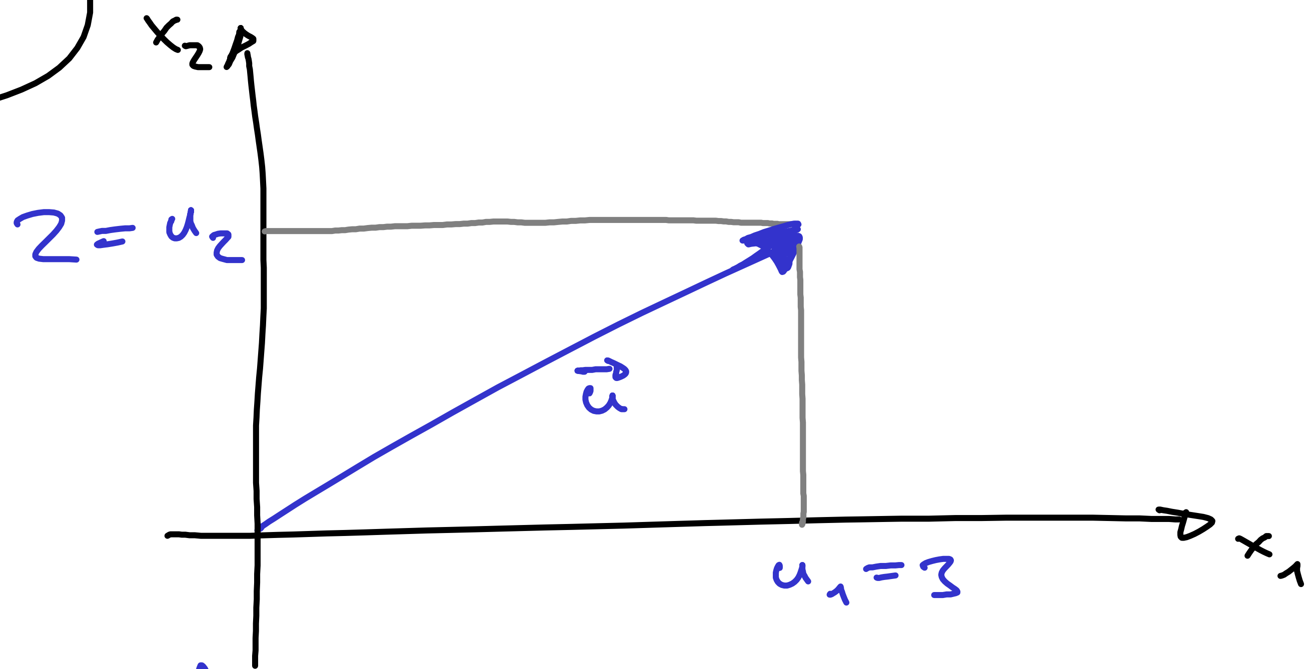
Sonnenlicht



$$\tan \phi = \frac{s}{b}$$

$$\Rightarrow \arctan \frac{s}{b} = \phi$$

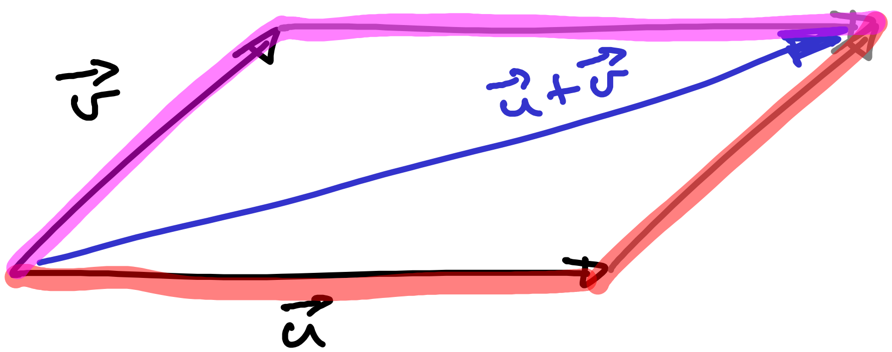
Vektoren



$$\vec{u} = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \end{pmatrix} \in \mathbb{R}^2$$

$$|\vec{u}| = \sqrt{3^2 + 2^2} = \sqrt{13}$$ (Länge, Betrag, Pythagoras)

Vektoraddition



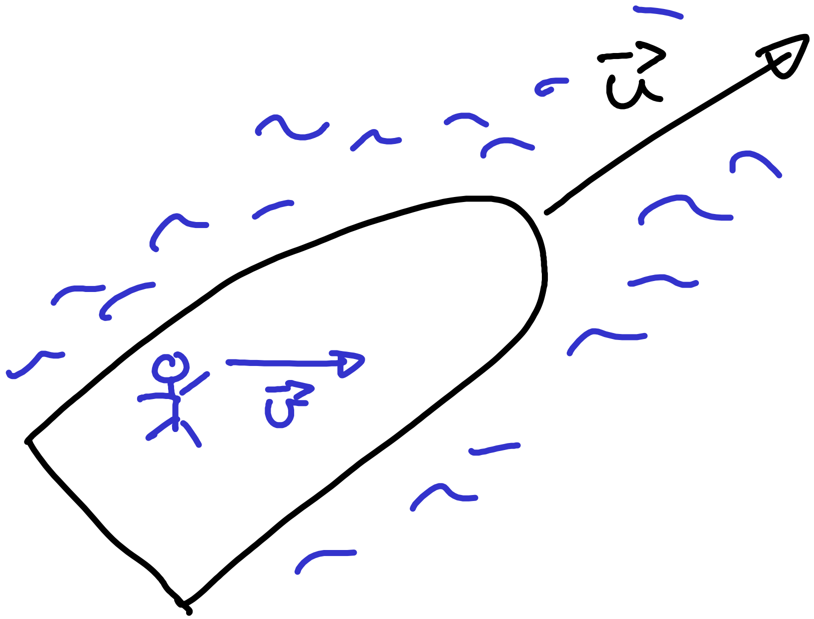
$$\vec{u}+\vec{v} = \vec{v}+\vec{u}$$

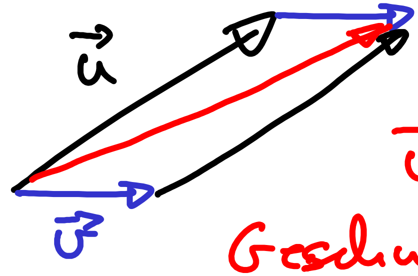
z.B.

$$\vec{u} = \begin{pmatrix}4\\0\end{pmatrix}, \quad \vec{v} = \begin{pmatrix}2\\3\end{pmatrix}, \quad \vec{u}+\vec{v} = \begin{pmatrix}6\\3\end{pmatrix}$$

$\vec{u}$: Geschw. Schiff (bzgl. Wasser)

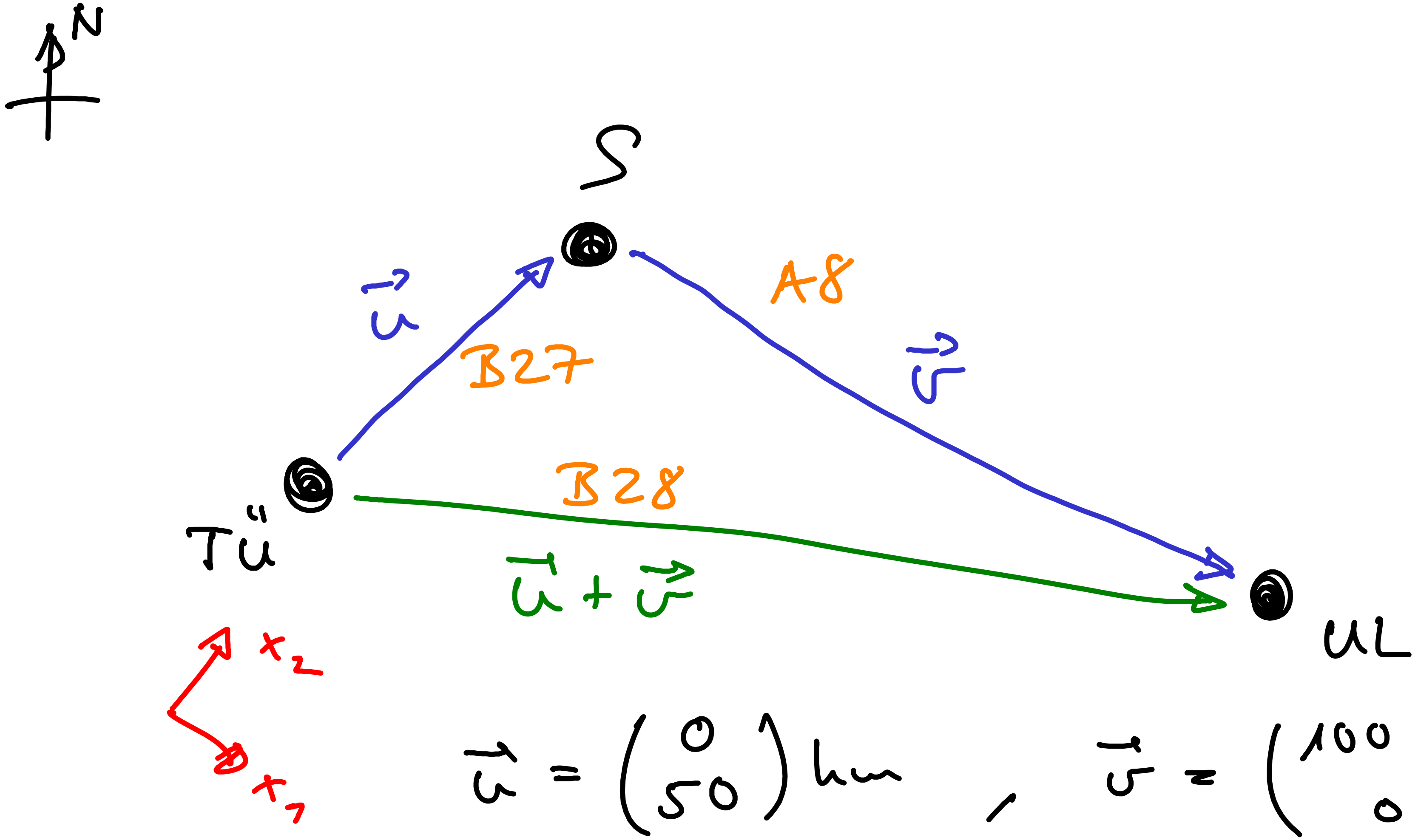
$\vec{v}$: Geschw. Person (bzgl. Schiff)





$\vec{u} + \vec{v}$

Geschw. Person bzgl. Wasser

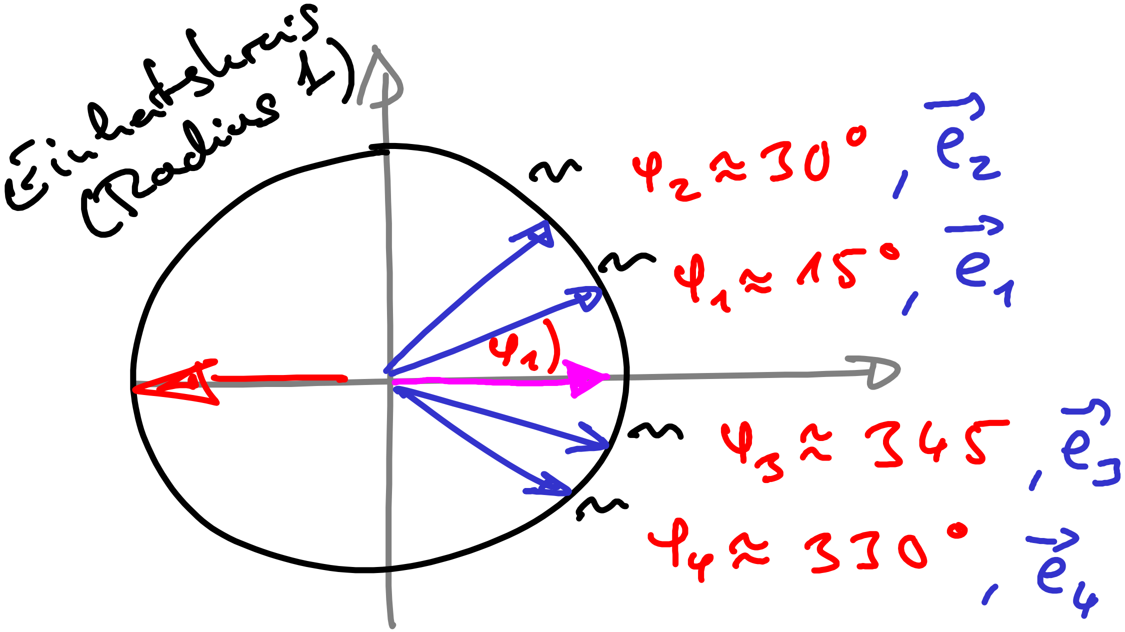


$$\vec{u} = \begin{pmatrix} 0 \\ 50 \end{pmatrix} \text{km}, \quad \vec{v} = \begin{pmatrix} 100 \text{ km} \\ 0 \end{pmatrix}$$

$$\vec{u} + \vec{v} = \begin{pmatrix} 100 \\ 50 \end{pmatrix} \text{km}$$

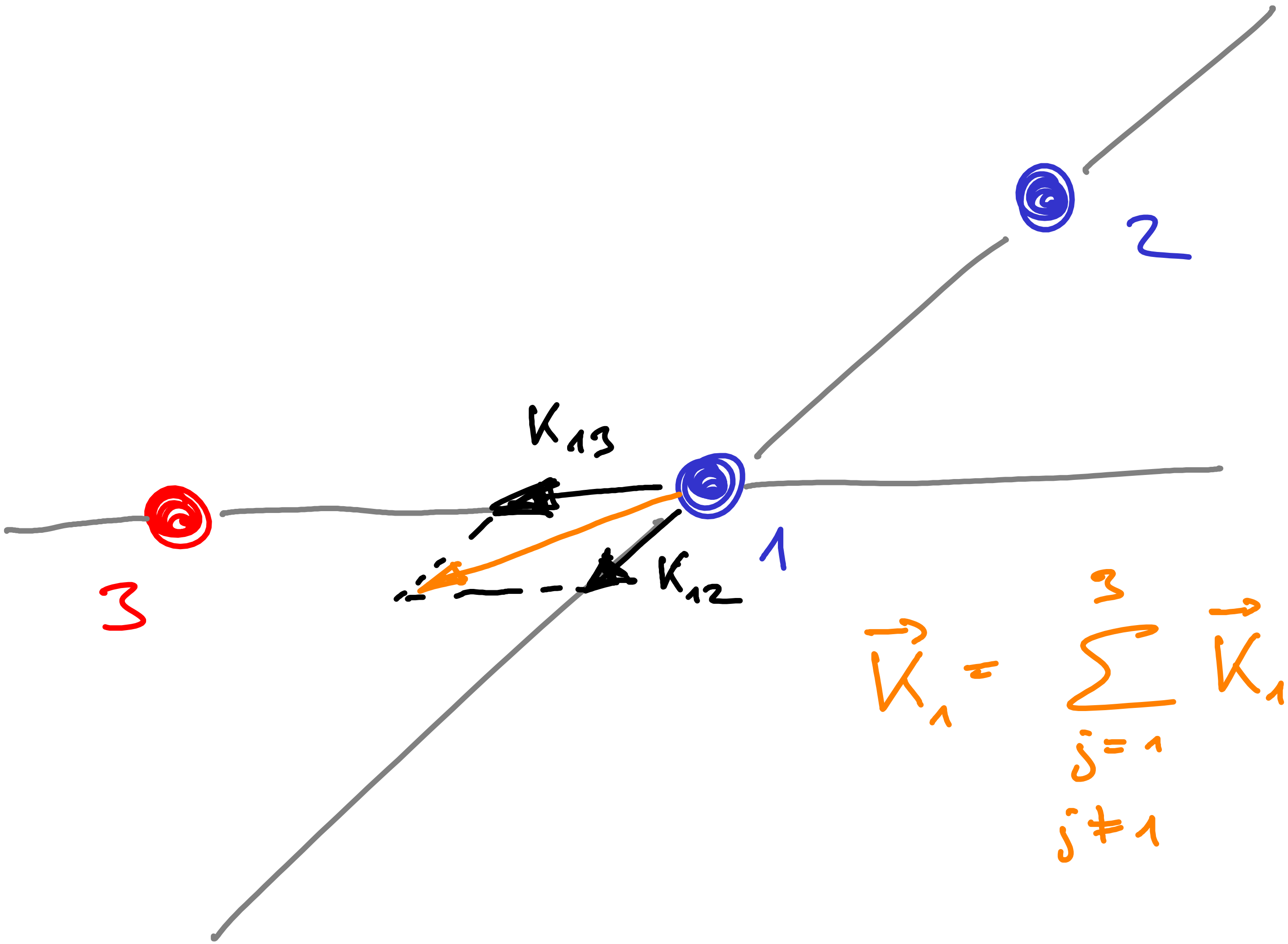
$$|\vec{u} + \vec{v}| = \sqrt{10\,000 \text{ km}^2 + 2500 \text{ km}^2} \approx 110 \text{ km}$$

Richtung: Betrag des Vektors interessiert (momentan) nicht ⤳ zeichne Vektoren gleicher Länge, z.B. 1

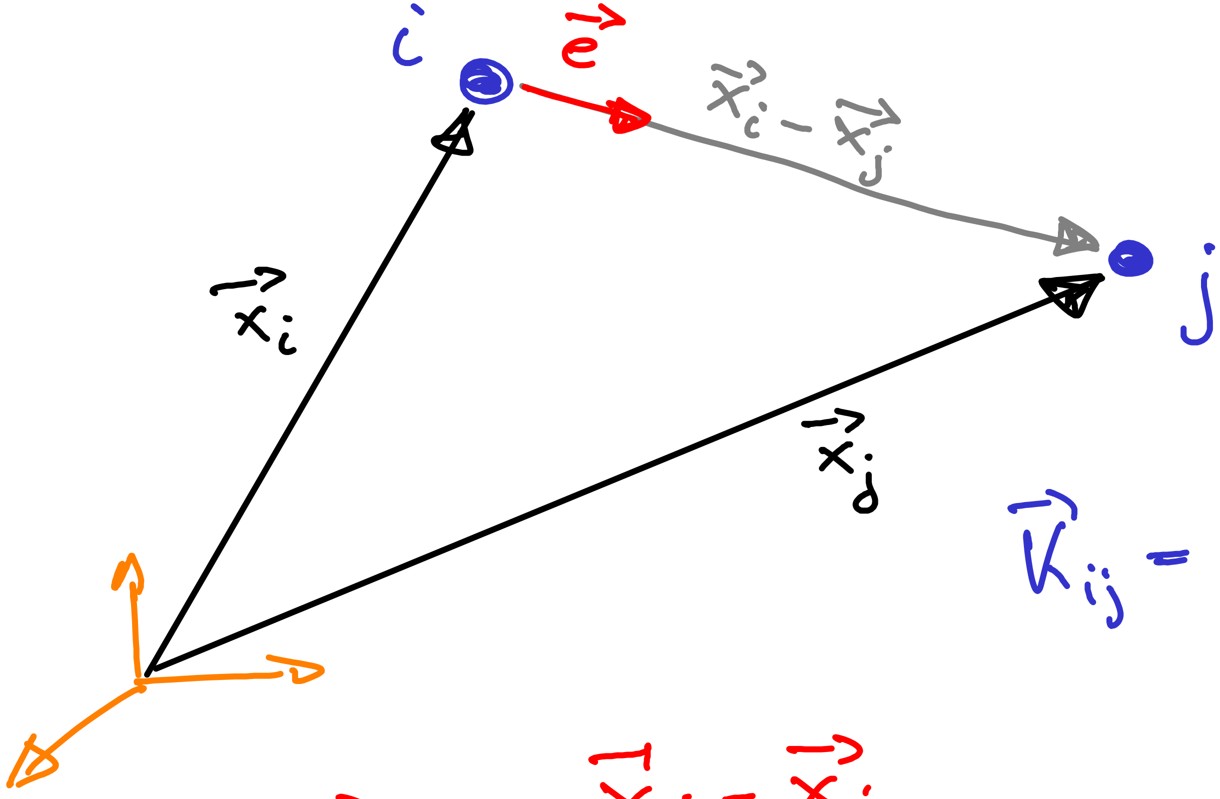


$$\overline{\varphi} = \frac{\varphi_1 + \varphi_2 + \varphi_3 + \varphi_4}{4} = 180° \;???$$

$$\overline{\vec{e}} = \frac{\vec{e}_1 + \vec{e}_2 + \vec{e}_3 + \vec{e}_4}{4}$$ besser!



$$\vec{K}_1 = \sum_{\substack{j=1 \\ j\neq 1}}^{3} \vec{K}_{1j} = \vec{K}_{12} + \vec{K}_{13}$$



$$|\vec{e}| = 1$$

$$\vec{K}_{ij} = \frac{q_i q_j}{|\vec{x}_i - \vec{x}_j|^2} \vec{e}$$

$$\vec{e} = \frac{\vec{x}_i - \vec{x}_j}{|\vec{x}_i - \vec{x}_j|}$$

donc

$$\vec{K}_{ij} = q_i q_j \frac{\vec{x}_i - \vec{x}_j}{|\vec{x}_i - \vec{x}_j|^3}$$

$\vec{u}$

$\alpha \vec{u}$, $\vec{u} \parallel (\alpha \vec{u})$

parallel

umgekehrt

$\vec{u}$, $\vec{v}$, $\vec{u} \parallel \vec{v}$

gesucht: α, so dass $\vec{v} = \alpha \cdot \vec{u}$

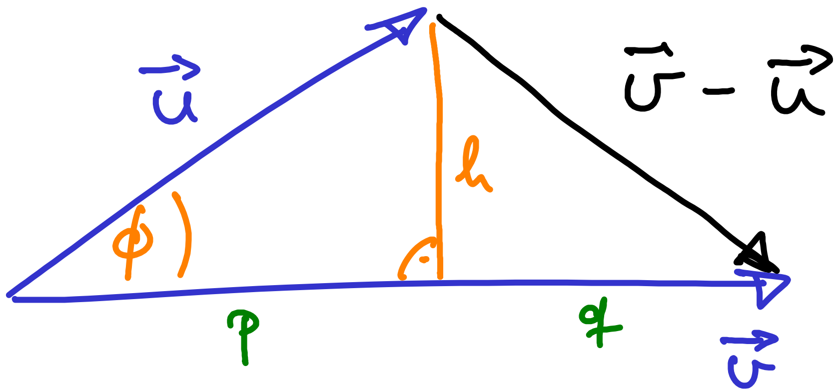
$$\vec{e} = \frac{\vec{u}}{|\vec{u}|} = \frac{\vec{v}}{|\vec{v}|} \Rightarrow \vec{v} = \frac{|\vec{v}|}{|\vec{u}|}\vec{u}$$

Skalarprodukt

z.B. $\vec{u} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$, $\vec{v} = \begin{pmatrix} 0 \\ -1 \\ 8 \end{pmatrix}$

$$\vec{u} \cdot \vec{v} = 1 \cdot 0 + 2 \cdot (-1) + 3 \cdot 8 = 22$$

Skalarprodukt anschaulich:



$$\cos\phi = \frac{p}{|\vec{u}|} \quad , \quad p^2 = |\vec{u}|^2 \cos^2\phi$$

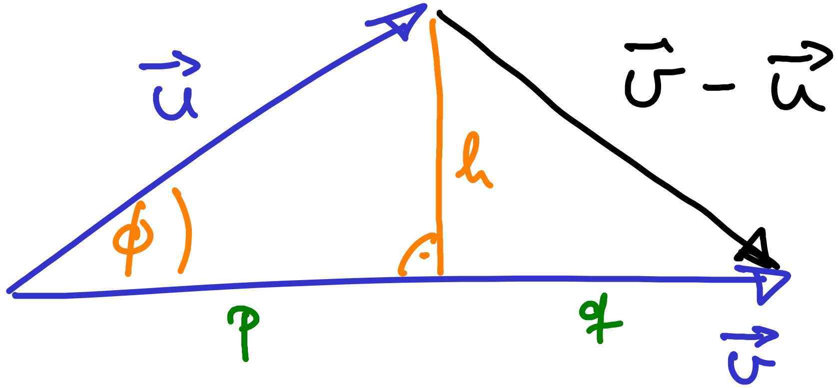
$$(\vec{v}-\vec{u})^2 = (\vec{u} - \vec{v})^2 = (\vec{u} - \vec{v})\cdot(\vec{u} - \vec{v}) = \vec{u}^2 - 2\vec{u}\cdot\vec{v} + \vec{v}^2$$

$$p^2 + h^2 = \vec{u}^2$$

$$q^2 + h^2 = |\vec{v} - \vec{u}|^2 = (\vec{v} - \vec{u})^2 = |\vec{u}|^2 + |\vec{v}|^2 + 2\vec{u}\cdot\vec{v}$$

$$|\vec{v}| = p + q \quad , \quad |\vec{v}|^2 = p^2 + q^2 + 2pq$$

Jetzt steht schon alles da, was man braucht, aber wenn wir nun den Wald vor lauter Bäumen nicht mehr sehen, probieren wir's auf der nächsten Seite nochmal übersichtlicher...



$$h^2 + q^2 = |\vec{v} - \vec{u}|^2$$ (Pythagoras rechts)

$$\iff h^2 + (|\vec{v}| - p)^2 = (\vec{v} - \vec{u}) \cdot (\vec{v} - \vec{u})$$

$$\iff \cancel{h^2} + \cancel{|\vec{v}|^2} + \cancel{p^2} - 2|\vec{v}|\,p = \cancel{|\vec{v}|^2} + \cancel{|\vec{u}|^2} - 2\,\vec{u} \cdot \vec{v} \qquad \Big| \cdot \left(-\tfrac{1}{2}\right)$$

(Pythagoras links)

$$\iff |\vec{v}|\,|\vec{u}| \cos\phi = \vec{u} \cdot \vec{v}$$

$$\cos\phi = \frac{p}{|\vec{u}|}$$