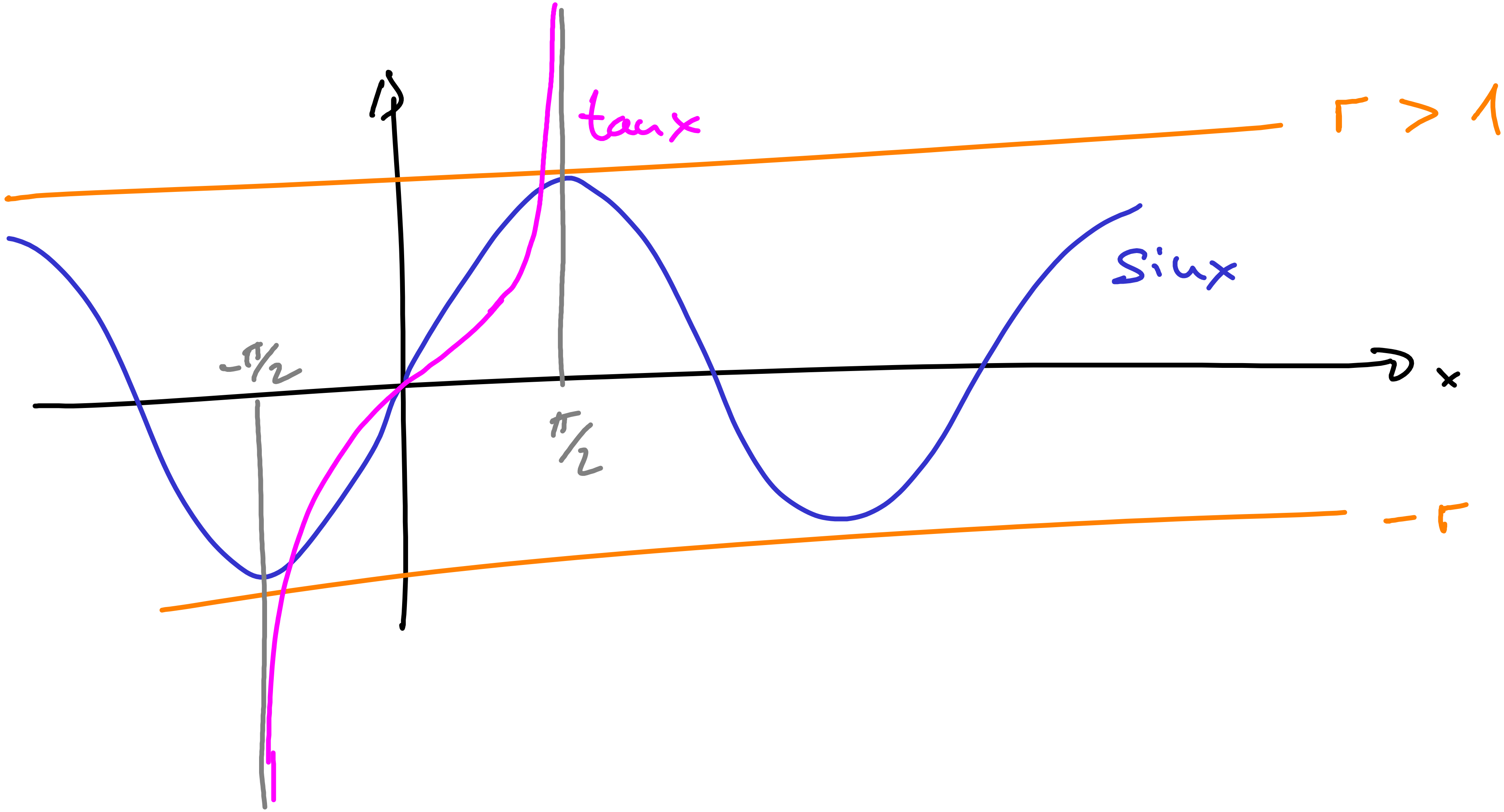
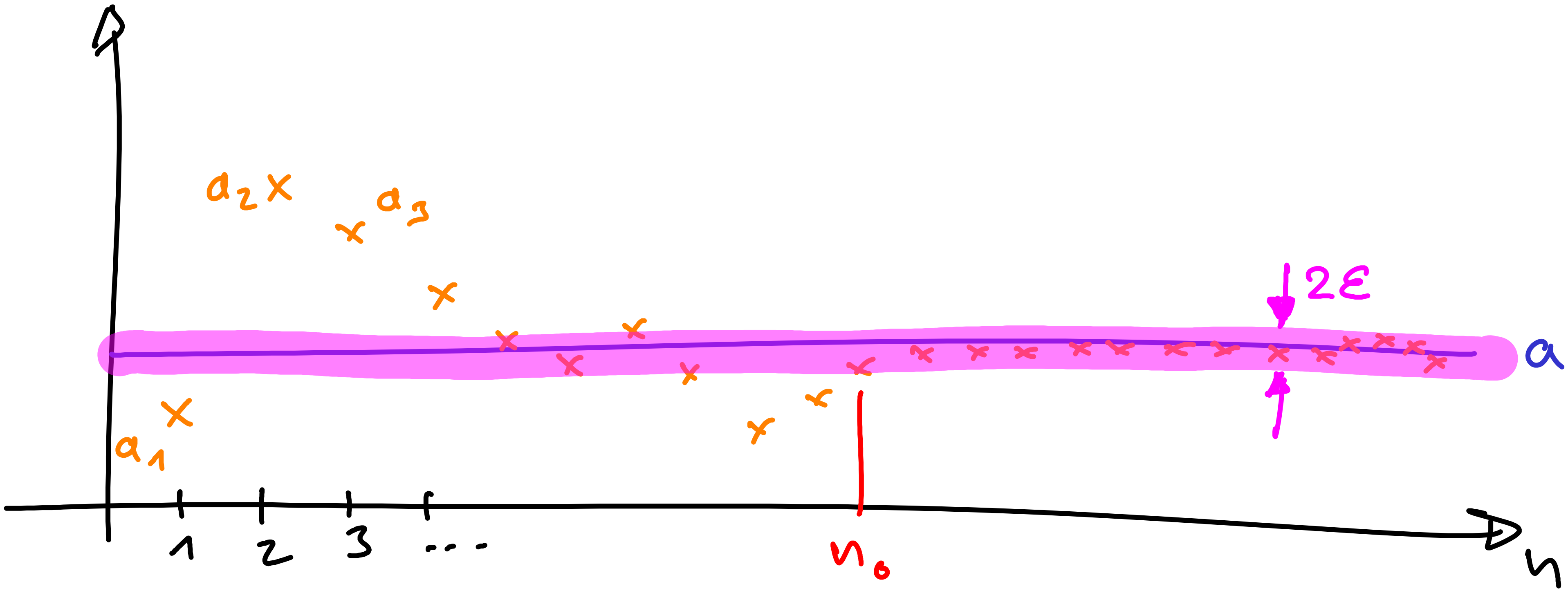


tan x
r > 1
sin x
$-\pi/2$
$\pi/2$
x
$-r$



a_2
a_3
a_1
2ε
a
1
2
3
...
n_0
n

$a_n = \frac{1}{n}$, $n \in \mathbb{N}$

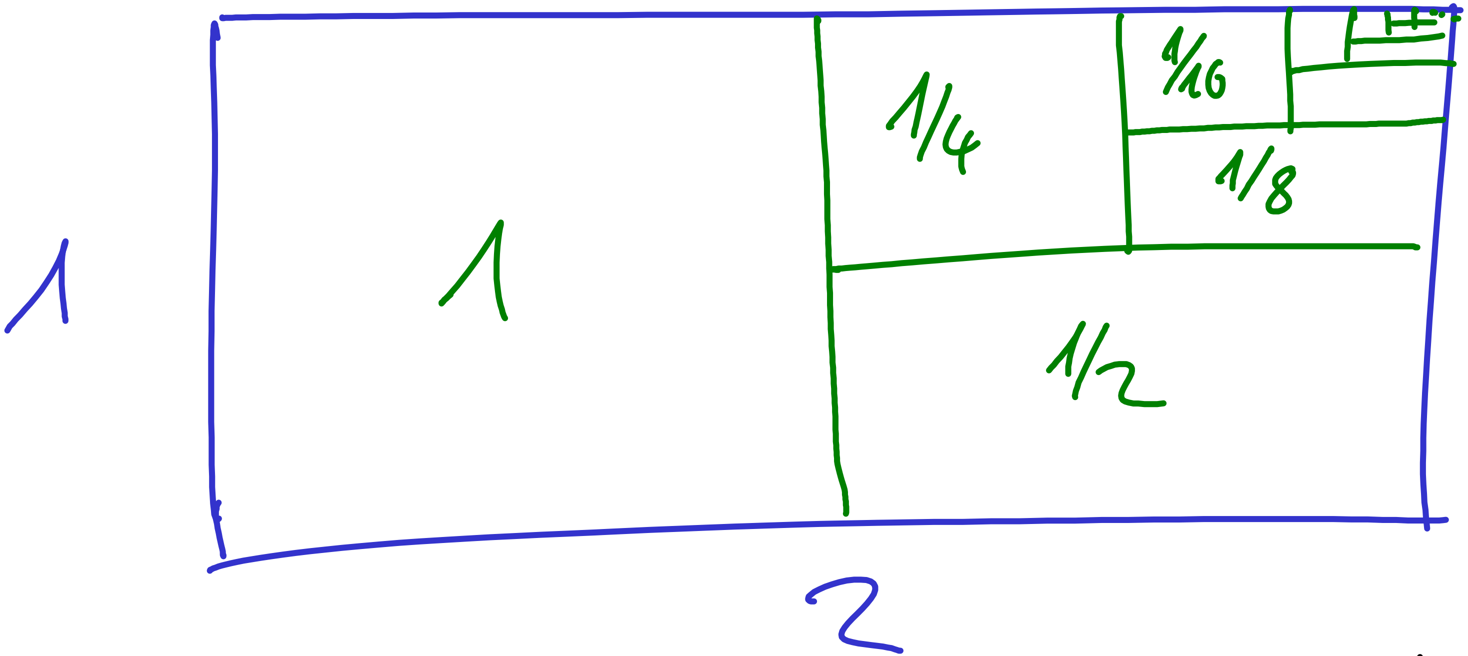
Behauptung: $\lim\limits_{n \to \infty} a_n = 0$

$$|a_n - 0| = \left|\frac{1}{n} - 0\right| = \left|\frac{1}{n}\right| = \frac{1}{n} \overbrace{< \varepsilon}^{\text{das hatten wir gerne}}$$

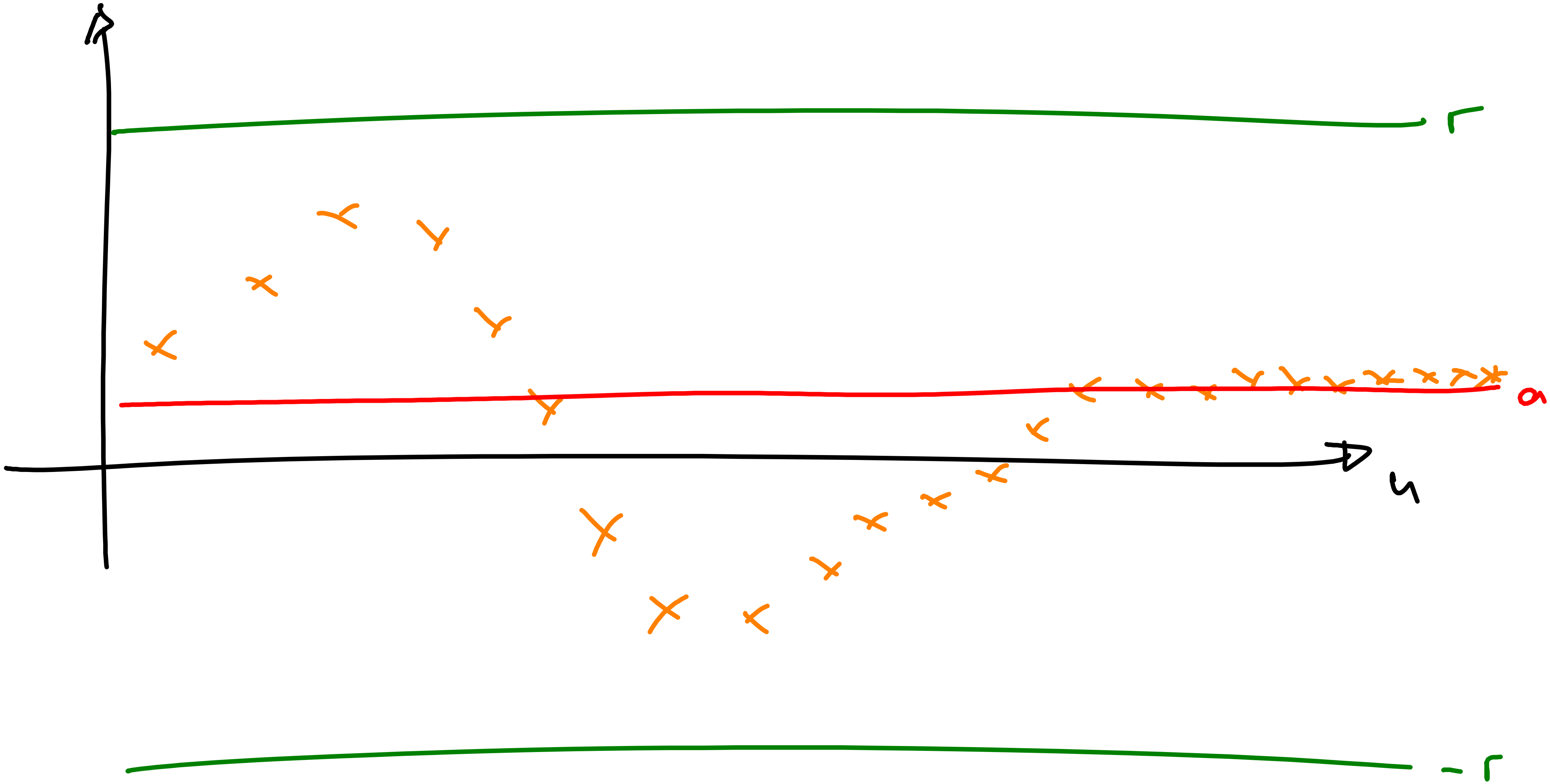
$$\Leftrightarrow \quad n > \frac{1}{\varepsilon}$$

Wähle $n_0 > \frac{1}{\varepsilon}$ $\square$

Rechteck



Fläche $2 = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \dots$

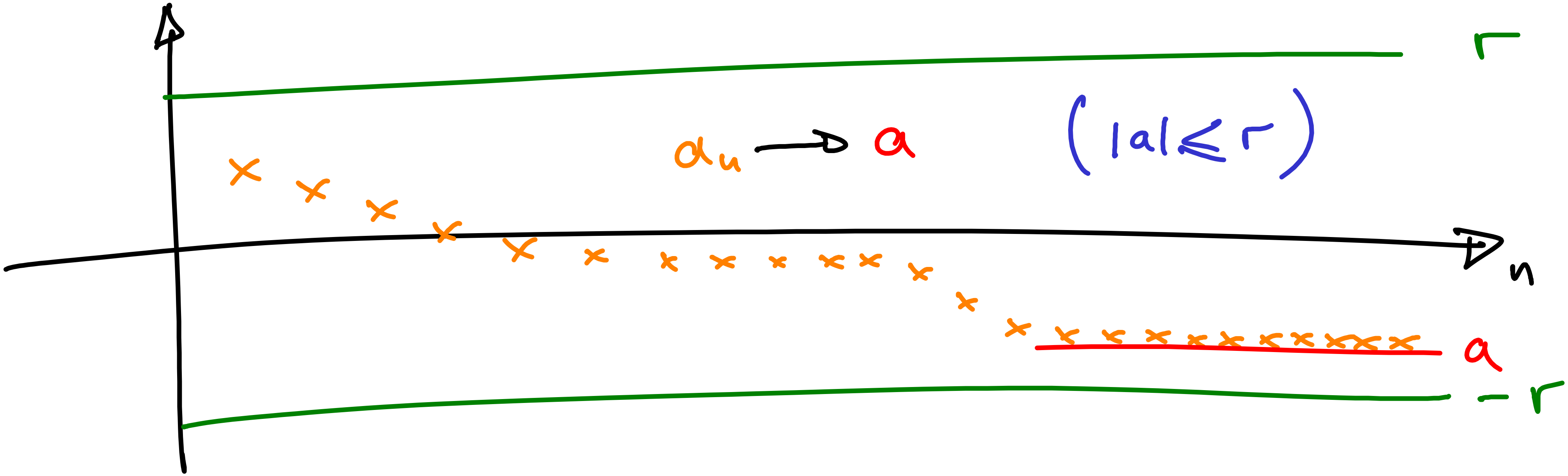


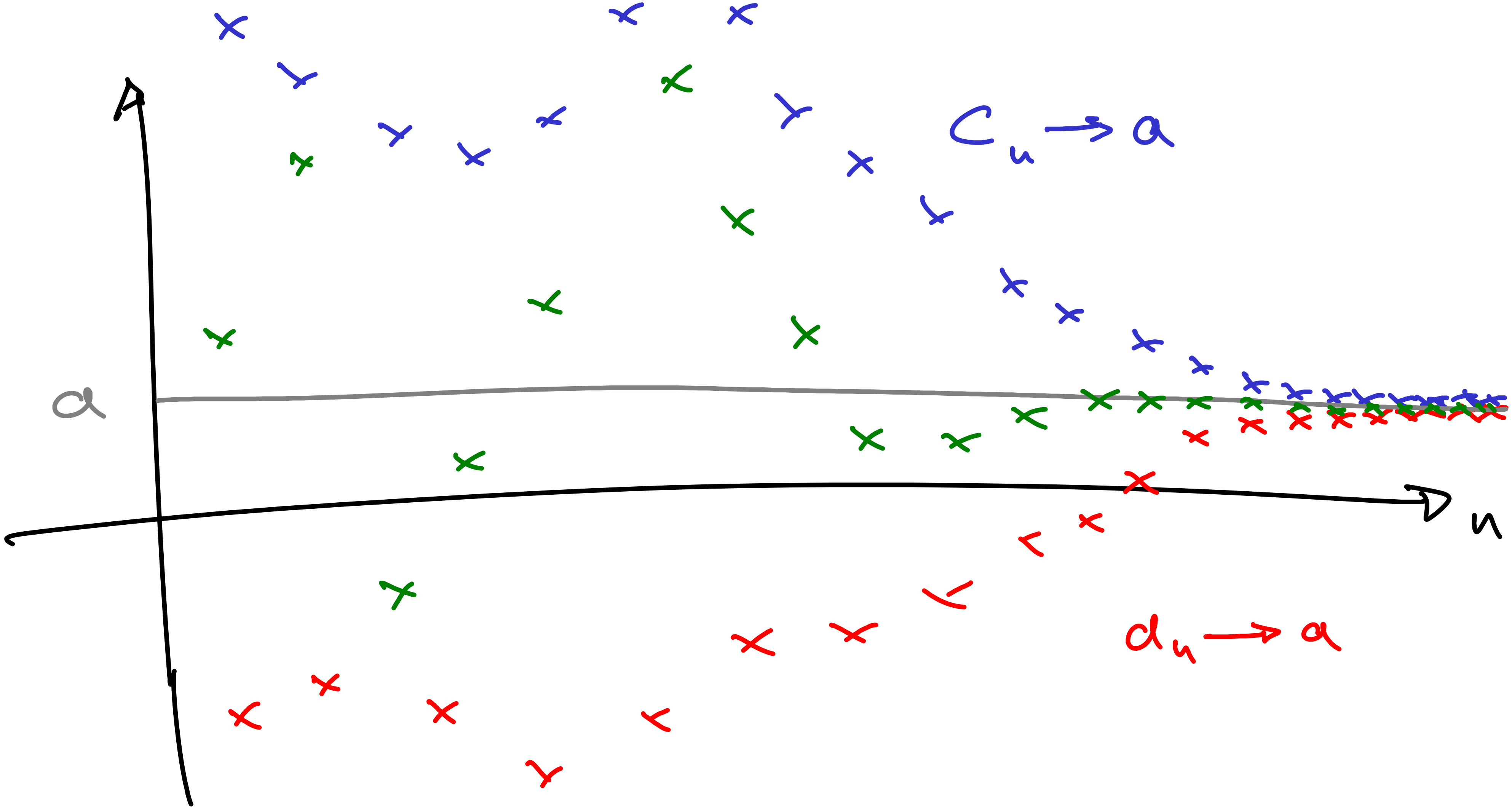
$$\lim_{n \to 0} a_n = a$$

Folge a_n sei monoton fallend, d.h.

$$a_{n+1} \leq a_n \quad \forall\, n \in \mathbb{N}$$

und beschränkt, d.h. $|a_n| \leq r \quad \forall\, n \in \mathbb{N} \ (r > 0)$





$$a_n \leq b_n \leq c_n \implies b_n \to a$$

konvergiert n^2 ?

$$n^2 = \frac{1}{n} \cdot n^3$$

$\frac{1}{n} \to 0$, $n^3 \to \infty$

---

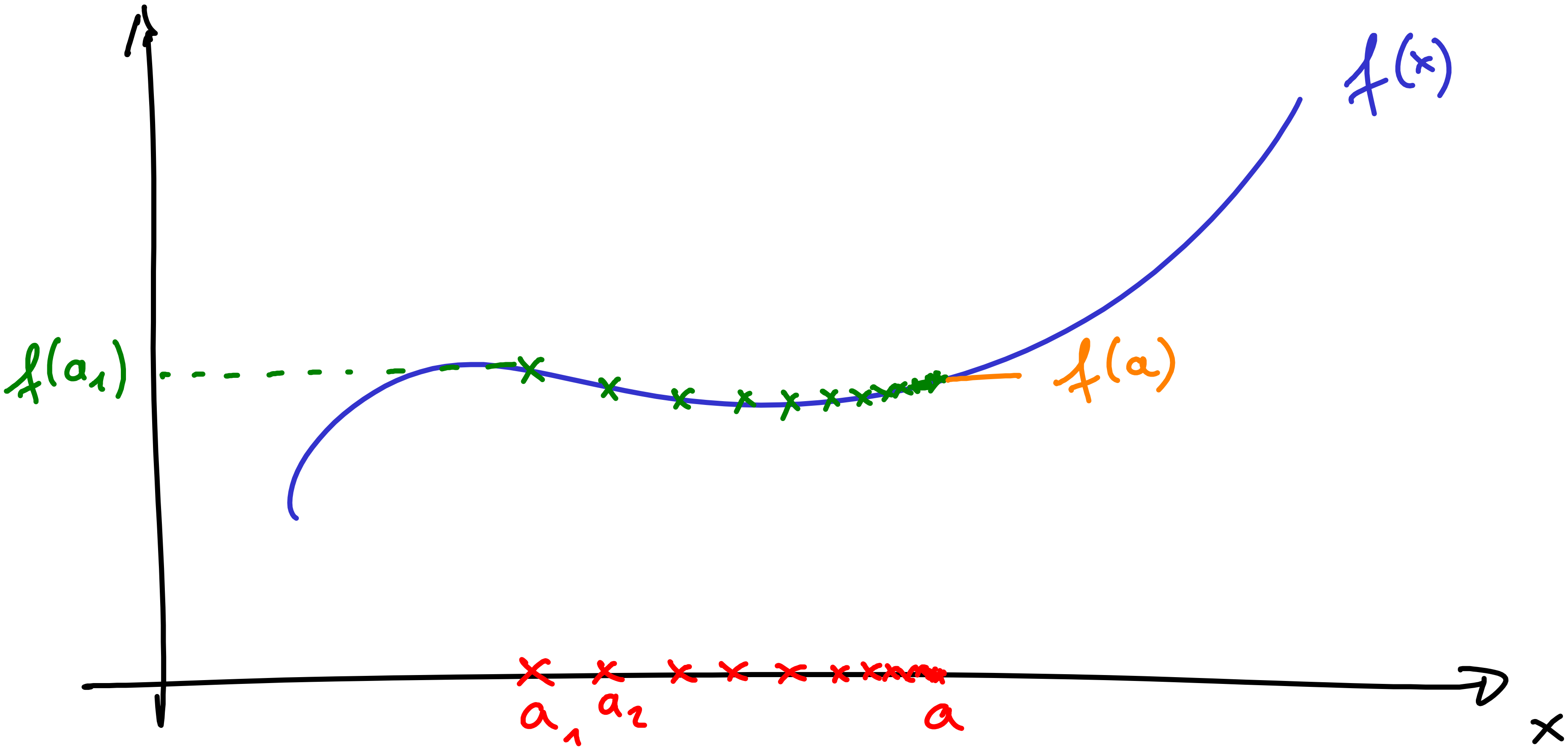
konvergiert $a_n = 5$ $(\forall n \in \mathbb{N})$?

$$5 = \lim_{n\to\infty} 5 \cancel{=} \underbrace{\left(\lim_{n\to\infty} \frac{5}{n}\right)}_{=0} \cdot \underbrace{\left(\lim_{n\to\infty} n\right)}_{=\infty} = 0 \cdot \infty = ???$$

$$\lim_{n\to\infty} \frac{n^{28} - n^{2}}{3n + 4n^{28}} = \lim_{n\to\infty} \frac{1 - \underbrace{\frac{1}{n^{26}}}_{\to 0}}{\underbrace{\frac{3}{n^{27}}}_{\to 0} + 4}$$

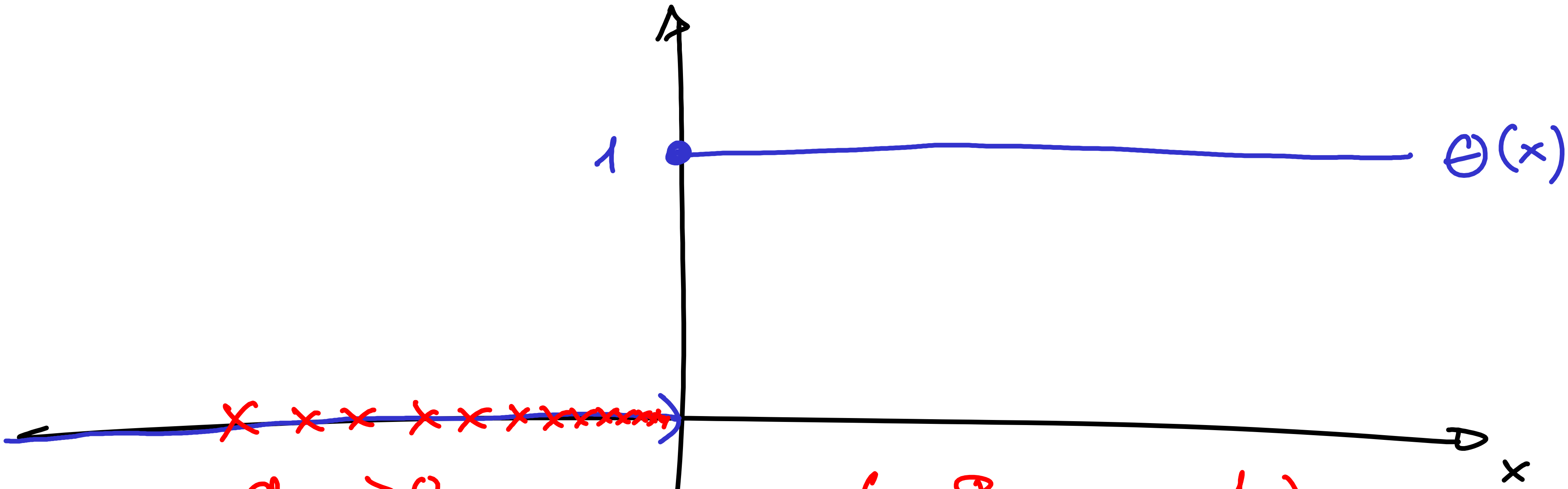
$$= \frac{\lim_{n\to\infty}\left(1 - \frac{1}{n^{26}}\right)}{\lim_{n\to\infty}\left(\frac{3}{n^{27}} + 4\right)} = \frac{\lim_{n\to\infty} 1 - \lim_{n\to\infty} \frac{1}{n^{26}}}{\lim_{n\to\infty} \frac{3}{n^{27}} + \lim_{n\to\infty} 4}$$

$$= \frac{1 - 0}{3 \cdot 0 + 4} = \frac{1}{4}$$



$$\lim_{n\to\infty} f(a_n) = f\left(\lim_{n\to\infty} a_n\right) = f(a)$$

Stetigkeit

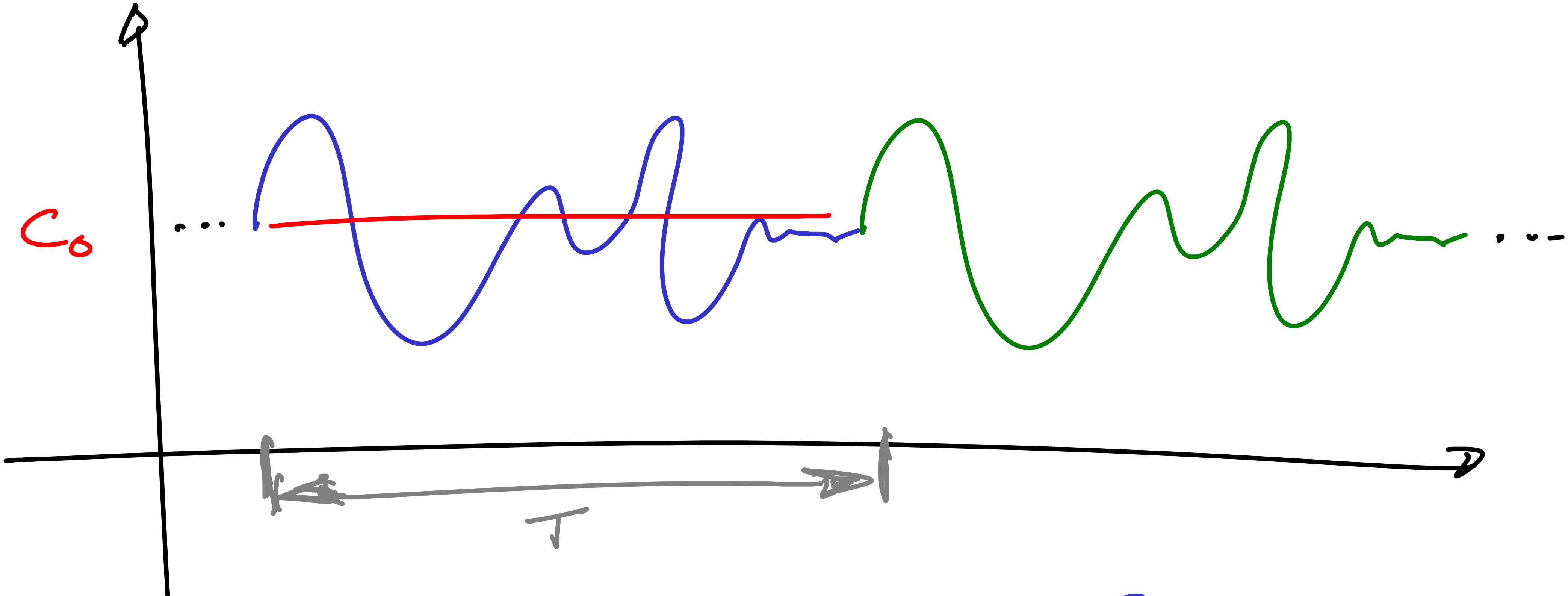


$a_n \to 0$ $\quad$ (z.B. $a_n = -\frac{1}{n}$)

$a_n < 0 \quad \forall n$

$$\lim_{n\to\infty} \Theta(a_n) = 0 \neq \Theta(0) = 1$$

$$\neq \Theta\left(\lim_{n\to\infty} a_n\right) = 1$$



Frequenz : $\frac{2\pi}{T}$, $\frac{2\pi}{T} \cdot 2$, $\frac{2\pi}{T} \cdot 3$, ...

geometrische Folge

① $q > 1$ Widerspruchsbeweis

Annahme: q^n beschränkt

d.h. $\exists\, r > 0 : |q^n| < r \quad \forall n$

$$\overset{}{\underset{q>0}{\Longleftrightarrow}} \quad q^n < r \quad \forall n$$

$$\underset{\text{log monoton wachsend}}{\Longleftrightarrow} \quad n \log q < \log r \quad \forall n \quad \Big| \cdot \frac{1}{\log q} \leftarrow \text{pos.}$$

$$\Longleftrightarrow \quad n < \frac{\log r}{\log q} \quad \forall n$$

↯ Widerspruch für n groß genug

$\Longrightarrow$ q^n <u>nicht</u> beschränkt $\Longrightarrow$ q^n <u>nicht</u> konvergent

② $|q| < 1$: Bew. $q^n \to 0$

$$|q^n - 0| = |q^n| = |q|^n \underbrace{}_{\text{Ziel}} < \varepsilon$$

$$\overset{}{\underset{q \neq 0}{\Longleftrightarrow}} \quad n \log|q| < \log \varepsilon \quad \Big| \cdot \frac{1}{\log|q|} \leftarrow \text{neg}$$

$$\Longleftrightarrow \quad n > \frac{\log \varepsilon}{\log|q|}$$

Wähle $n_0 \in \mathbb{N}$ mit $n_0 > \frac{\log \varepsilon}{\log|q|}$:

$$|q^n - 0| < \varepsilon \quad \forall\, n > n_0 \qquad \square$$

geometrische Reihe

$$S_n = \sum_{k=0}^{n} q^k = 1 + q + q^2 + q^3 + \dots + q^n \quad \Big|\, \cdot q$$

$$q S_n = q + q^2 - - - - - + q^n + q^{n+1}$$

Differenz:

$$S_n - q S_n = 1 - q^{n+1} \quad \Big|\, \frac{1}{1-q} \quad (q \neq 1)$$

$$S_n = \frac{1-q^{n+1}}{1-q} \quad \forall\, q \neq 1$$

$$\sum_{k=0}^{\infty} q^k = \lim_{n\to\infty} \sum_{k=0}^{n} q^k = \lim_{n\to\infty} \frac{1-q^{n+1}}{1-q} \underset{|q|<1}{=} \frac{1}{1-q} \quad \square$$

$$\lim_{n\to\infty} q^{n+1} = 0$$