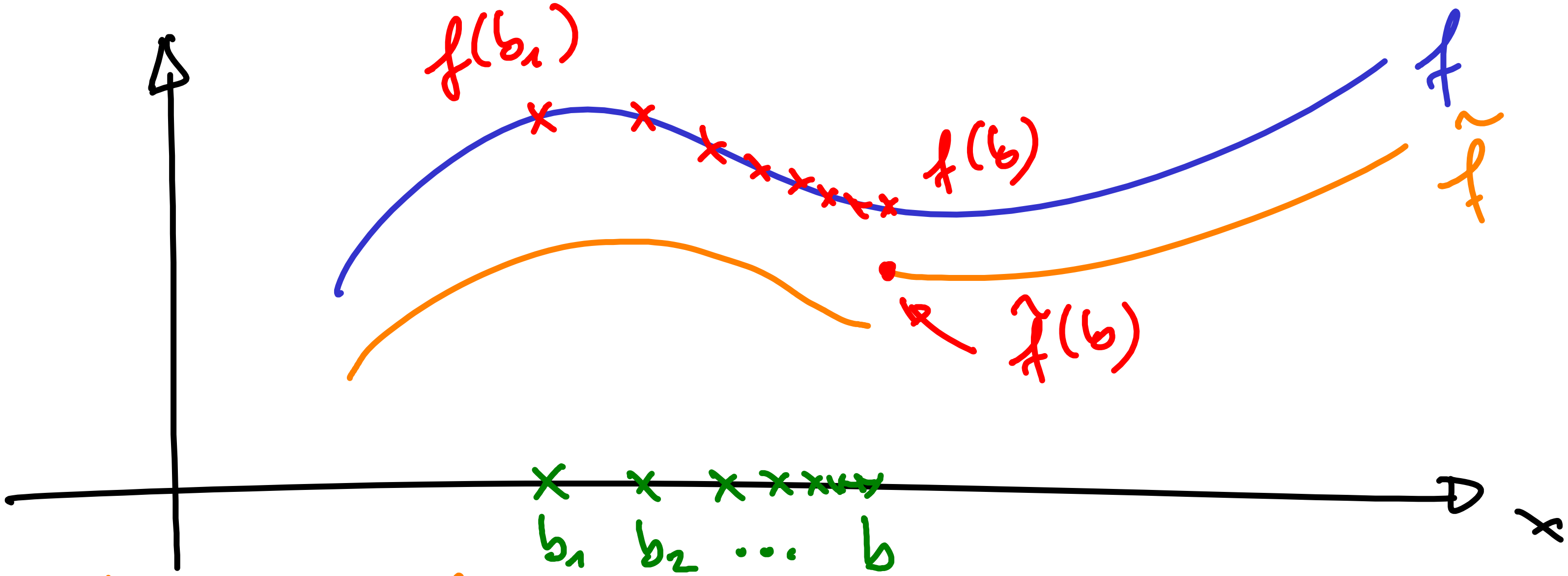
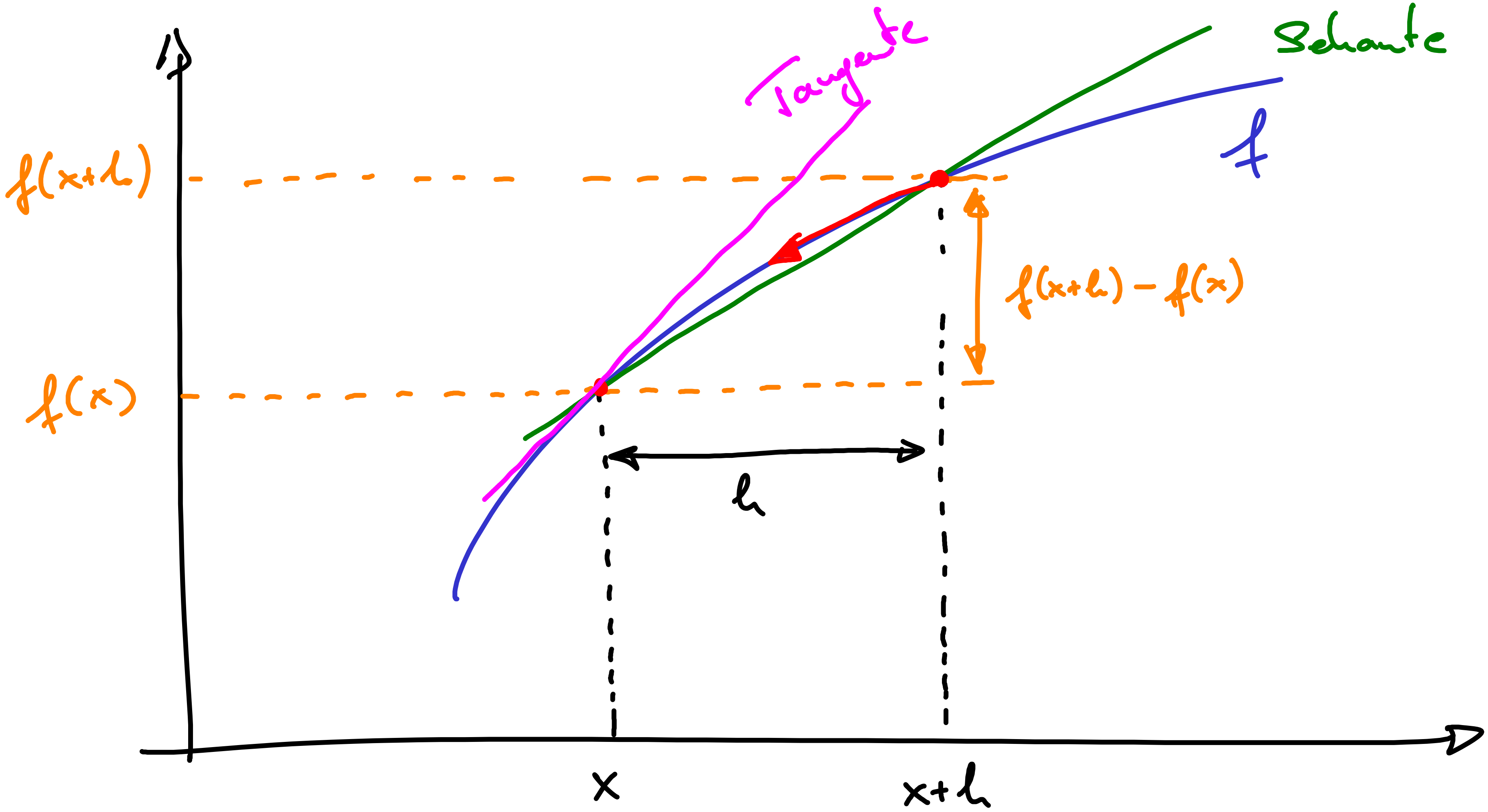
irgendeine Folge b_n mit $\lim_{n\to\infty} b_n = b$

$$\lim_{x\to b} f(x) := \lim_{n\to\infty} f(b_n) \underset{\text{stetig}}{=} f(b)$$

wenn $\lim_{n\to\infty} b_n = b$



nicht stetig z.B. $\tilde{f}$



Sekante $\xrightarrow[h \to 0]{}$ Tangente

Ableitung $f'(x) =$ Steigung der Tangente an der Stelle x

Notation

$$f' = \frac{df}{dx} = \left(\frac{d}{dx}\right) f$$

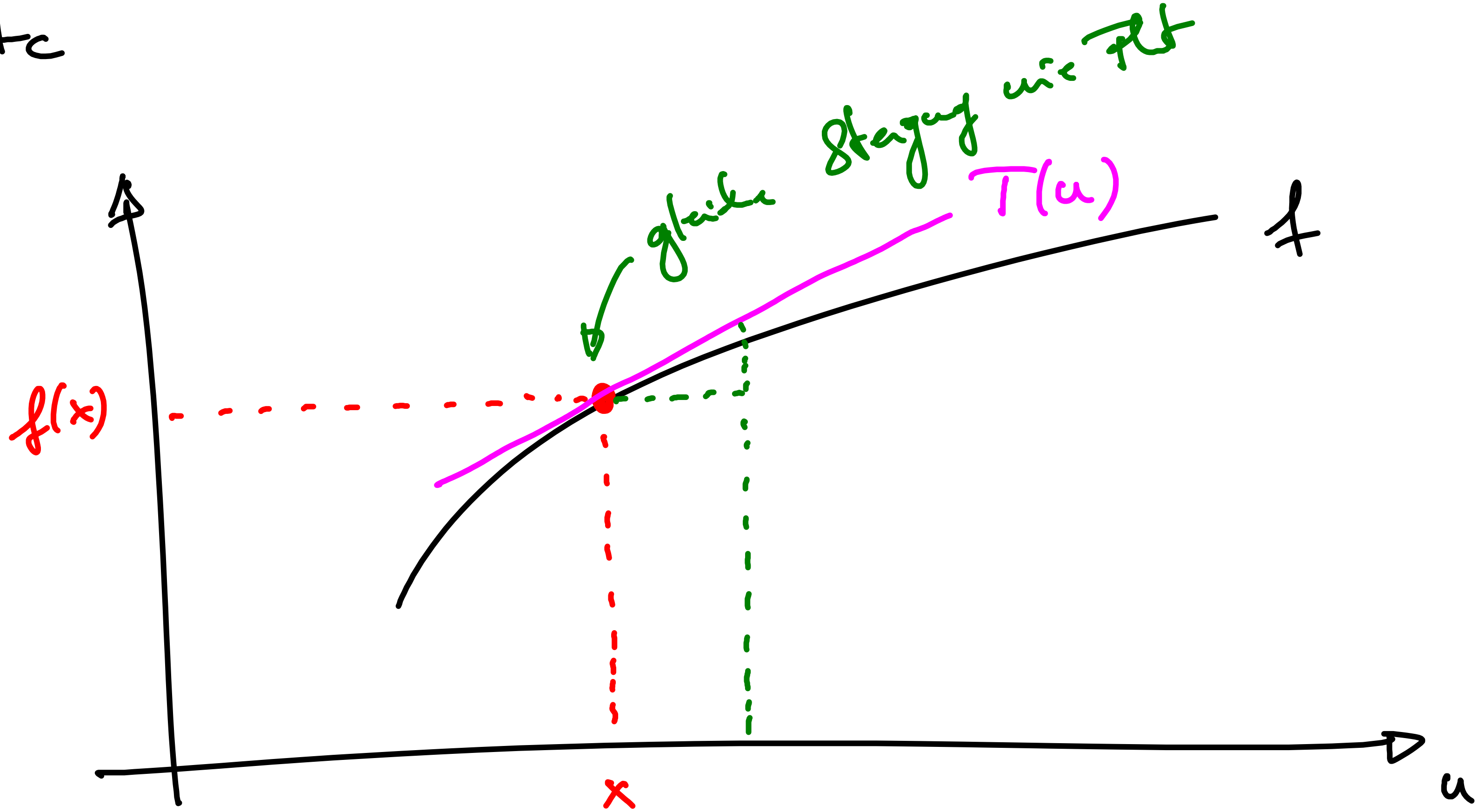
$$f'' = f^{(2)} = \frac{d^2 f}{dx^2} = \left(\frac{d}{dx}\right)^2 f$$

$$f''' = f^{(3)}$$

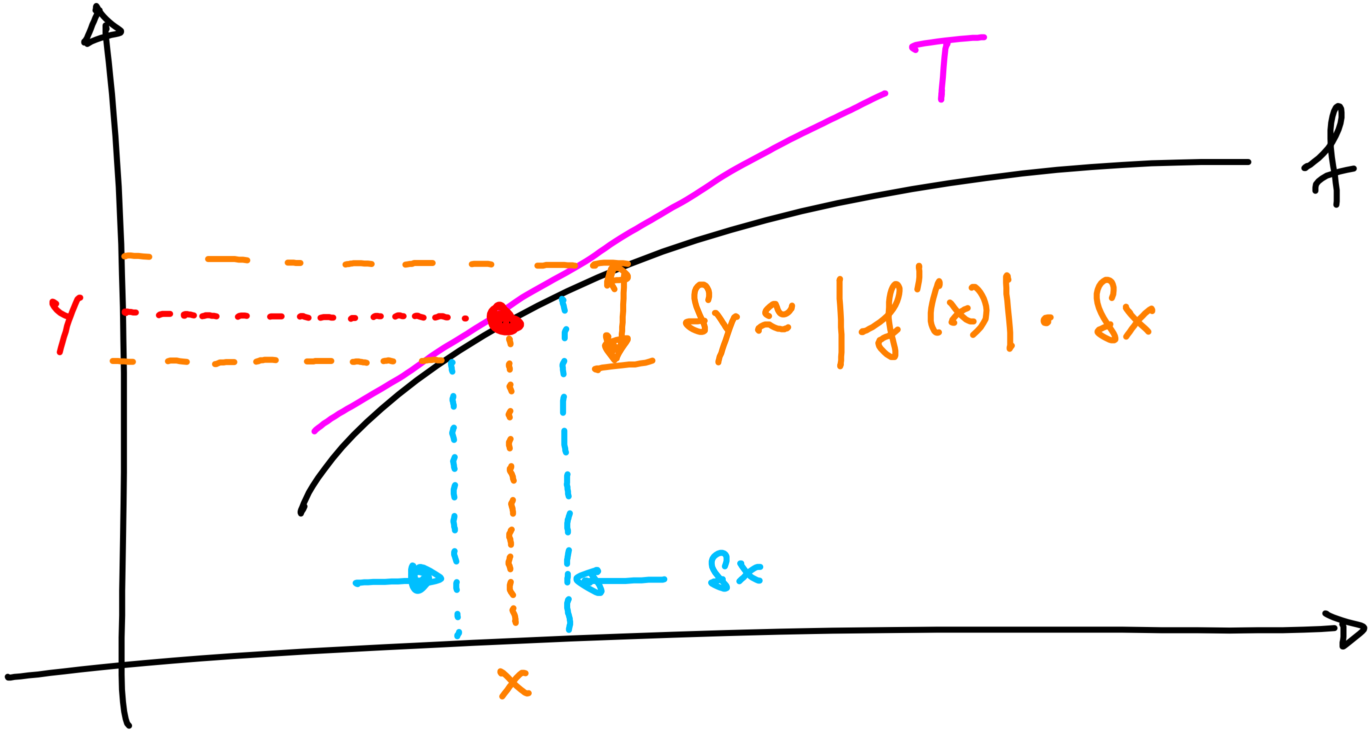
$$f''''''' = f^{(7)}$$

$$f'(x) = \frac{df}{dx}(x)$$

Tangente

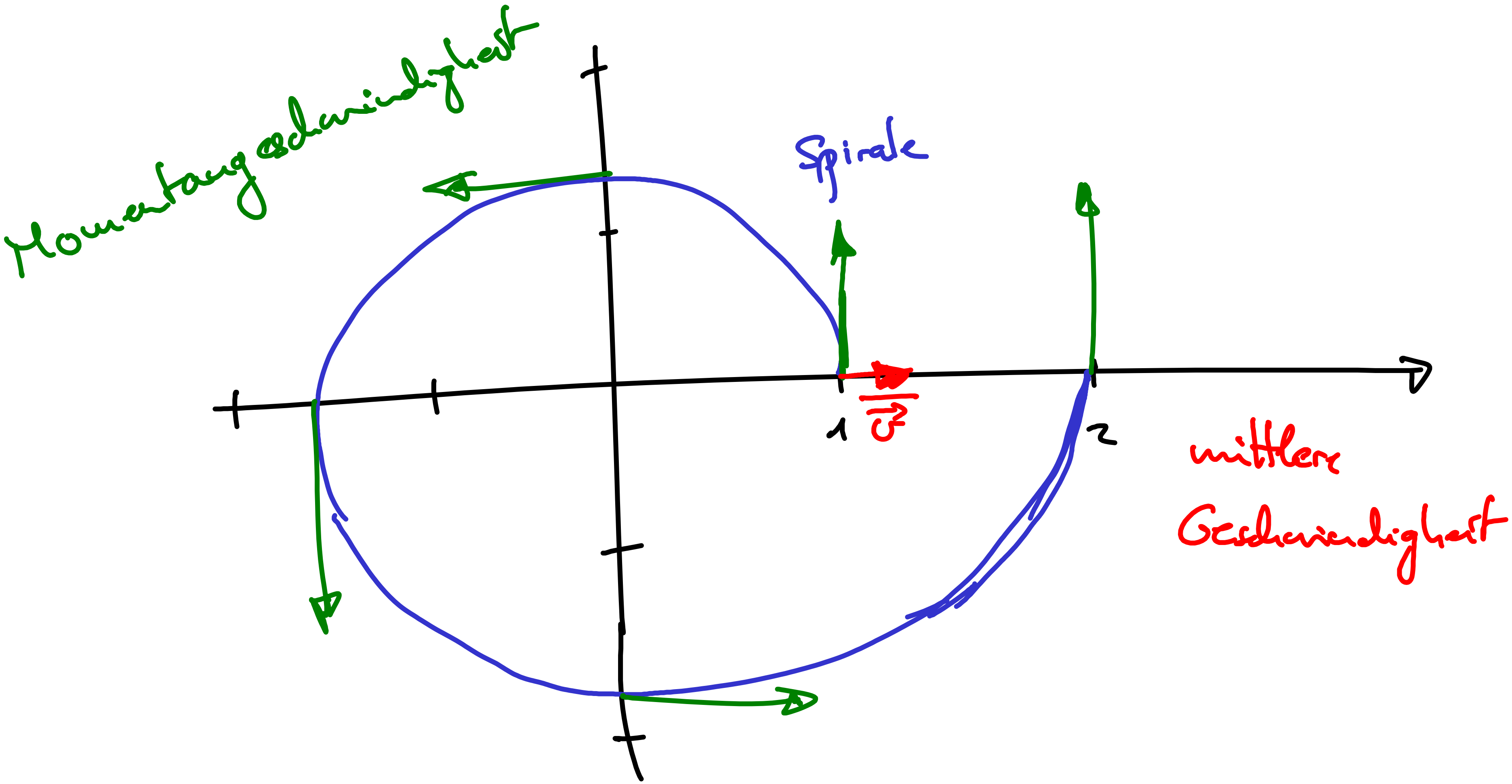


Größtfehlerabschätzung



suche eigentlich max / min von f auf Intervall $\left[x - \frac{\delta x}{2}, x + \frac{\delta x}{2}\right]$

Kurve: $\vec{x} : [0, 2\pi] \longrightarrow \mathbb{R}^2$

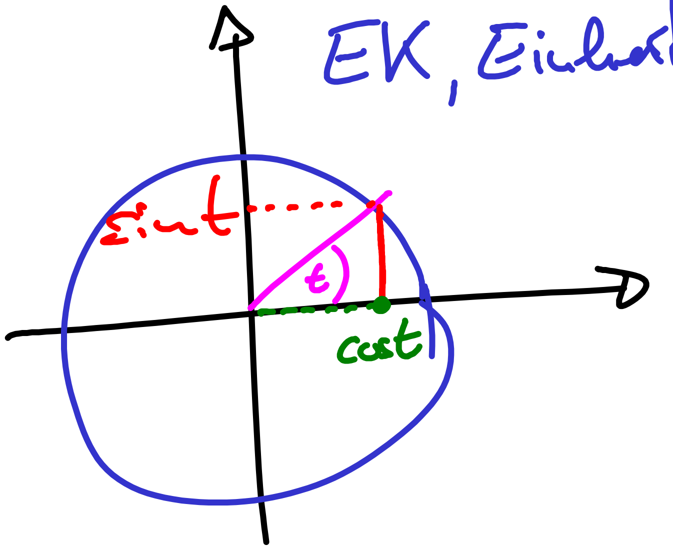


in Formeln?

zunächst Kreis $\vec{x}(t) = \begin{pmatrix} \cos t \\ \sin t \end{pmatrix} \cdot r$

Radius

EK, Einheitskreis ($r=1$)



lasse r wachse (z.B. linear): Spirale

$$\vec{x}(t) = \begin{pmatrix} \left(1+\frac{t}{2\pi}\right)\cdot \cos t \\ \left(1+\frac{t}{2\pi}\right)\cdot \sin t \end{pmatrix}$$

$$\overline{\vec{v}} = \frac{\vec{x}(2\pi) - \vec{x}(0)}{2\pi} = \frac{\begin{pmatrix} 2 \\ 0 \end{pmatrix} - \begin{pmatrix} 1 \\ 0 \end{pmatrix}}{2\pi} = \frac{1}{2\pi}\begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

Momentangeschw.

$$\frac{d\vec{x}}{dt} = \begin{pmatrix} \frac{1}{2\pi}\cos t + \left(1+\frac{t}{2\pi}\right)(-\sin t) \\ \frac{1}{2\pi}\sin t + \left(1+\frac{t}{2\pi}\right)\cos t \end{pmatrix}$$

Beispiel für Ableitung ① $f(x) = \frac{1}{x}$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h}$$

$$= \lim_{h\to 0} \frac{1}{h} \cdot \frac{x-(x+h)}{x(x+h)} = \lim_{h\to 0} \frac{-\cancel{h}}{\cancel{h}\cdot x\cdot(x+h)}$$

$$= -\frac{1}{x^2}$$

② $f(x) = e^x = \exp(x)$

$$f'(x) = \lim_{h \to 0} \frac{e^{x+h} - e^x}{h} = \lim_{h \to 0} \frac{e^x \cdot e^h - e^x}{h}$$

$$= e^x \underbrace{\lim_{h \to 0} \frac{e^h - 1}{h}}_{\text{hängt nicht von } x \text{ ab, Konstante}}$$

und $e = 2{,}718\ldots$ gilt $\lim_{h \to 0} \frac{e^h - 1}{h} = 1$

damit $(e^x)' = e^x$

Beweis Produktregel

$$(f\cdot g)'(x) = \lim_{h\to 0} \frac{f(x+h)\,g(x+h) - f(x)\,g(x)}{h}$$

$$= \lim_{h\to 0} \frac{[f(x+h) - f(x)]\,g(x+h) + f(x)\,g(x+h) - f(x)\,g(x)}{h}$$

$$= \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} \cdot g(x+h) + \lim_{h\to 0} f(x)\,\frac{g(x+h) - g(x)}{h}$$

$$= f'(x)\,g(x) + f(x)\,g'(x)$$

Beweisidee Kettenregel

nähern g durch Tangente

$$[f(g(x))]' = \lim_{h \to 0} \frac{f(g(x+h)) - f(g(x))}{h}$$

$$\approx \lim_{\substack{h \to 0 \\ hg'(x) \to 0}} \frac{f(g(x) + g'(x) \cdot h) - f(g(x))}{h\, g'(x)} \cdot g'(x)$$

$$= \lim_{\tilde{h} \to 0} \frac{f(g(x) + \tilde{h}) - f(g(x))}{\tilde{h}} \cdot g'(x)$$

$$= f'(g(x)) \cdot g'(x)$$

äußere / innere Ableitung

Umkehr-fkt. ableiten

$$f^{-1}(f(x)) = x \qquad \text{ableiten:}$$

$$f^{-1\prime}(f(x)) \cdot f'(x) = 1 \qquad \Big| \cdot \frac{1}{f'(x)}$$

$$f^{-1\prime}(\underbrace{f(x)}_{=:y}) = \frac{1}{f'(x)} \qquad x = f^{-1}(y)$$

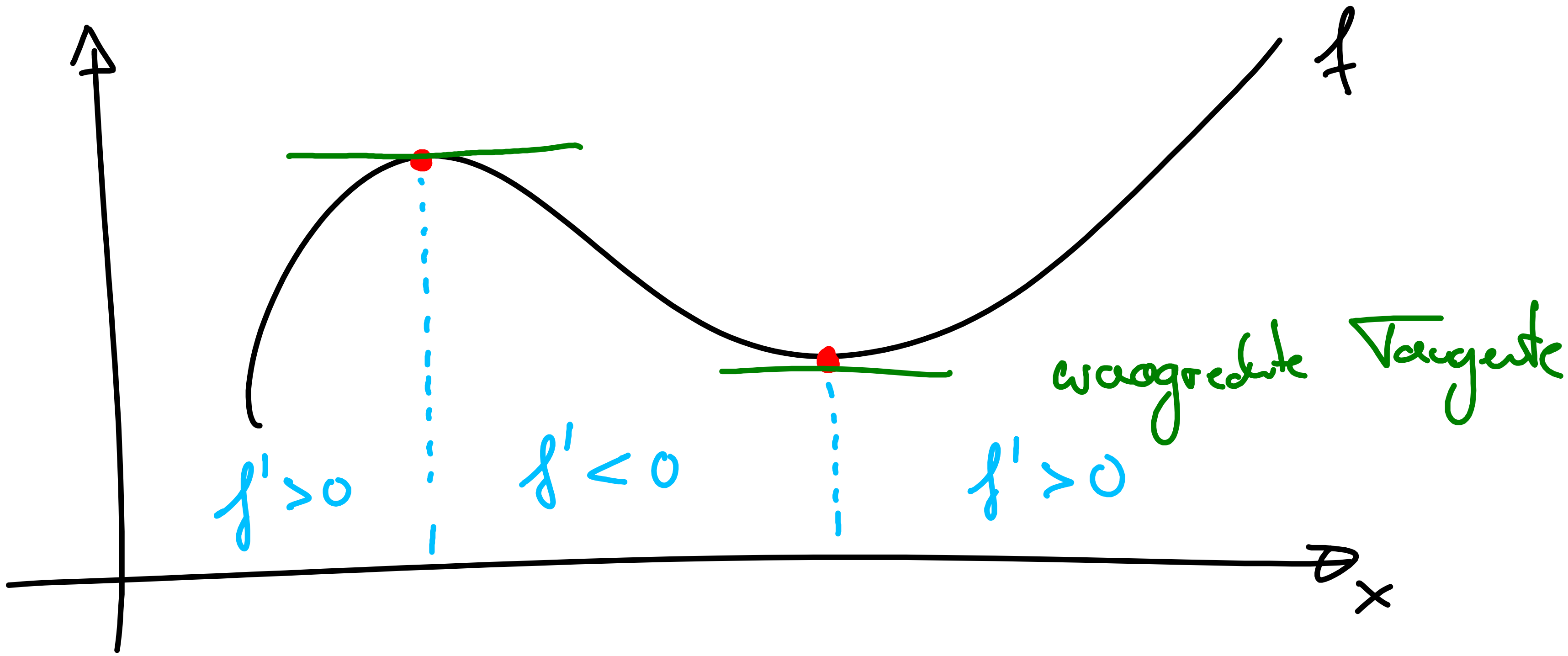
$$\Leftrightarrow \quad f^{-1\prime}(y) = \frac{1}{f'(f^{-1}(y))}$$

nenne y wieder x

$$f^{-1\prime}(x) = \frac{1}{f'(f^{-1}(x))}$$

Bsp: $f(x) = \log x$

$$(\log x)' = \frac{1}{\exp'(\log x)} = \frac{1}{\exp(\log x)} = \frac{1}{x}$$



lokales Extremum $\Longrightarrow$ $f' = 0$ (an der Stelle)

z.B.