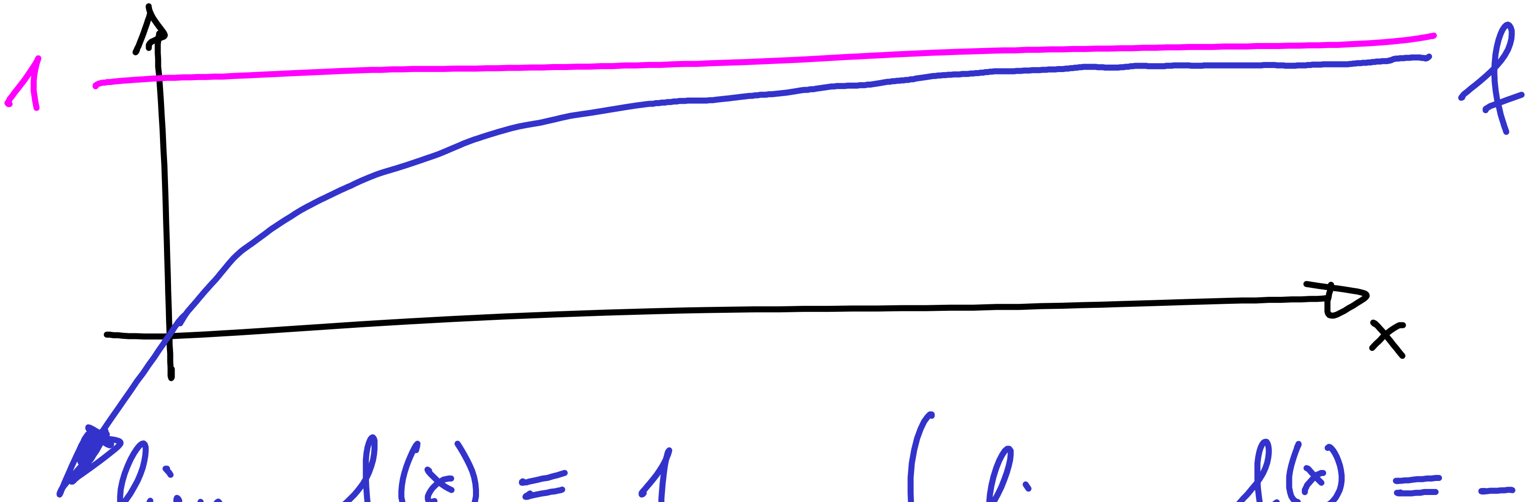
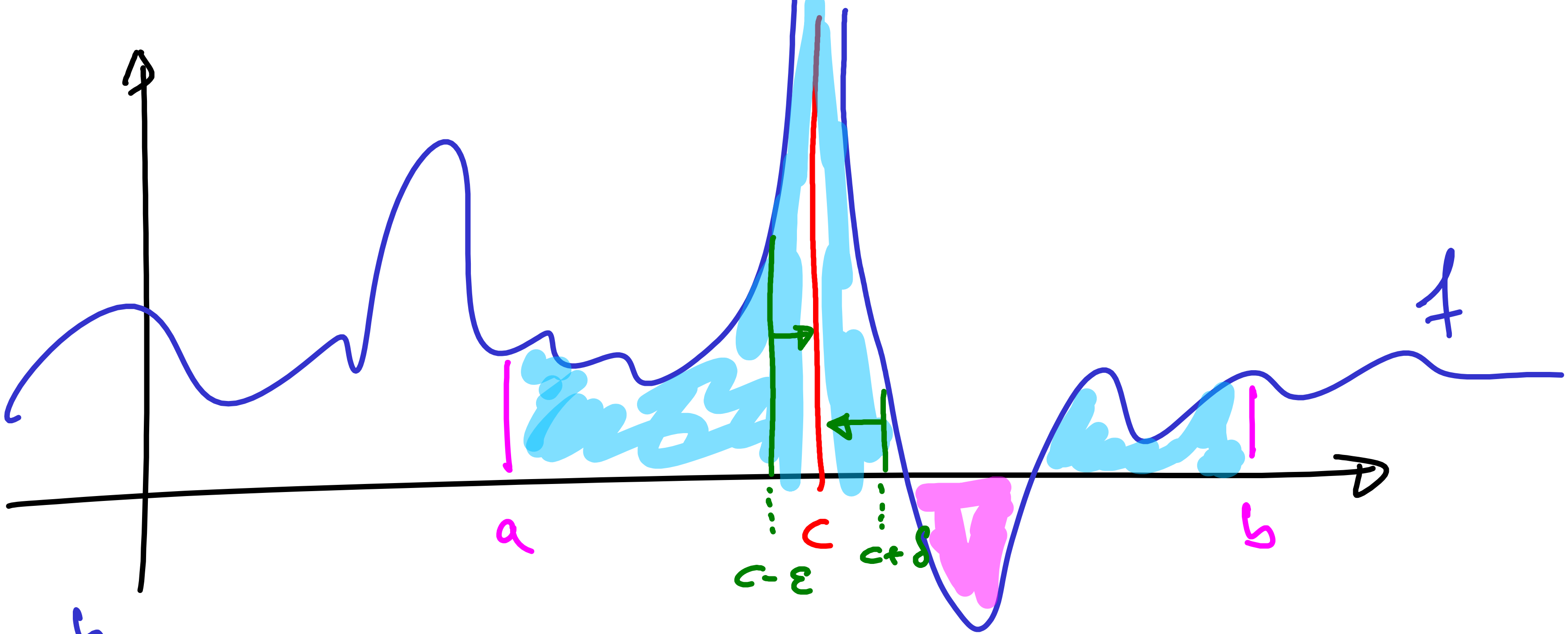
$$f(x) = 1 - e^{-\lambda x} \quad , \quad \lambda > 0$$



$$\lim_{x \to \infty} f(x) = 1 \qquad \left(\lim_{x \to -\infty} f(x) = -\infty \right)$$



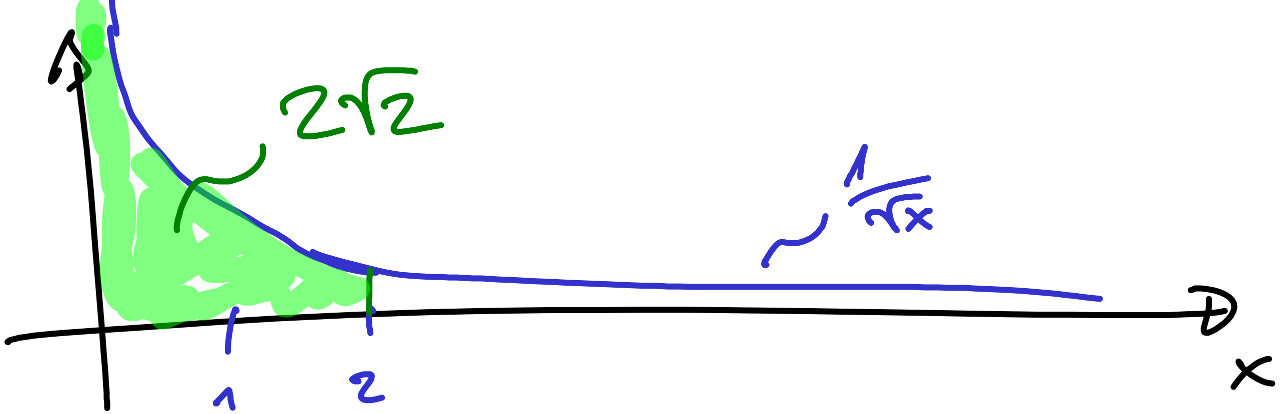
$$\int_a^b f(x)\,dx$$

Beispiel:

$$\int_0^2 \frac{dx}{\sqrt{x}} = \lim_{\varepsilon \to 0+} \int_\varepsilon^2 x^{-1/2}\,dx$$

$$= \lim_{\varepsilon \to 0+} \left[2x^{1/2} \right]_\varepsilon^2 = \lim_{\varepsilon \to 0+} \left(2\sqrt{2} - 2\sqrt{\varepsilon} \right)$$

$$= 2\sqrt{2}$$



anderes Beispiel:

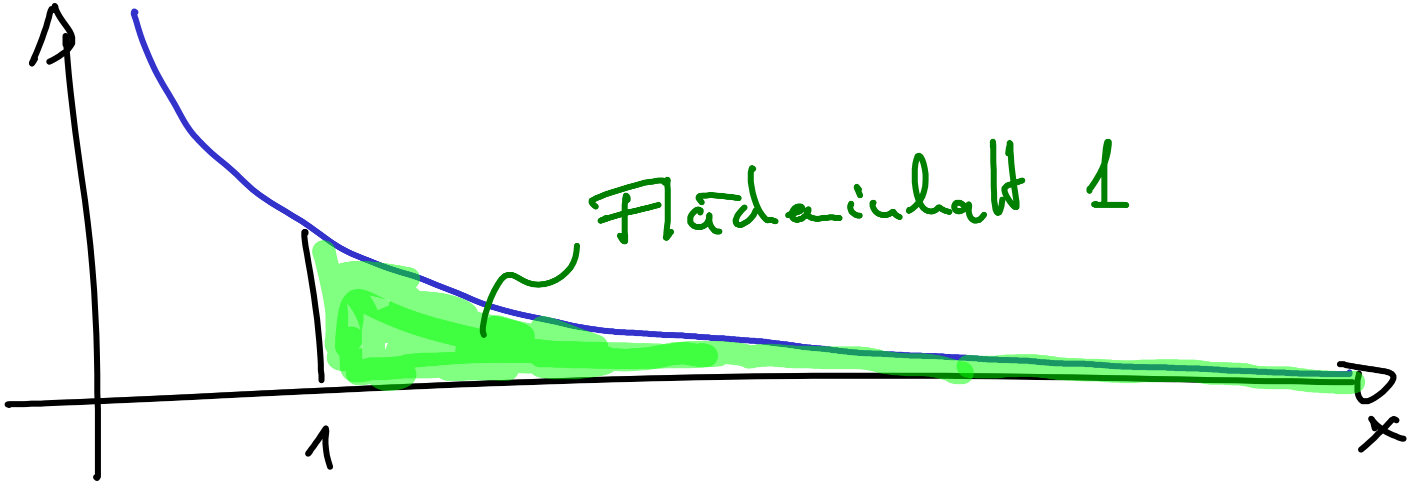
$$\int_0^2 \frac{dx}{x} = \lim_{\varepsilon \to 0+} \int_\varepsilon^2 \frac{dx}{x} = \lim_{\varepsilon \to 0+} \Big[\log x\Big]_\varepsilon^2$$

$$= \lim_{\varepsilon \to 0+} \left(\log 2 - \log \varepsilon\right) = \infty$$

Skizze sieht aus wie oben, jetzt aber unendlich groß.

$$\int_1^\infty \frac{dx}{x^2} = \lim_{b\to\infty} \int_1^b \frac{dx}{x^2} = \lim_{b\to\infty} \left[-\frac{1}{x} \right]_1^b$$

$$= \lim_{b\to\infty} \left(-\frac{1}{b} - \left(-\frac{1}{1} \right) \right) = 1$$



Bsp. für partielle Ableitungen

$$f(s,t) = s e^{t} + \sin(st)$$

$$\frac{\partial f}{\partial s} = e^{t} + \cos(st) \cdot t$$

$$\frac{\partial f}{\partial t} = s e^{t} + \cos(st) \cdot s$$

Calc-zilla

$$\frac{\partial}{\partial x} y^{x} = \frac{\partial}{\partial x} e^{\log(y^{x})} = \frac{\partial}{\partial x} e^{x \log y}$$

$$= \log y \cdot e^{x \log y} = y^{x} \cdot \log y$$

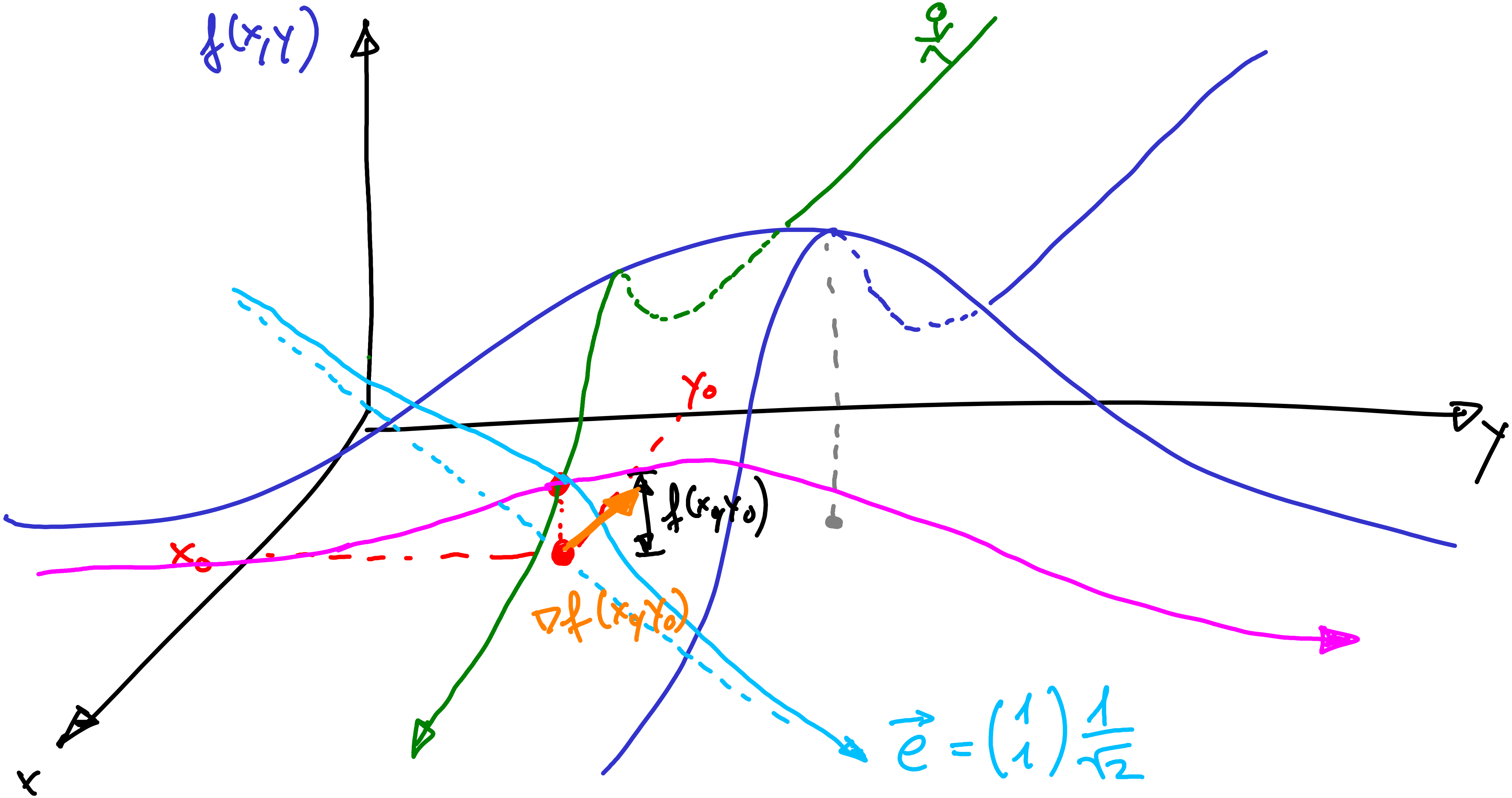
$$f : \mathbb{R}^2 \longrightarrow \mathbb{R}$$

$$(x_1, x_2) \longmapsto f(x_1, x_2)$$

$$\frac{\partial f}{\partial x_1} = \lim_{h \to 0} \frac{f(\vec{x} + h\vec{e}_1) - f(\vec{x})}{h}$$

$$\vec{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, \quad \vec{e}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \vec{x} + h\vec{e}_1 = \begin{pmatrix} x_1 + h \\ x_2 \end{pmatrix}$$

$$= \lim_{h \to 0} \frac{f(x_1 + h, x_2) - f(x_1, x_2)}{h}$$



$\frac{\partial f}{\partial x}(x_0,y_0)$ = Steigung des grünen Weges < 0

an der Stelle (x_0,y_0)

$$\frac{\partial f}{\partial y}(x_0, y_0) = \text{Steigung des rosa Weges an der Stelle } (x_0, y_0) > 0$$

$$\frac{\partial f}{\partial \vec{e}}(x_0, y_0) = \text{Steigung des blauen Weges an der Stelle } (x_0, y_0) \approx 0 \text{ (hier)}$$

Beispiel: Richtungsableitung

$$f(s,t) = s e^t + \sin(st)$$

$$\vec{e} = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}$$

$$\frac{\partial f}{\partial s} = e^t + \cos(st)\cdot t$$

$$\frac{\partial f}{\partial t} = s e^t + \cos(st)\cdot s$$

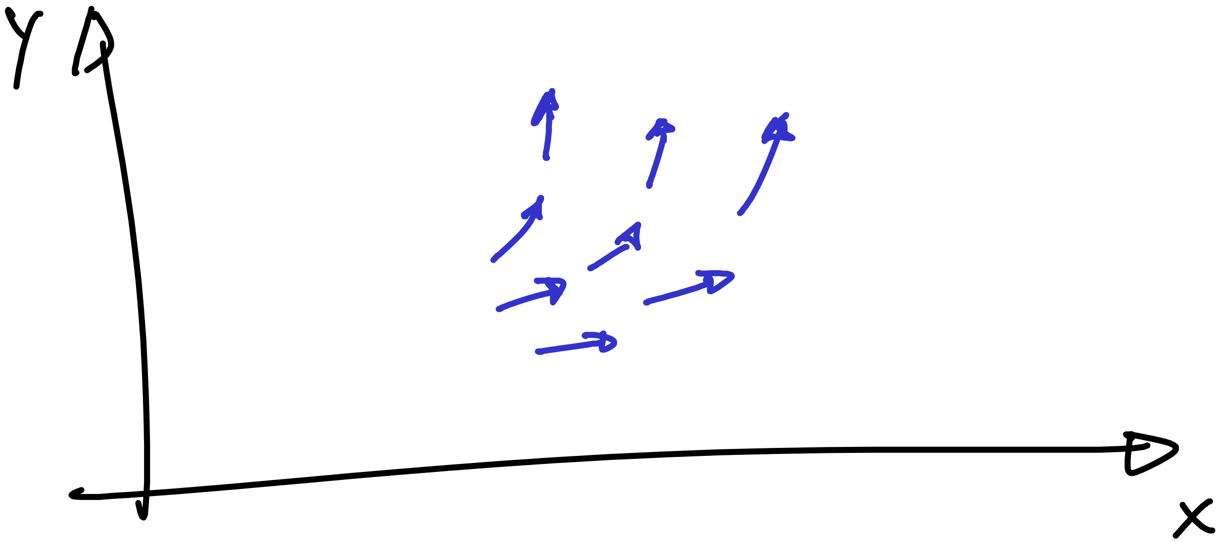
$$\frac{\partial f}{\partial \vec{e}}(0,0) = \lim_{h\to 0} \left.\frac{f\left(\binom{s}{t} + \frac{h}{\sqrt{2}}\binom{1}{1}\right) - f\left(\binom{s}{t}\right)}{h}\right|_{(s,t)=(0,0)}$$

$$= \left(\frac{\partial f}{\partial s}(0,0), \frac{\partial f}{\partial t}(0,0)\right)\cdot \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}$$

$$= (1,0)\cdot\frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix} = \frac{1}{\sqrt{2}}(1\cdot 1 + 0\cdot 1) = \frac{1}{\sqrt{2}}$$

$$f : \mathbb{R}^2 \to \mathbb{R}$$
$$(x,y) \mapsto f(x,y)$$

$$\nabla f : \mathbb{R}^2 \to \mathbb{R}^2$$
$$(x,y) \mapsto \left(\frac{\partial f}{\partial x}(x,y), \frac{\partial f}{\partial y}(x,y) \right)$$



Bsp. für 2. part. Abl.

$$f(s,t) = s e^{t} + \sin(st)$$

$$\frac{\partial f}{\partial s} = e^{t} + \cos(st) \cdot t$$

$$\frac{\partial f}{\partial t} = s e^{t} + \cos(st) \cdot s$$

$$\frac{\partial^2 f}{\partial s \partial t} = \frac{\partial}{\partial s} \frac{\partial f}{\partial t} = \frac{\partial}{\partial s}\left(s e^{t} + \cos(st) \cdot s \right)$$

$$= e^{t} - \sin(st) \cdot st + \cos(st)$$

$$\frac{\partial^2 f}{\partial t \partial s} = \frac{\partial}{\partial t} \frac{\partial f}{\partial s} = \frac{\partial}{\partial t}\left(e^{t} + \cos(st) \cdot t \right)$$

$$= e^{t} - \sin(st) \cdot ts + \cos(st)$$

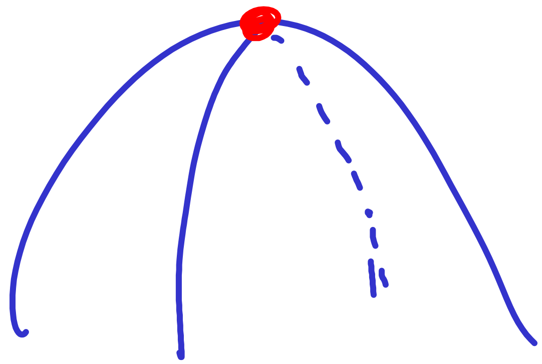
nicht immer gleich (hier schon), aber immer, falls 2. Ableitung stetig sind

$$\frac{\partial^2 f}{\partial t^2} = \frac{\partial}{\partial t}\left(se^t + \cos(st)\cdot s\right)$$

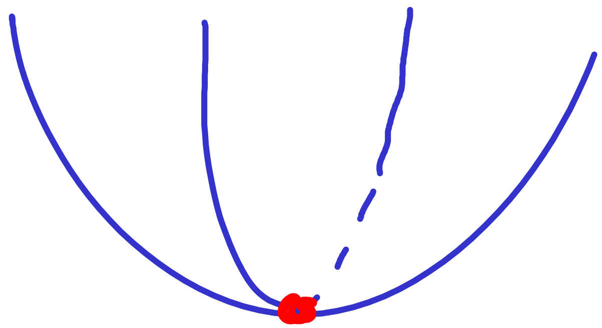
$$= se^t - \sin(st)\cdot s^2$$

$$\frac{\partial^2 f}{\partial s^2} = \frac{\partial}{\partial s}\left(e^t + \cos(st)\cdot t\right)$$

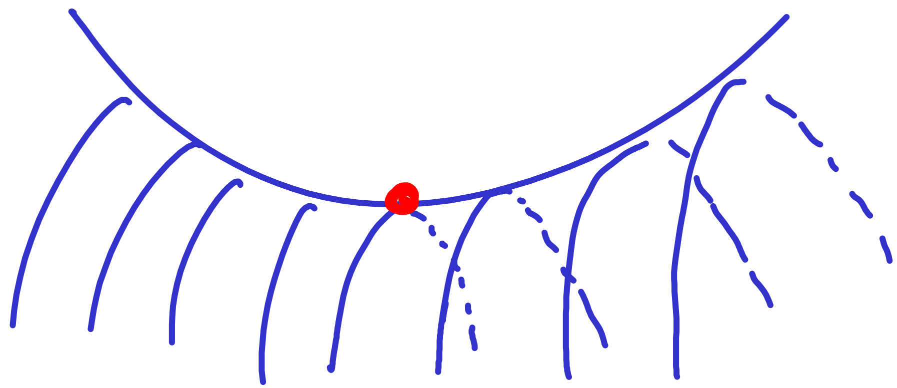
$$= -\sin(st)\cdot t^2$$



Maximum



Minimum



Sattel