Nachklausur 08/09

$\boxed{1}$ d) $x^2 y z^3 = 7^3$, $xy^2 = 7^9$, $xyz = ?$

multiplizieren: $x^3 y^3 z^3 = 7^{12} \Rightarrow xyz = 7^4$

$\boxed{6}$ b) $\begin{pmatrix} N_1 \\ L_1 \end{pmatrix} = \begin{pmatrix} 2000 \\ 2000 \end{pmatrix}$, $W = \frac{1}{200} \begin{pmatrix} 191 & 3 \\ 200x & 197 \end{pmatrix}$

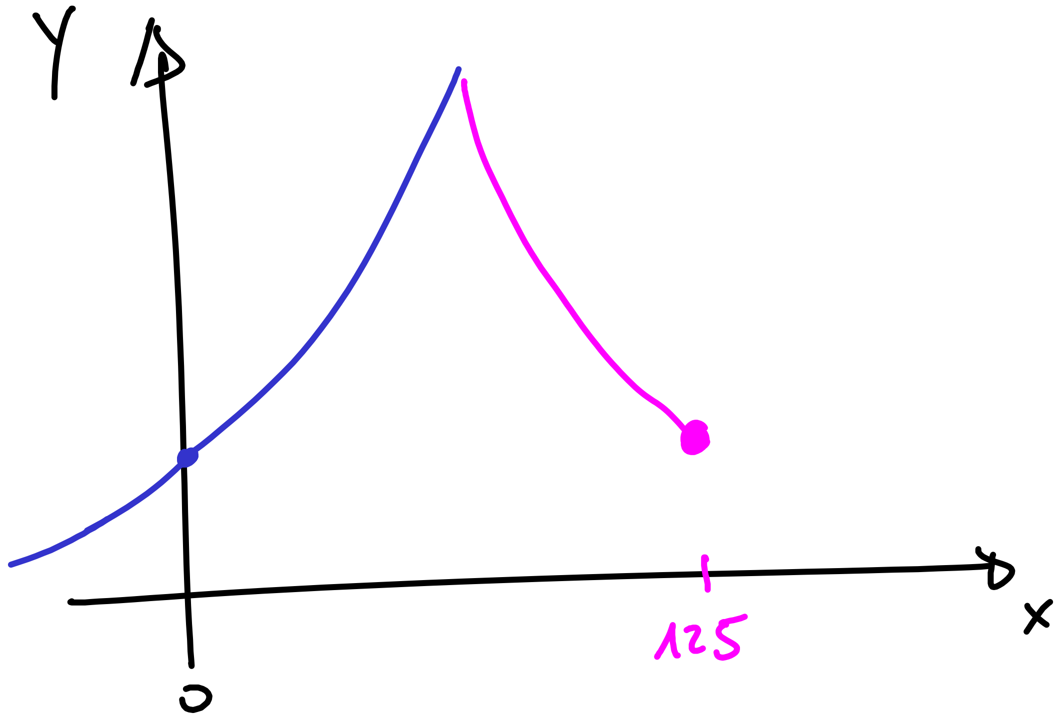
$$W \begin{pmatrix} N_1 \\ L_1 \end{pmatrix} = \begin{pmatrix} \frac{191}{200} & \frac{3}{200} \\ x & \frac{197}{200} \end{pmatrix} \begin{pmatrix} 2000 \\ 2000 \end{pmatrix}$$

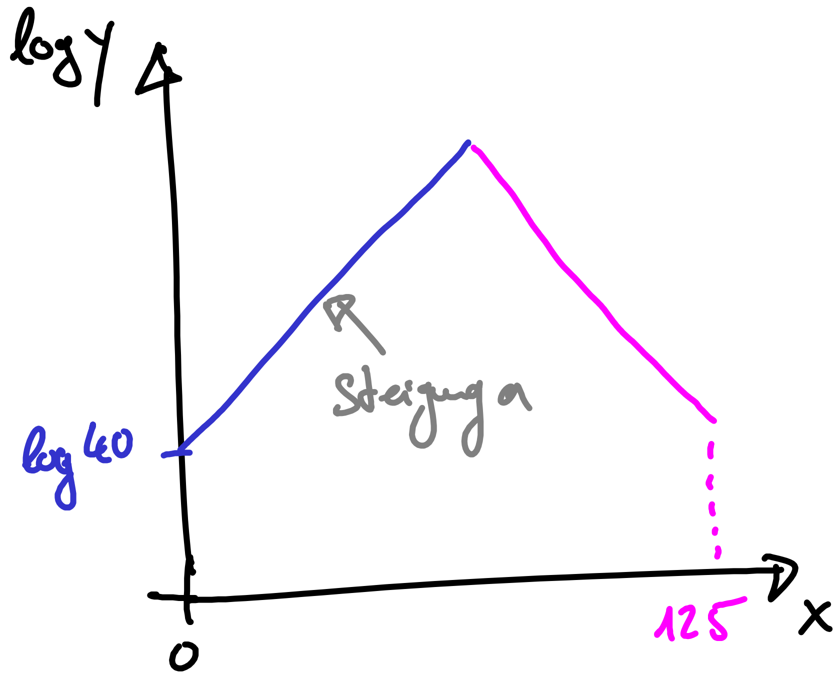
$$= \frac{1}{200} \left[\begin{pmatrix} 191 & 3 \\ 200x & 197 \end{pmatrix} \begin{pmatrix} 2000 \\ 2000 \end{pmatrix} \right]$$

$$= \begin{pmatrix} 191 & 3 \\ 200x & 197 \end{pmatrix} \begin{pmatrix} 10 \\ 10 \end{pmatrix}$$

ÜA 23 c) linke Flanke: $y = 40\, e^{ax}$, $\log y = \log 40 + ax$

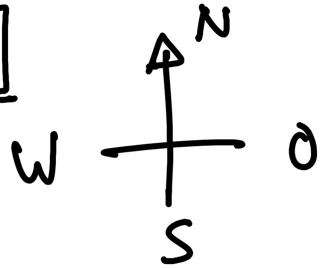
rechte Flanke: $y = 40\, e^{-a(x-125)}$

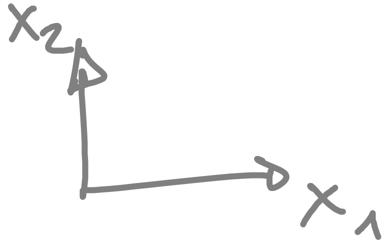




rechte Flanke: $\log y = \log 40 - a(x-125)$

$\boxed{\text{ÜA 44}}$





Wind $\uparrow$, $\vec{w} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} 95$ (alles in km/h)

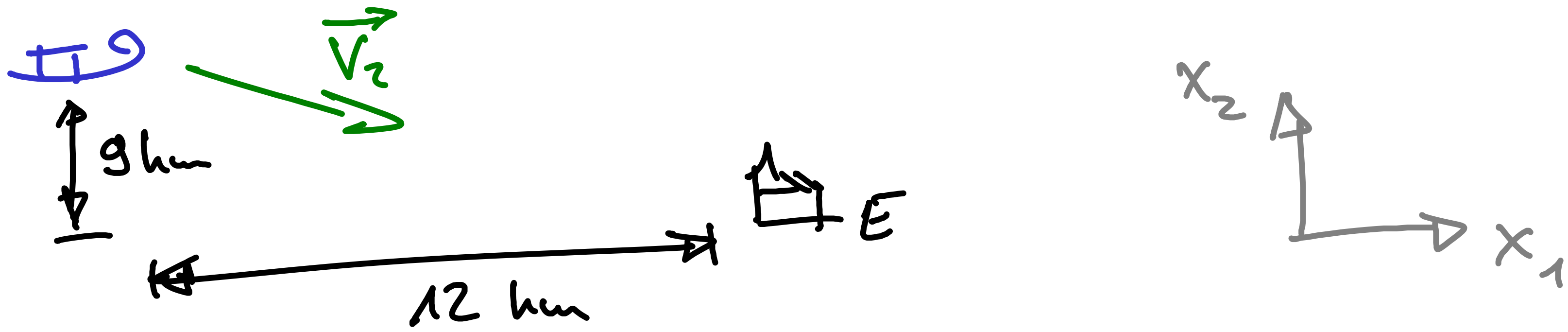
Schlitten über Grund $\searrow$, $\vec{v}_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix} 180$

a) $\vec{u}_1 + \vec{w} = \vec{v}_1 \Rightarrow \vec{u}_1 = \vec{v}_1 - \vec{w} = \begin{pmatrix} 180/\sqrt{2} \\ -180/\sqrt{2} - 95 \end{pmatrix}$

$$|\vec{u}_1| = \sqrt{\left(\frac{180}{\sqrt{2}}\right)^2 + \left(\frac{180}{\sqrt{2}} + 95\right)^2}$$

b) Für Rudolph kommt der Wind aus $-\frac{\vec{u}_1}{|\vec{u}_1|} =$ einsetzen ...

c)



$$\vec{w} = \begin{pmatrix}1\\0\end{pmatrix} 20$$

"max. mögl. Geschw." ⇝ $|\vec{u}_2| = |\vec{u}_1|$

"geradl. auf das Ziel" ⇝ $\vec{v}_2 = |\vec{v}_2| \frac{1}{5} \cancel{\begin{pmatrix}12\\-9\end{pmatrix}} \begin{pmatrix}4\\-3\end{pmatrix}$

$$\vec{u}_2 + \vec{w} = \vec{v}_2 \Leftrightarrow \vec{u}_2 = \vec{v}_2 - \vec{w}$$

mit $|\vec{u}_1| = |\vec{u}_2| = |\vec{v}_2 - \vec{w}|$

$$\underbrace{|\vec{u}_1|}_{\text{Zahl aus (a)}} = \underbrace{\sqrt{\vec{v}_2^{\,2} - 2\vec{v}_2\vec{w} + \vec{w}^{\,2}}}_{\text{hier steckt } |\vec{v}_2|}$$

besser $\vec{u}_1^{\,2} = |\vec{v}_2|^2 + \vec{w}^{\,2} - \underbrace{\cancel{16}^{\,32} |\vec{v}_2|}$

gesucht

gekannt

quadrat. Glu. für $|\vec{v}_2|$

$$-2\,\vec{v}_2\,\vec{w} = -2\,|\vec{v}_2|\,\frac{1}{5}\underbrace{\begin{pmatrix}4\\-3\end{pmatrix}\cdot\begin{pmatrix}1\\0\end{pmatrix}}_{4\cdot 1-3\cdot 0=4}\,20 = \cancel{16}^{\,-32}\,|\vec{v}_2|$$

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geom. Reihe $\quad \sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$ für $|x| < 1$

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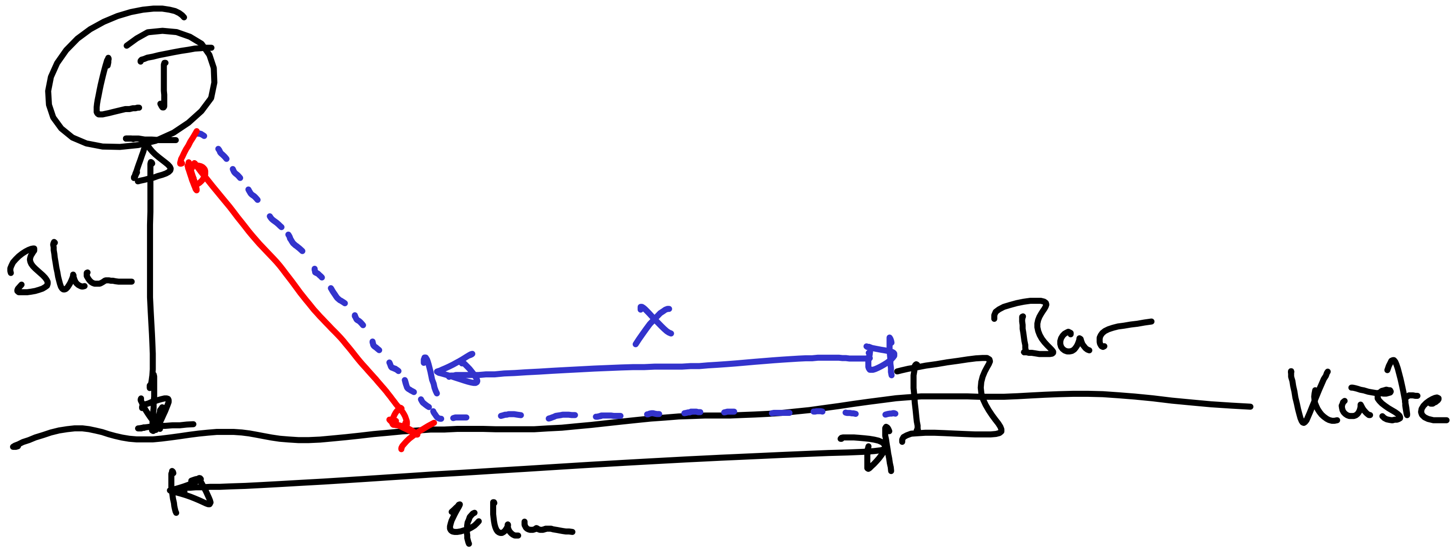
$\boxed{2}$



$v_{\text{radeln}} = 3$ (alles in km/h ... km)

$v_{\text{lauf}} = 5$

Zeit bis zur Bar

$$t(x) = \frac{1}{3}\sqrt{3^2 + (4-x)^2} + \frac{x}{5}$$

$$t'(x) = \frac{1}{3}\,\frac{1}{\cancel{2}}\,\frac{\cancel{2}(4-x)(-1)}{\sqrt{3^2+(4-x)^2}} + \frac{1}{5} \overset{!}{=} 0$$

$+\dfrac{4-x}{3\sqrt{\ldots}}$

dann

$\cdot \sqrt{}$

$$\Longleftrightarrow \quad \frac{1}{5}\sqrt{9+(4-x)^2} = \frac{4-x}{3}$$

$$\Longrightarrow \quad \frac{1}{25}\left(9+(4-x)^2\right) = \frac{(4-x)^2}{9} \qquad \Big| \cdot 9$$

$$\Longleftrightarrow \quad \frac{81}{25} + \frac{9(4-x)^2}{25} = (4-x)^2\,\frac{25}{25} \qquad \Big| - \frac{9}{25}(4-x^2)$$

$$\Big| \cdot 25/16$$

$$\Longleftrightarrow \quad (4-x)^2 = \frac{81}{16} \quad \Longleftrightarrow \quad |4-x| = \frac{9}{4}$$

$4-x$ pos., also $x = 4 - \frac{9}{4} = \frac{7}{4}$, $t\left(\frac{7}{4}\right) = \ldots$

$$t(0) = \frac{5}{3}$$

$$t(4) = 1 + \frac{4}{5} = \frac{9}{5} \qquad \Big|\ t(x) \text{ stetig} \ \Big|$$

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$\boxed{6}$ c) exponentieller Zerfall mit Faktor 0,95

$$L_t = (0{,}95)^t L_0$$

oder mit Hilfe von $\tilde{W}$

$$\tilde{W} = \begin{pmatrix} \frac{19}{20} & 0 \\ \dots & \dots \end{pmatrix}$$

$$\begin{pmatrix} L_{t+1} \\ N_{t+1} \end{pmatrix} = \tilde{W} \begin{pmatrix} L_t \\ N_t \end{pmatrix} = \begin{pmatrix} \frac{19}{20} \cdot L_t \\ \dots \end{pmatrix}$$

$$\Rightarrow L_{t+1} = \left(\frac{19}{20}\right) L_t = \left(\frac{19}{20}\right)^{t+1} L_0$$

zurück zu (a)

$$x = \frac{9}{10} \cdot \frac{1}{20} = \frac{9}{200}$$

Nadelbaum wächst nach

Laubbaum stirbt ab

c)

$$\tilde{W} = \begin{pmatrix} \frac{19}{20} & 0 \\ \frac{1}{20} & 1 \end{pmatrix}$$

$$N_{t+1} = \frac{1}{20} L_t + N_t$$

(iii) $L_t = (0{,}95)^t L_0 \overset{!}{=} \frac{1}{2} \cdot L_0$

$$\Leftrightarrow 0{,}95^t = \frac{1}{2} \Leftrightarrow e^{t \log(0{,}95)} = \frac{1}{2}$$

$$\Leftrightarrow \quad t \log(0{,}95) = -\log 2$$

$$\Leftrightarrow \quad t = -\frac{\log(2)}{\log(0{,}95)}$$

oder $\left(\frac{19}{20}\right)^t = \frac{1}{2} \Leftrightarrow e^{t \log\left(\frac{19}{20}\right)} = \frac{1}{2}$

$$\Leftrightarrow \quad t \log(19/20) = -\log 2$$

$$\Leftrightarrow \quad t = -\frac{\log 2}{\log(19/20)} = \frac{\log 2}{\log 20 - \log 19}$$

$$\underbrace{\log\left(\tfrac{1}{2}\right)}_{= \underbrace{\log 1}_{=0} - \log 2} = -\log 2$$

$$\log\left(\tfrac{1}{x}\right) = -\log x$$