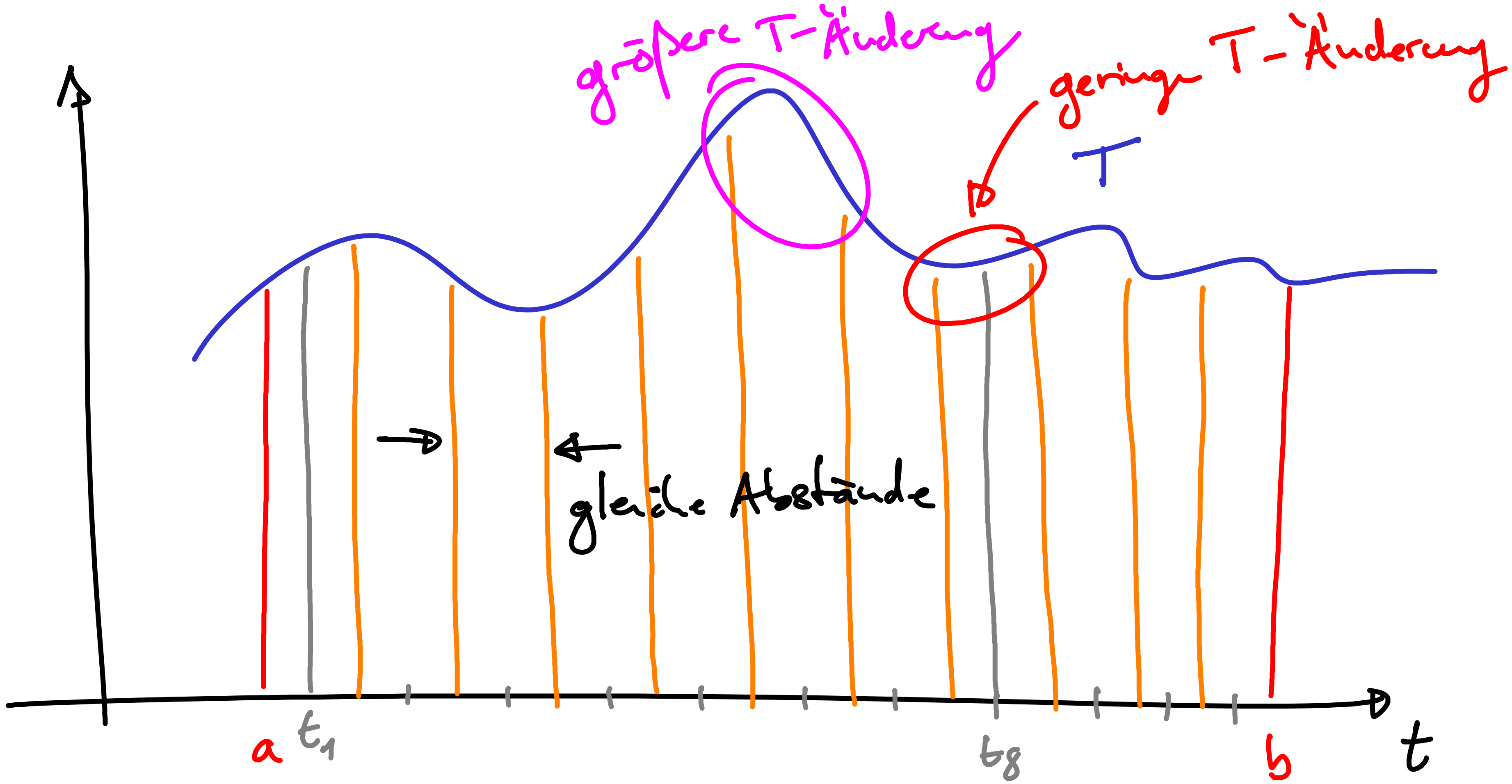
$$f: [0,\pi] \longrightarrow \mathbb{R}^2$$
$$x \longmapsto f(x) = \begin{pmatrix} \sin x \\ e^x + 1 \end{pmatrix}$$

$$F: [0,\pi] \longrightarrow \mathbb{R}^2$$
$$x \longmapsto F(x) = \begin{pmatrix} -\cos x \\ e^x + x \end{pmatrix}$$

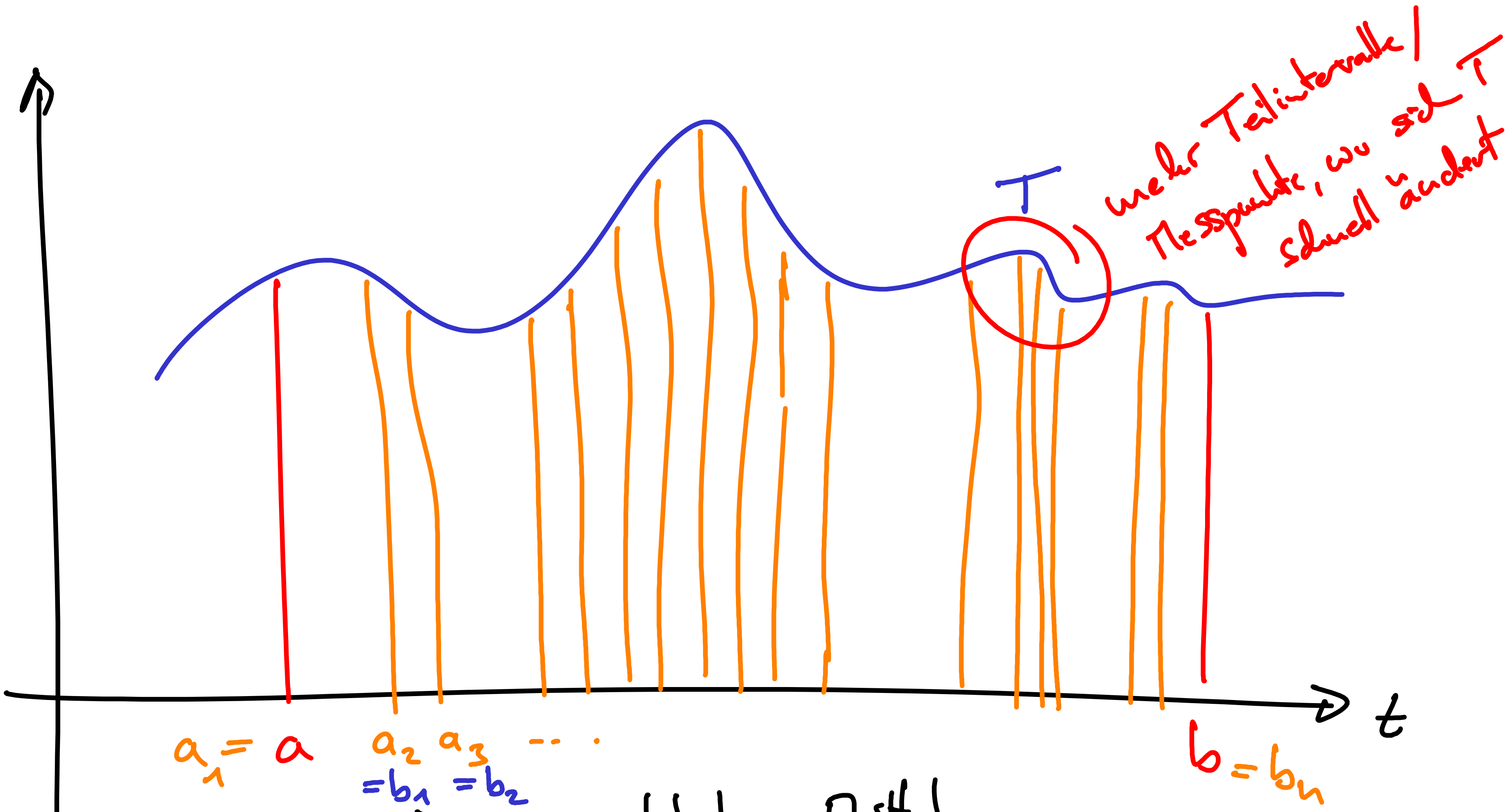
d.h. $F' = f$ bzw. F ist Stammflt. von f

$$\tilde{F}(x) = \begin{pmatrix} 25 - \cos x \\ e^x + 37 + x \end{pmatrix}$$ hat auch f als Ableitung

hier $C = \begin{pmatrix} 25 \\ 37 \end{pmatrix} \in \mathbb{R}^2$

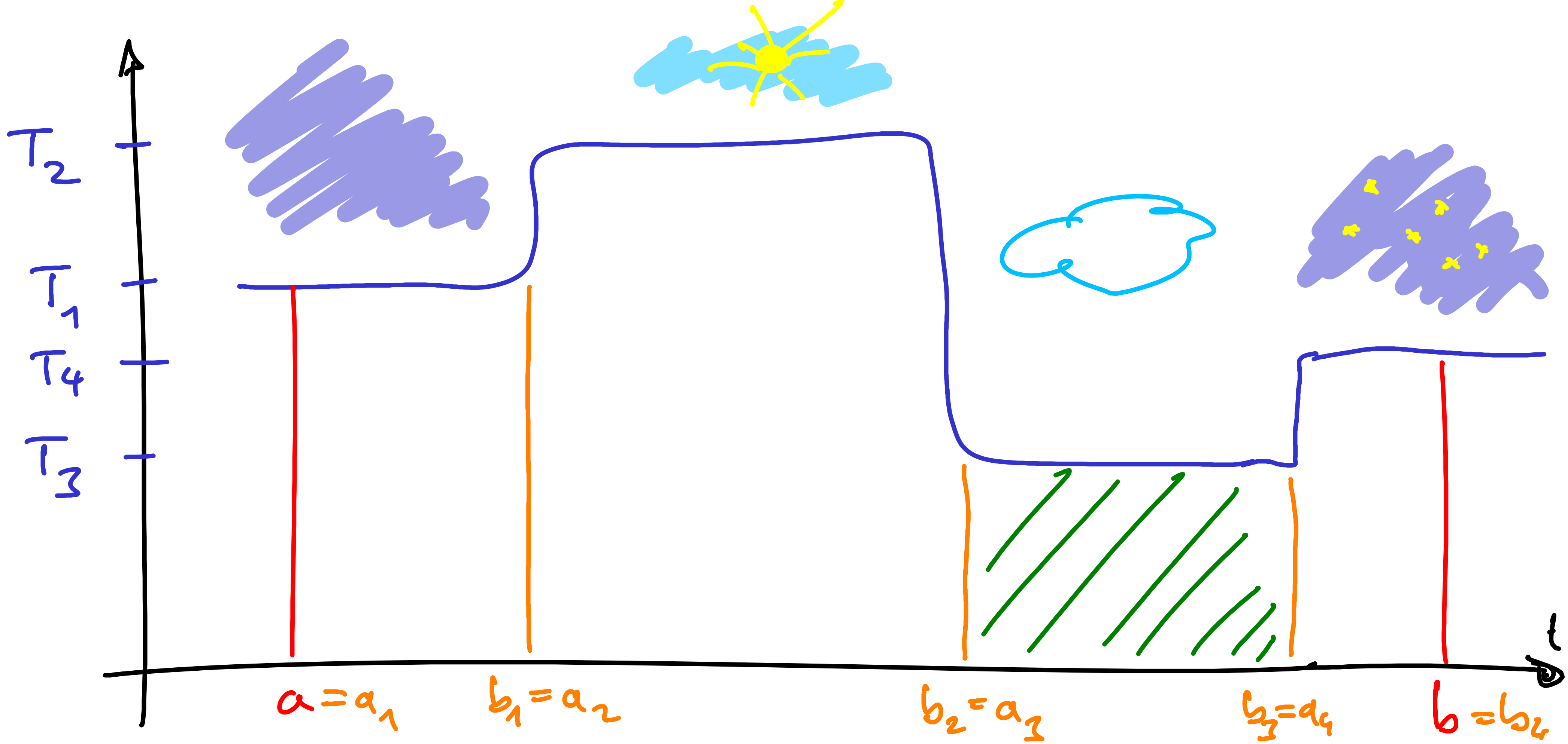


$$\overline{T} \approx \frac{1}{11} \sum_{i=1}^{11} T(t_i)$$

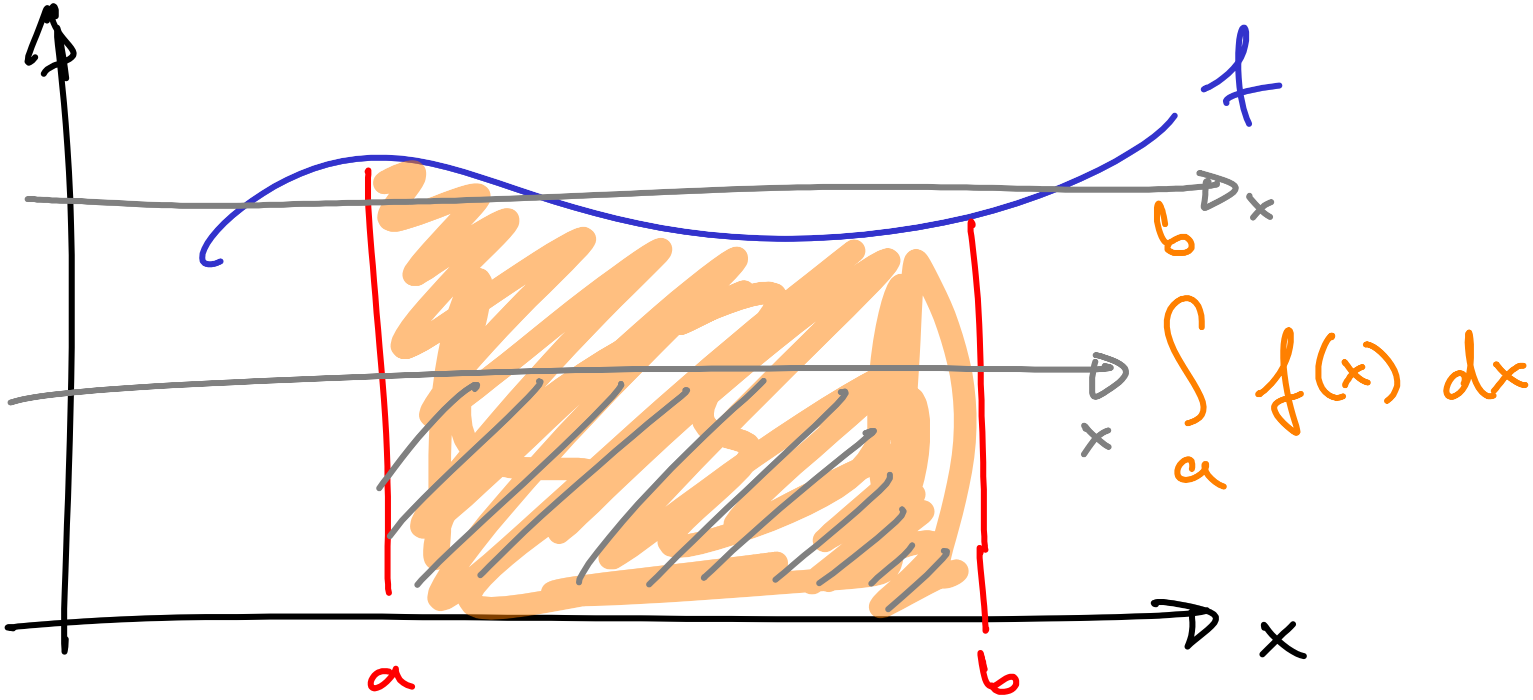


$\longrightarrow$ gewichtetes Mittel

Gewicht prop. zu Länge des Intervalls

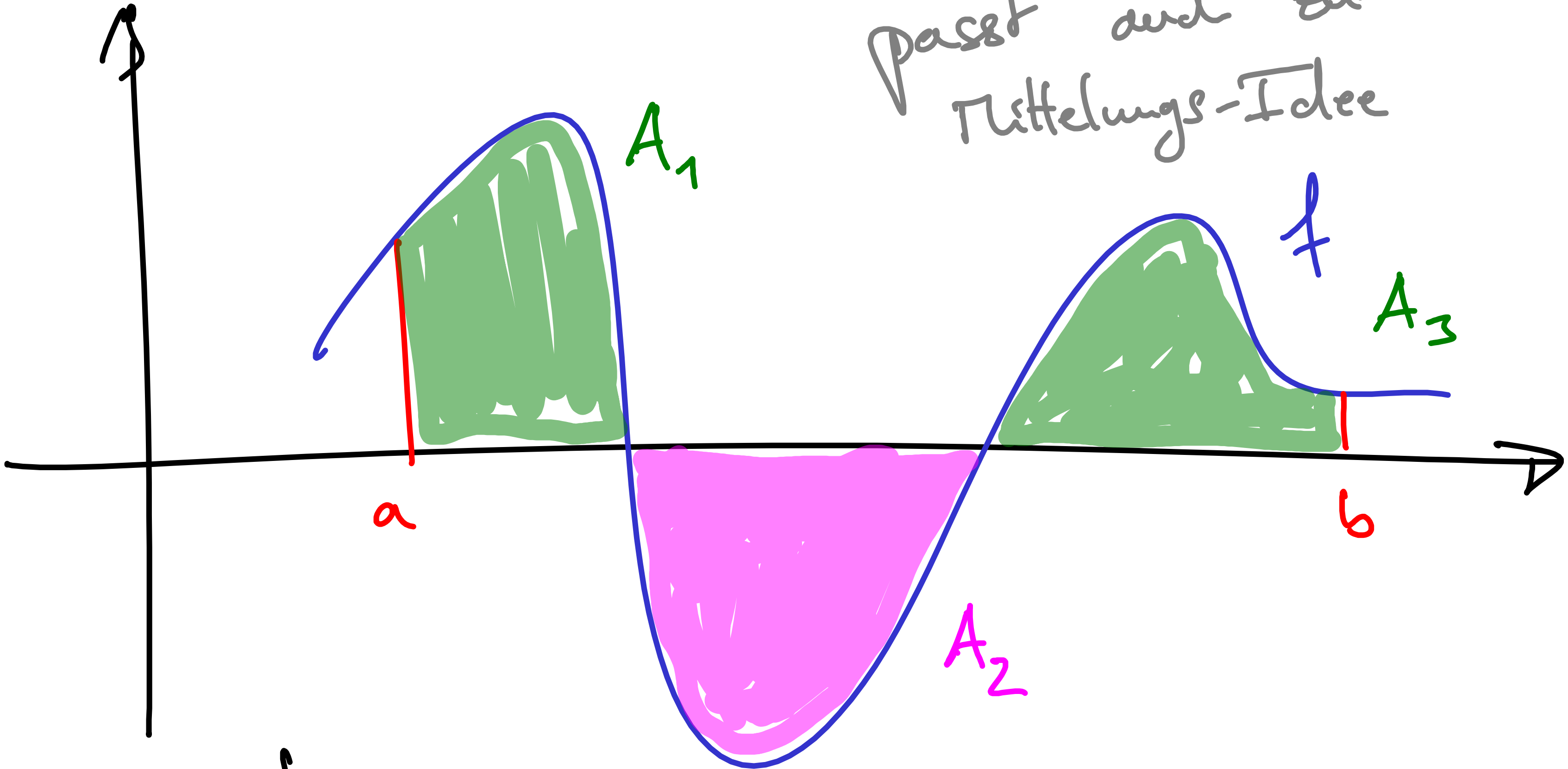


$$T = \frac{b_1 - a_1}{b-a} T_1 + \frac{b_2 - a_2}{b-a} T_2 + \frac{b_3 - a_3}{b-a} T_3 + \frac{b_4 - a_4}{b-a} T_4$$

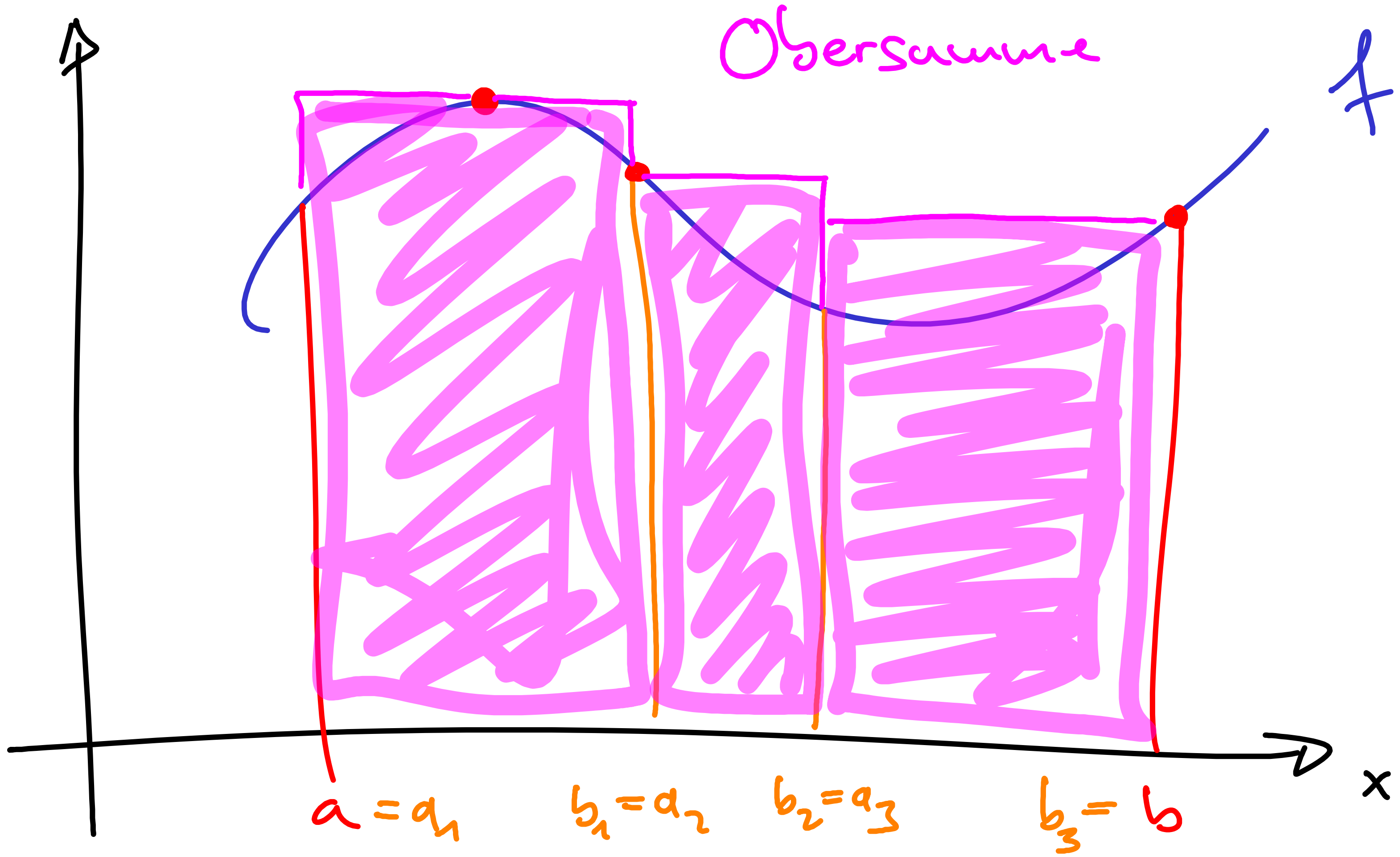


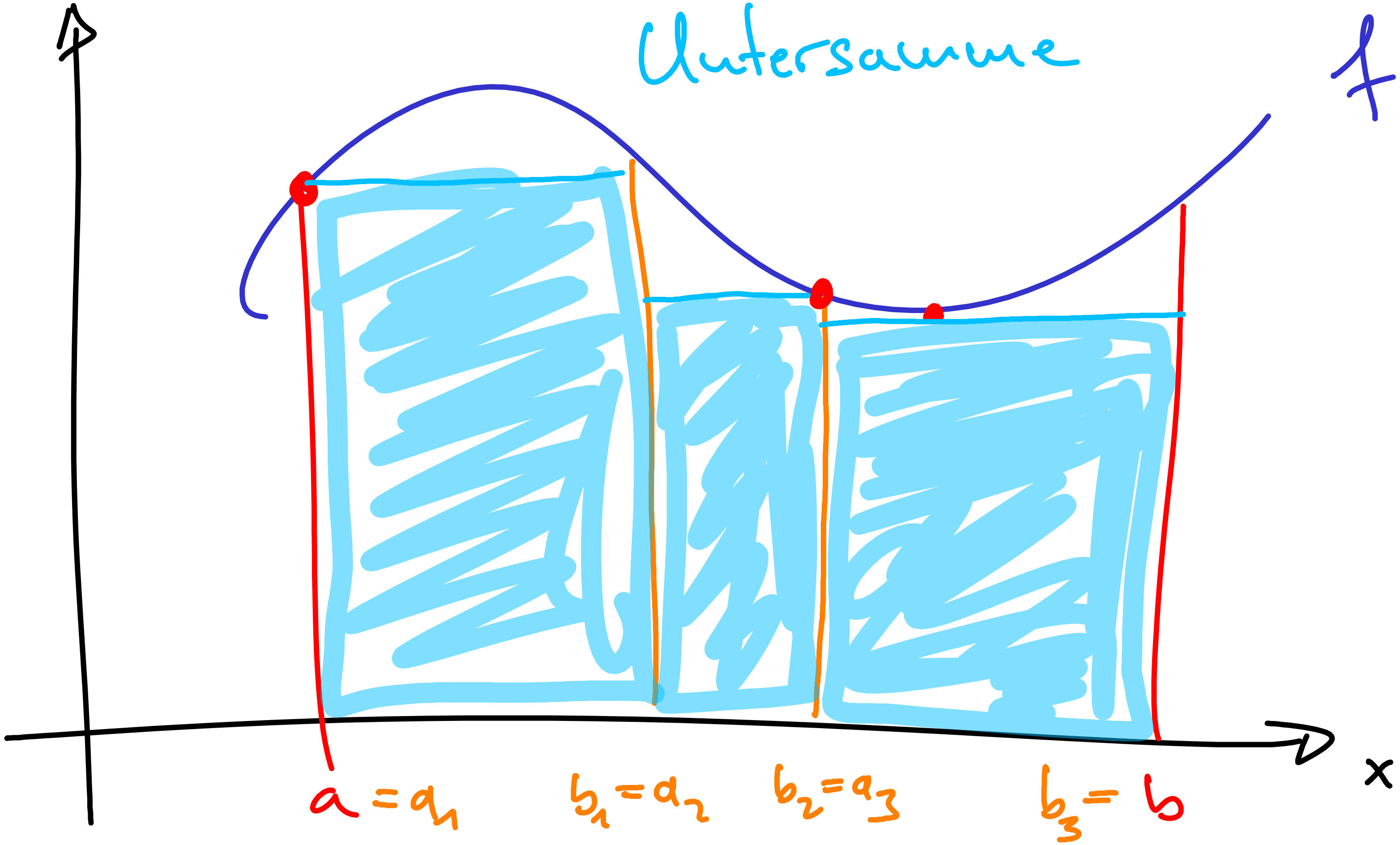
"Definition" für "$\int$"

passt auch zur Mittelungs-Idee



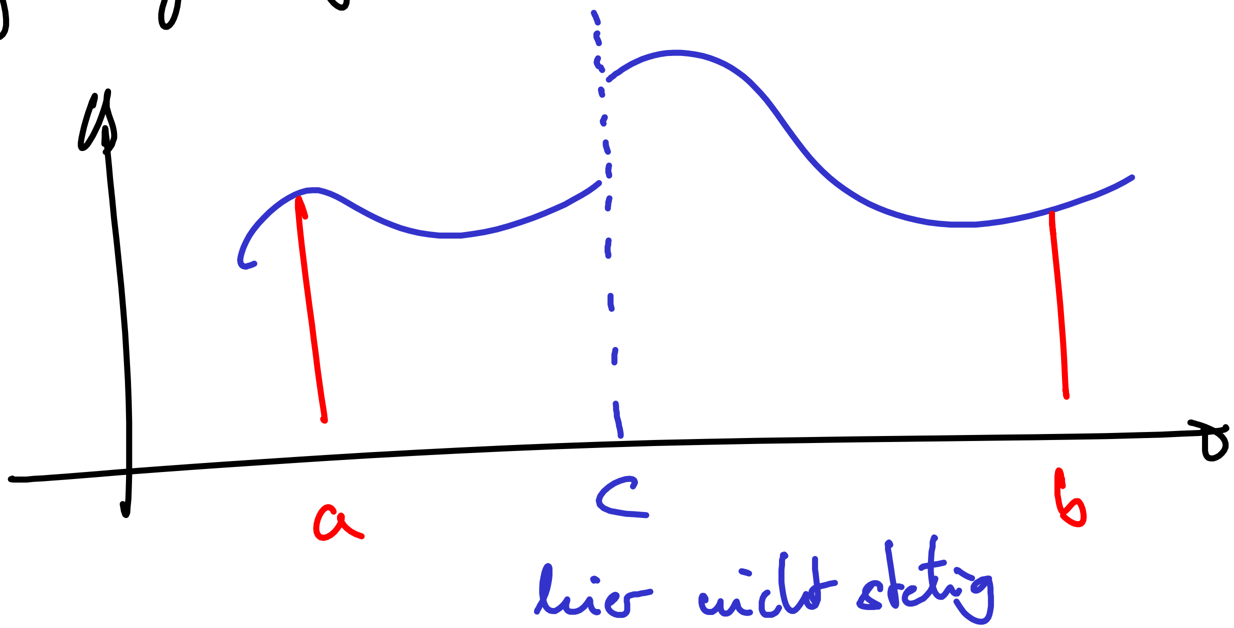
$$\int_a^b f(x)\,dx = A_1 - A_2 + A_3$$


Obersumme
f
x
a = a1
b1 = a2
b2 = a3
b3 = b



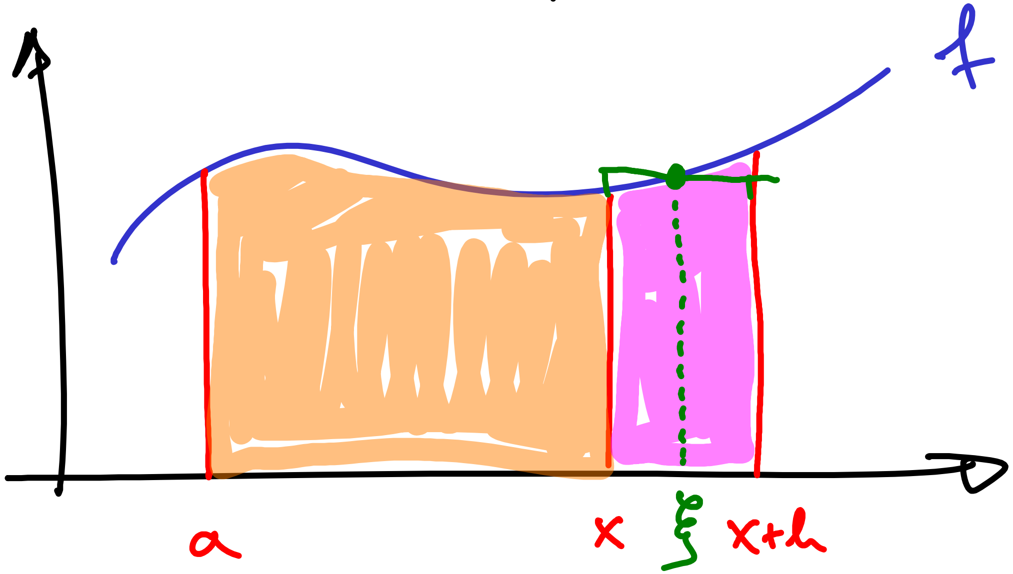
Untersumme
f
x
a = a1
b1 = a2
b2 = a3
b3 = b

Bemerkung: Stückweise Stetigkeit von f genügt für Existenz des Integrals



$$\int_a^b f(x)\,dx := \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$$

Beweisidee zum Hauptsatz



$$F(x) = \int_a^x f(y)\,dy$$

z.z.: $F' = f$

es gibt ein $\xi \in [x, x+h]$,
so dass $f(\xi)\cdot h =$
da f stetig

$$F'(x) = \lim_{h\to 0} \frac{F(x+h) - F(x)}{h} = \lim_{h\to 0} \frac{f(\xi)\cdot \cancel{h}}{\cancel{h}}$$

$$\xi \xrightarrow[h\to 0]{} x$$

$$= f(x) \qquad \square$$

Beispiele

$$\int_0^1 \left(7x^5 - 8x^2 + 3x\right) dx$$

$$= \left[\frac{7}{6}x^6 - \frac{8}{3}x^3 + \frac{3}{2}x^2\right]_0^1$$

Schreibweise für

$$= \frac{7}{6}\cdot 1^6 - \frac{8}{3}\cdot 1^3 + \frac{3}{2}\cdot 1^2 - \left(\frac{7}{6}\cdot 0^6 - \frac{8}{3}\cdot 0^3 - \frac{3}{2}\cdot 0\right)$$

$$= \frac{7 - 16 + 9}{6} = 0$$

gesucht: $F(x)$ mit $F'(x) = e^{-x^2}$

- keine "schöne" Antwort
- aber die Fkt. existiert (laut Hauptsatz)

$$\left(\int_{-12}^{x} e^{-t^2}\, dt \right)' = e^{-x^2}$$

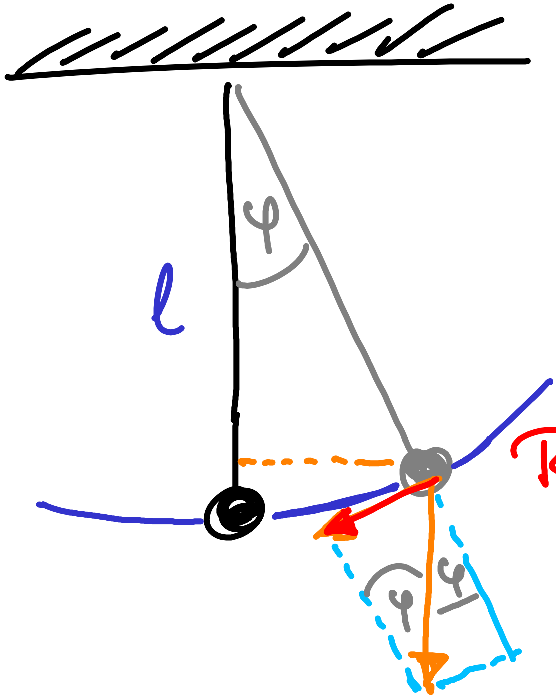
$$= \left(F(x) - F(-12) \right)'$$

$$\int_1^5 e^{-3x}\,dx = \left[\frac{e^{-3x}}{(-3)}\right]_1^5 = \frac{e^{-3}-e^{-15}}{3}$$

$$\int_0^{\pi} \sin(2x)\,dx = \left[-\cos(2x)\cdot\frac{1}{2}\right]_0^{\pi}$$

$$= \left(-\frac{1}{2}\right)\cdot(1-1) = 0$$

$$\int e^{-x^2}\cdot x\,dx = \frac{e^{-x^2}}{(-2)}$$



$$\text{Rückstellkraft} = \underbrace{m \cdot g}_{\text{Gewichtskraft}} \cdot \sin\varphi$$

Energie / Arbeit die aufgewendet wird, um das Pendel aus der Ruhelage ($\varphi = 0$) bis $\varphi = \frac{\pi}{4}$ ($\hat{=}\, 45°$) auszulenken:

$$A = \int_0^{\pi/4} \underbrace{mg\sin\varphi}_{\text{Kraft}} \, \underbrace{l\, d\varphi}_{\text{Weg}} = m \cdot g \cdot l \left[-\cos\varphi\right]_0^{\pi/4}$$

$$= mgl\left(\cos(0) - \cos\frac{\pi}{4}\right) = mgl\underbrace{\left(1 - \frac{1}{2}\sqrt{2}\right)}_{\approx 0{,}3}$$