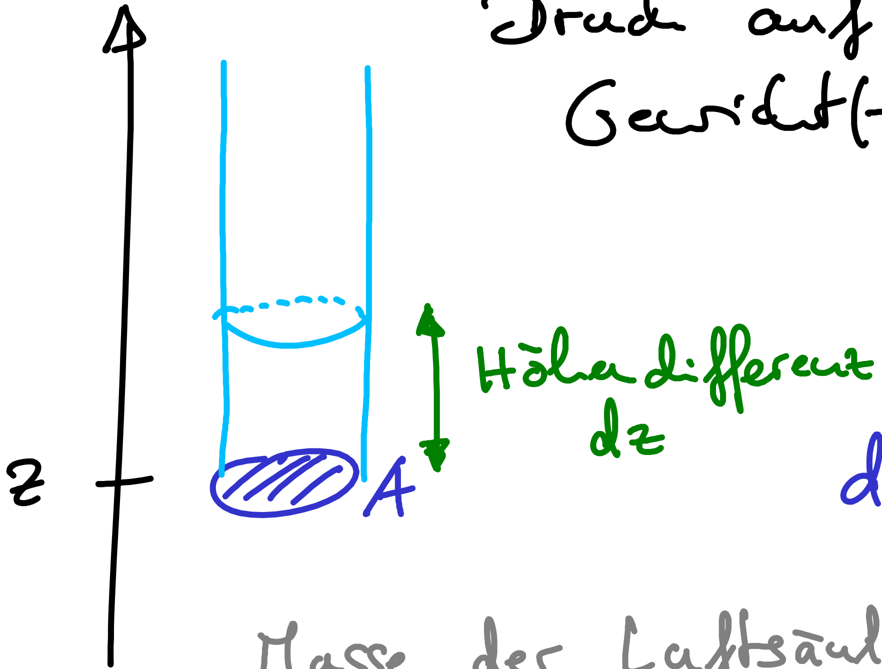


Druck auf Fläche A in Höhe z:
Gewicht(-skraft) der Luftsäule darüber

Druckänderung $z \to z+dz$

$$dp = -g\, m_{LS}/A$$

Volumen   Dichte

Masse der Luftsäule: $m_{LS} = V \cdot \rho = A \cdot dz \cdot \rho$

d.h. $$\frac{dp}{dz} = p'(z) = -g\,\rho(z) \qquad (*)$$

ideales Gas: $pV = nRT$ $\qquad V = \frac{m}{\rho}$

$$p\,\frac{m}{\rho} = nRT \iff \rho = \underbrace{\frac{m}{n}}_{=:M}\,\frac{p}{RT}$$

=:M molare Masse

in (*):

$$p'(z) = -\frac{gM}{R\,T(z)}\,p(z)$$

Konstanten

$$\boxed{p'(z) = -\frac{gM}{R\,T(z)}\,p(z)}$$ $$p' = \frac{dp}{dz}$$

① $T(z) = T$ konstant

$$\frac{dp}{dz} = -\frac{gM}{RT}\,p \qquad \Big|\cdot \frac{1}{p}\cdot dz$$

$$\int \frac{dp}{p} = -\int \frac{gM}{RT}\,dz$$

$$\log p = -\frac{gM}{RT}\cdot z + C \qquad \text{mit } C \in \mathbb{R} \text{ beliebig}$$

$$\Big|\ \exp(\dots)$$

$$p = \underbrace{e^{C}}_{=:p_0}\cdot e^{-\frac{gM}{RT}z}\cdot e^{+\frac{gM}{RT}z_0} \qquad \text{mit } z_0 \text{ beliebig}$$

$$= p_0\, e^{-\frac{gM}{RT}(z-z_0)}$$

$$\boxed{p'(z) = -\frac{gM}{R\,T(z)}\,p(z)} \qquad p' = \frac{dp}{dz}$$

② $T(z)$ linear: $T(z) = T_0 + \gamma(z - z_0)$

$$\int \frac{dp}{p} = -\frac{gM}{R} \int \frac{1}{T_0 + \gamma(z-z_0)}\,dz$$

$$\log p = -\frac{gM}{R\gamma} \log\left(T_0 + \gamma(z-z_0)\right) + C \quad \Big|\exp(\ldots)$$

$$p = \left(T_0 + \gamma(z-z_0)\right)^{-\frac{gM}{R\gamma}} \cdot e^{C}$$

klammere T_0 aus

$$= p_0 \left(1 + \frac{\gamma}{T_0}(z-z_0)\right)^{-\frac{gM}{R\gamma}}$$

$\left(p_0 = e^{C} \cdot T_0\, e^{-\frac{gM}{R\gamma}} \leftarrow \text{Vergessen}\right)$

Druck in der Höhe z_0

Bspe für Diff.-Gln.

① $\dot{x} + x = 0$ autonome DGL 1. Ordnung

Lösung: $x(t) = x(0) \cdot e^{-t}$

$\dot{x} = f(x,t) = -x$ $d=1$, $n=1$

② $\dot{x} + x = \sin t$ zeitabh. DGL 1. Ordnung

$\dot{x} = f(x,t) = \sin t - x$ $d=1$, $n=1$

③ $\ddot{x} + x = 0$ autonome DGL 2. Ordnung

$\ddot{x} = f(x,t) = -x$ $d=1$, $n=2$

(4) $\dot{x}_1 = x_2$ autonomes System 1. Ordnung

$\dot{x}_2 = -x_1$ $d=2,\ h=1$

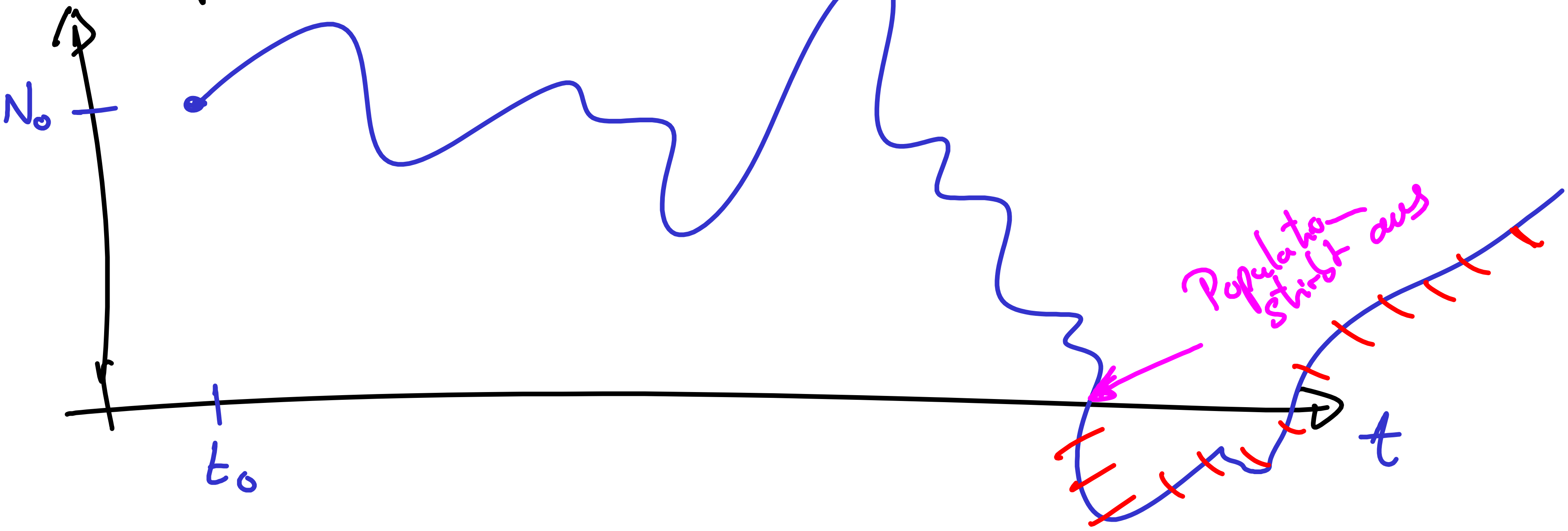
in Vektorschreibweise $\vec{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$

$$\dot{\vec{x}} = \begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} x_2 \\ -x_1 \end{pmatrix} = \vec{f}(x_1, x_2, t) = \vec{f}(\vec{x}, t)$$

$$= \underbrace{\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}}_{=:A} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

$$f(\vec{x}, t) = A \cdot \vec{x}$$

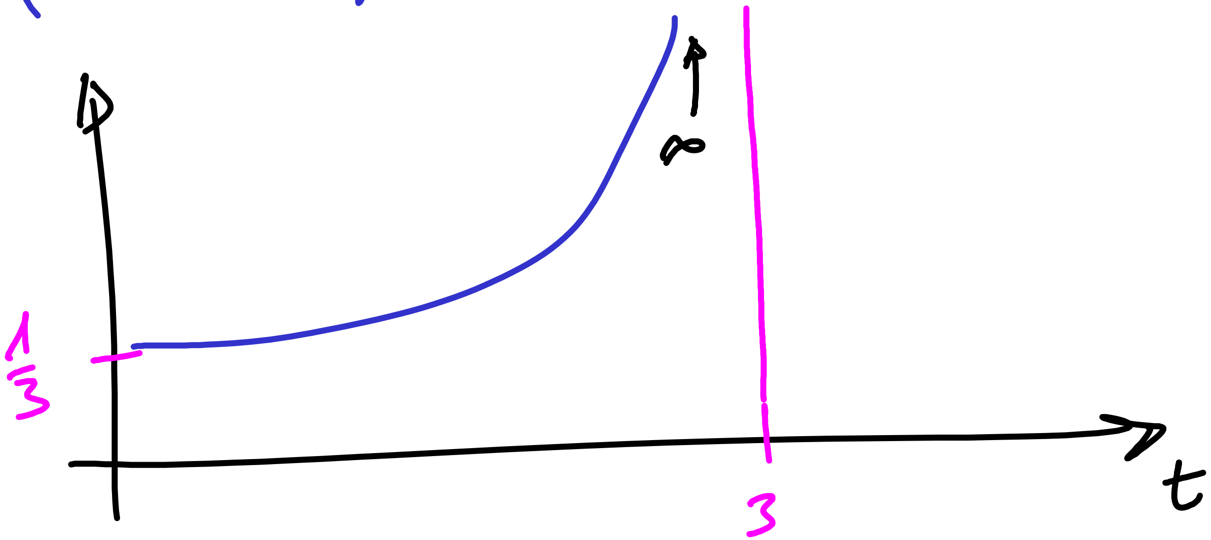
Populationsgröße $N(t)$



AWP: $\dot{x} = x^2 \quad , \quad x(0) = \frac{1}{3}$

Lösung: $x(t) = \frac{1}{3-t}$

$$\dot{x} = \left((3-t)^{-1}\right)^{\bullet} = (-1)\,(3-t)^{-2}\cdot(-1) = \left(\frac{1}{3-t}\right)^2 = x^2$$



$$\left(\begin{array}{l} \dot{x} = x^2 \quad , \quad \frac{dx}{dt} = x^2 \\ \int \frac{dx}{x^2} = \int dt \quad , \quad -\frac{1}{x} = t + C \\ \Rightarrow \quad x = -\frac{1}{t+C} = \frac{1}{-C-t} \end{array} \right)$$

Reduktion $\ddot{x} + \omega^2 x = 0$

$$x_1 = x \qquad \Longrightarrow \qquad \dot{x}_1 = \dot{x} = x_2$$

$$x_2 = \dot{x} \qquad\qquad\qquad \dot{x}_2 = \ddot{x} = -\omega^2 x = -\omega^2 x_1$$

$$\vec{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

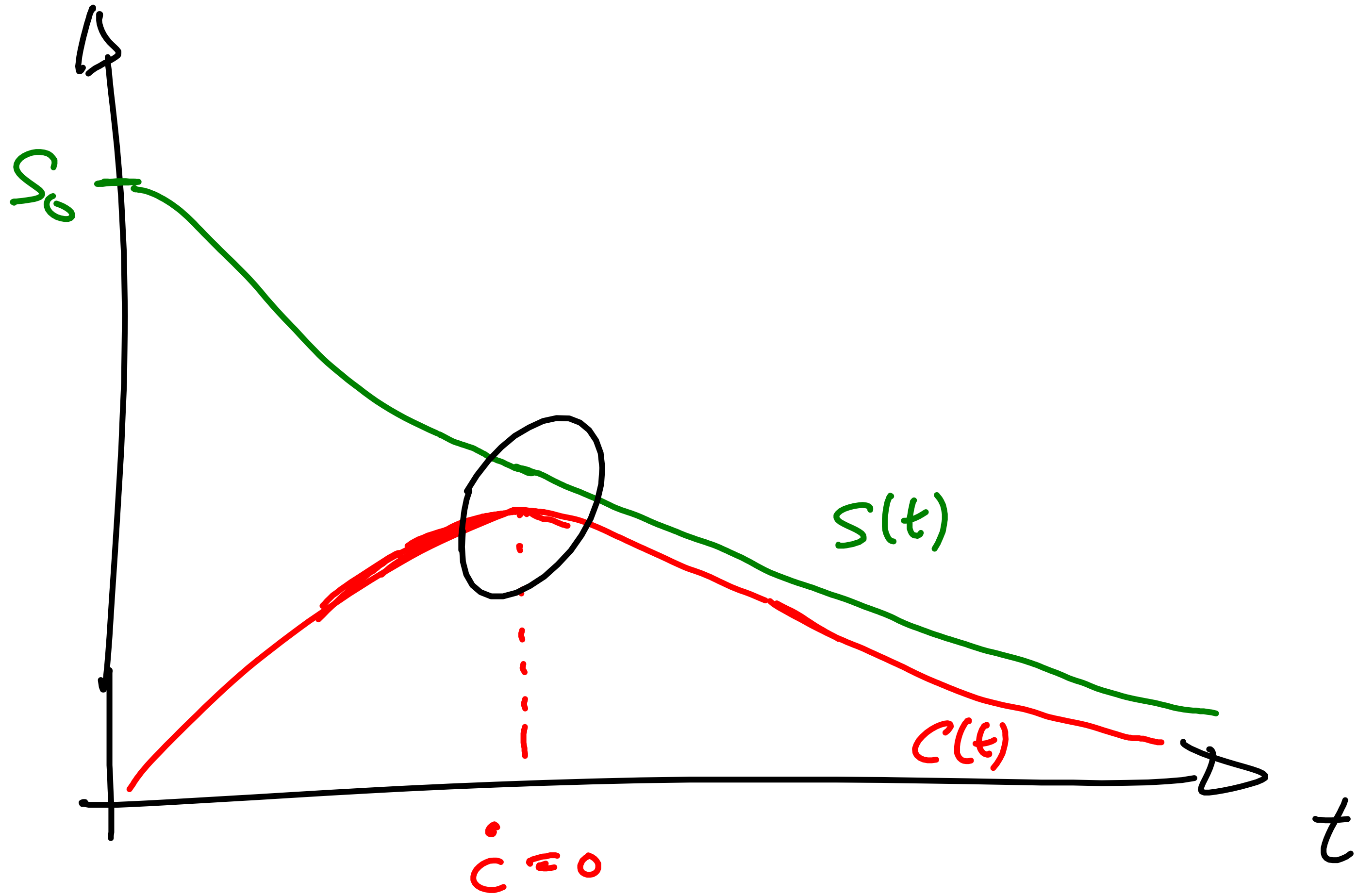
$$\dot{\vec{x}} = \underbrace{\begin{pmatrix} x_2 \\ -\omega^2 x_1 \end{pmatrix}}_{\vec{f}(\vec{x},t)} = \begin{pmatrix} 0 & 1 \\ -\omega^2 & 0 \end{pmatrix} \vec{x}$$

$$S + E \underset{k_{-1}}{\overset{k_1}{\rightleftharpoons}} SE \qquad c = \text{Konz}(SE)$$

$$\dot{s} = -k_1 s e + k_{-1} c$$

$$SE \xrightarrow{k_2} P + E$$

$$\dot{e} = -k_1 s e + k_{-1} c \overbrace{+ k_2 c}^{\text{aus 2. Reaktionsgln.}}$$



S_0
$S(t)$
$C(t)$
t
$\dot{C} = 0$