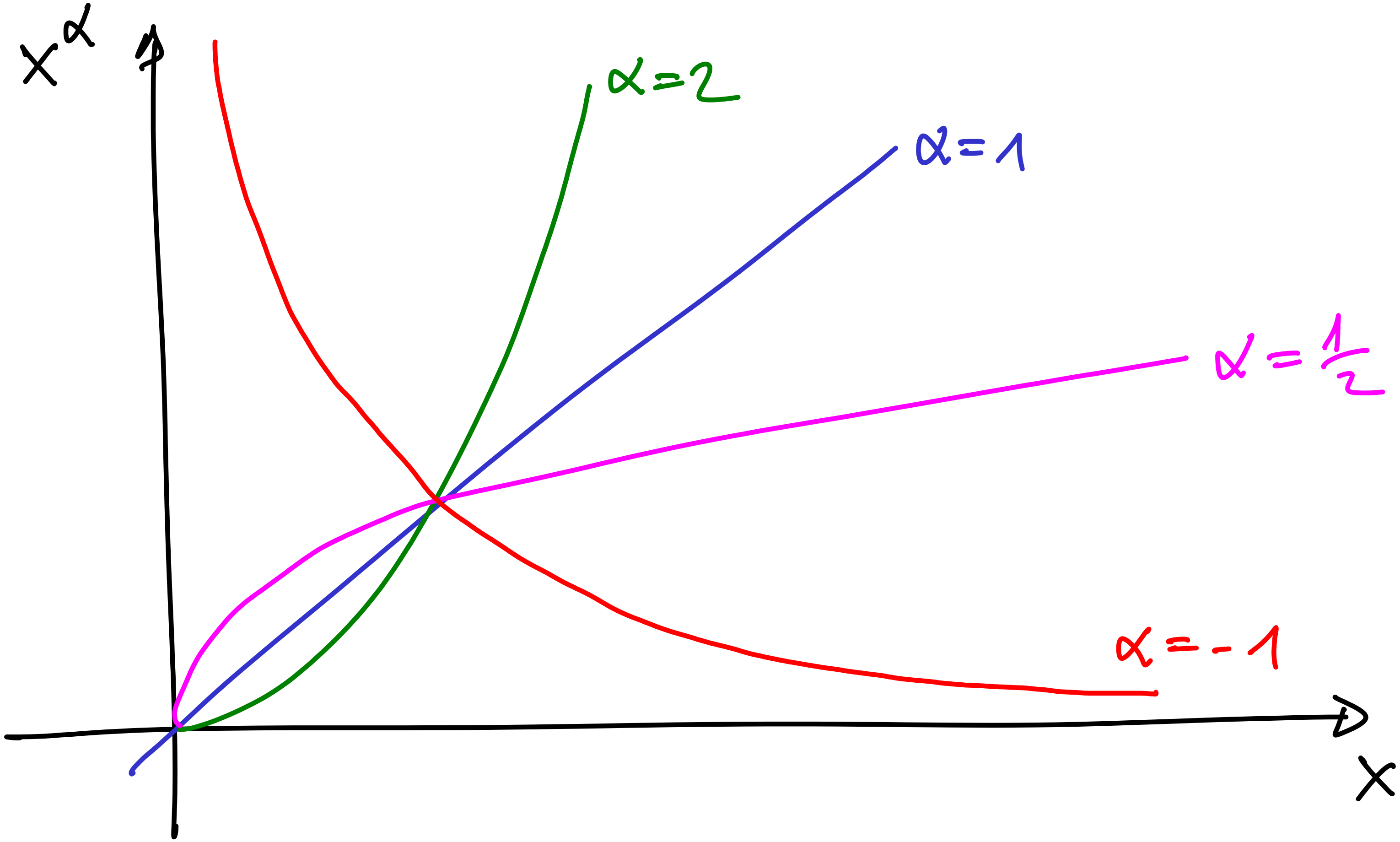
$$\sqrt[3]{9^{-2} \cdot 3} = \left(\frac{1}{9^2} \cdot 3 \right)^{1/3}$$

$$= \left(\frac{1}{(3^2)^2} \cdot 3 \right)^{1/3} = \left(\frac{1}{3^4} \cdot 3 \right)^{1/3} = \left(\frac{1}{3^3} \right)^{1/3} = \frac{1}{3}$$

$$= 3^{-1}$$



x^α
$\alpha = 2$
$\alpha = 1$
$\alpha = \frac{1}{2}$
$\alpha = -1$
x

$G(0) = 100€$

$\alpha = 1{,}06$ (6% Zinsen)

Schuld nach halben Jahr (t in Jahren)

$G(\frac{1}{2}) = \alpha^{1/2} \cdot G(0)$

$= \sqrt{1{,}06} \cdot 100€$

$\approx 1{,}0296 \cdot 100€ = 102{,}96€$

(nicht 3%, d.h. nicht $\frac{1}{2} \cdot 6\%$)

$$\alpha^t = e^{\gamma t} = (e^\gamma)^t$$

also $e^\gamma = \alpha$

$$\underset{\alpha = e^\gamma}{\alpha^{t/T} =} (e^\gamma)^{t/T} = e^{\gamma t/T} = e^{\frac{\gamma}{T} \cdot t} \underset{\frac{\gamma}{T} = \lambda}{=} e^{\lambda t}$$

$[t] = \text{Zeit}$ (Dimension)

$[\lambda] = \frac{1}{\text{Zeit}}$

$$G\left(\frac{1}{\lambda}\right) = e^{\lambda \cdot \frac{1}{\lambda}} G(0) = e \cdot G(0)$$

$$G(t) = e^{\lambda t} G(0)$$

Radioaktiver Zerfall

$G(t)$ Menge zu Beginn des Intervalls $[t, t+T]$

$G(t+T)$ " am Ende ——— " ———

messe z.B. G in # Atome, dann ist # Zerfälle:

$$Z(t) = G(t) - G(t+T)$$

$$= e^{-\lambda t} G(0) - \underbrace{e^{-\lambda(t+T)}}_{= e^{-\lambda t} \cdot e^{-\lambda T}} G(0)$$

$$= e^{-\lambda t} \underbrace{G(0)\left(1 - e^{-\lambda T}\right)}_{= Z(0),\ \#\ \text{Zerfälle in } [0, T]}$$

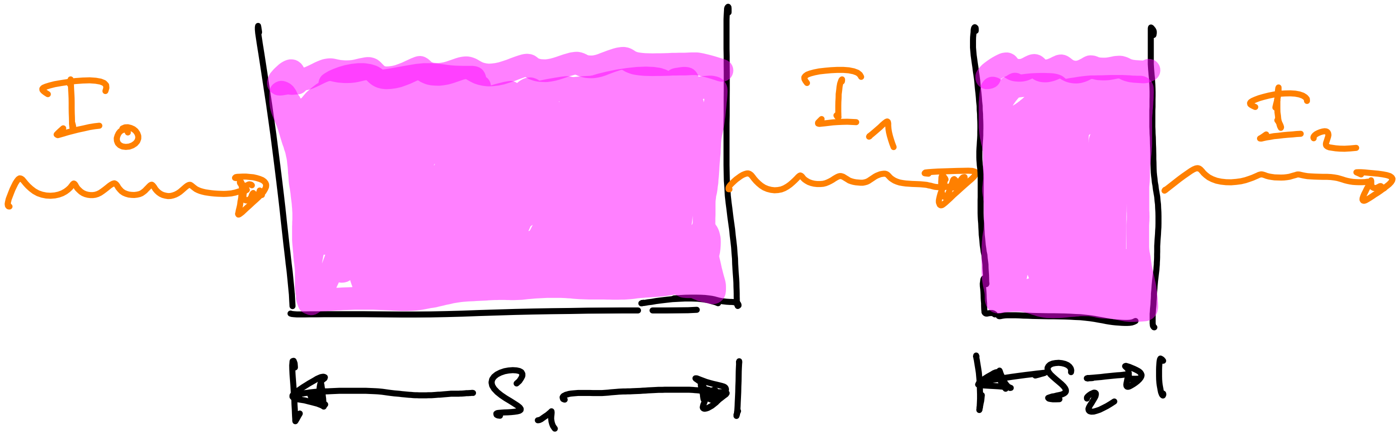
Anteil der zerfällt

$$\frac{G(t)-G(t+T)}{G(t)} = \frac{e^{-\lambda t}\cancel{G(0)} - e^{-\lambda(t+T)}\cancel{G(0)}}{e^{-\lambda t}\cancel{G(0)}}$$

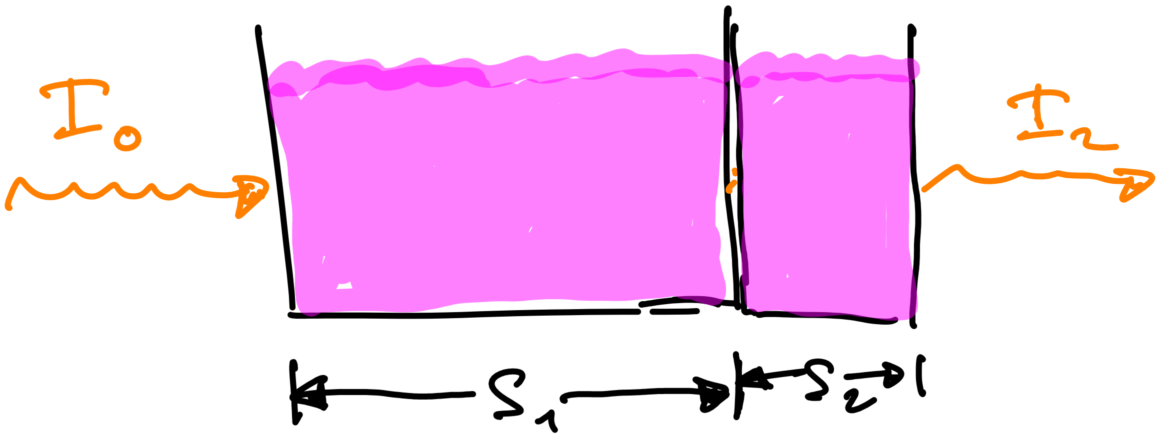
$$= 1 - e^{-\lambda T}$$ ← hängt nicht von t ab!

$$\text{Zähler} = \underline{e^{-\lambda t} G(0)} \left(1 - e^{-\lambda T}\right)$$

zu Lambert-Beer

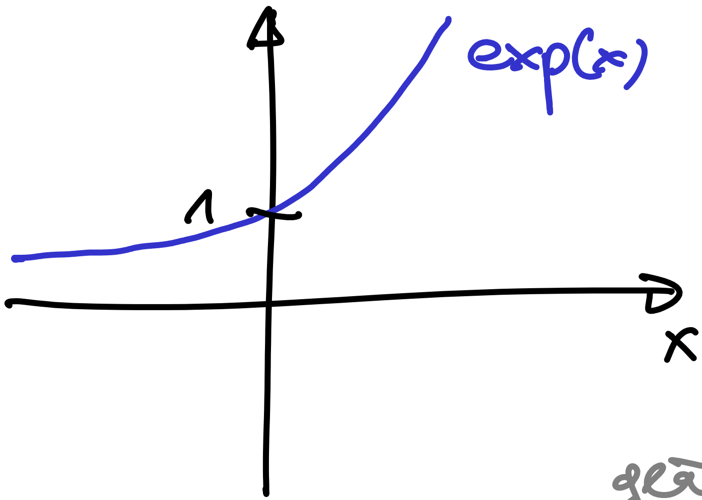


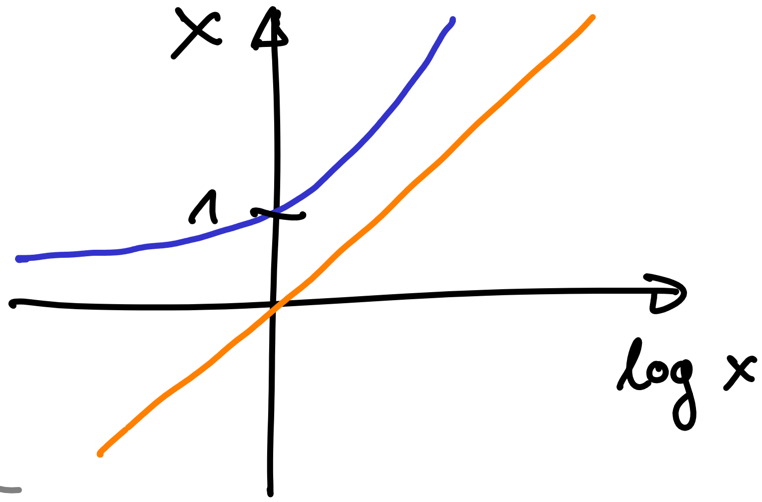
$$I_1 = \alpha_{s_1} \cdot I_0 \quad , \quad I_2 = \alpha_{s_2} \cdot I_1 = \alpha_{s_1} \cdot \alpha_{s_2} \cdot I_0$$



$$I_2 = \alpha_{s_1+s_2} \, I_0$$

d.h. $\alpha_{s_1+s_2} = \alpha_{s_1} \cdot \alpha_{s_2}$ also exponentieller Zerfall

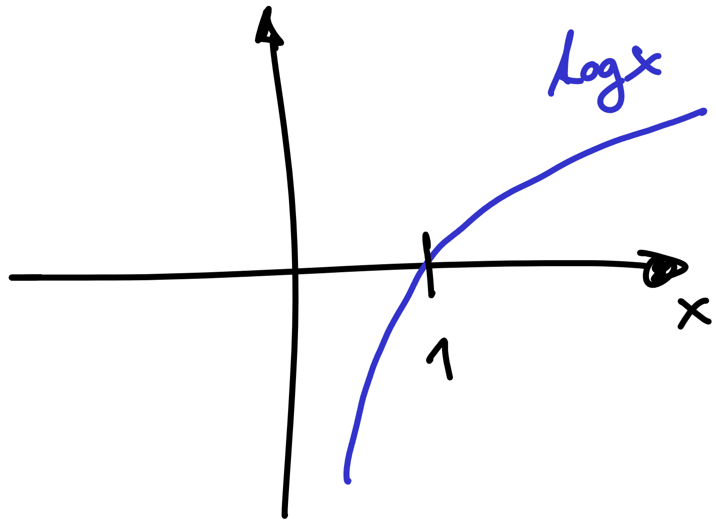




geänderte Achsenbeschriftung

$$\log(e^x) = \log(\exp(x)) = x$$

$$\exp(\log x) = x$$



log-Rechenregeln

① $\log(xy) = \log x + \log y$ Warum?

$$x = e^a,\ y = e^b \iff \log x = a,\ \log y = b$$

$$\log(xy) = \log(e^a \cdot e^b) \overset{P.R.}{=} \log(e^{a+b}) = a+b$$

$\uparrow$ log ist Umkehrfkt. von $e^{\dots}$

$$= \log x + \log y$$

② $\log(x^\alpha) = \alpha \cdot \log x$ Warum?

$$x = e^\gamma \iff \log x = \gamma$$

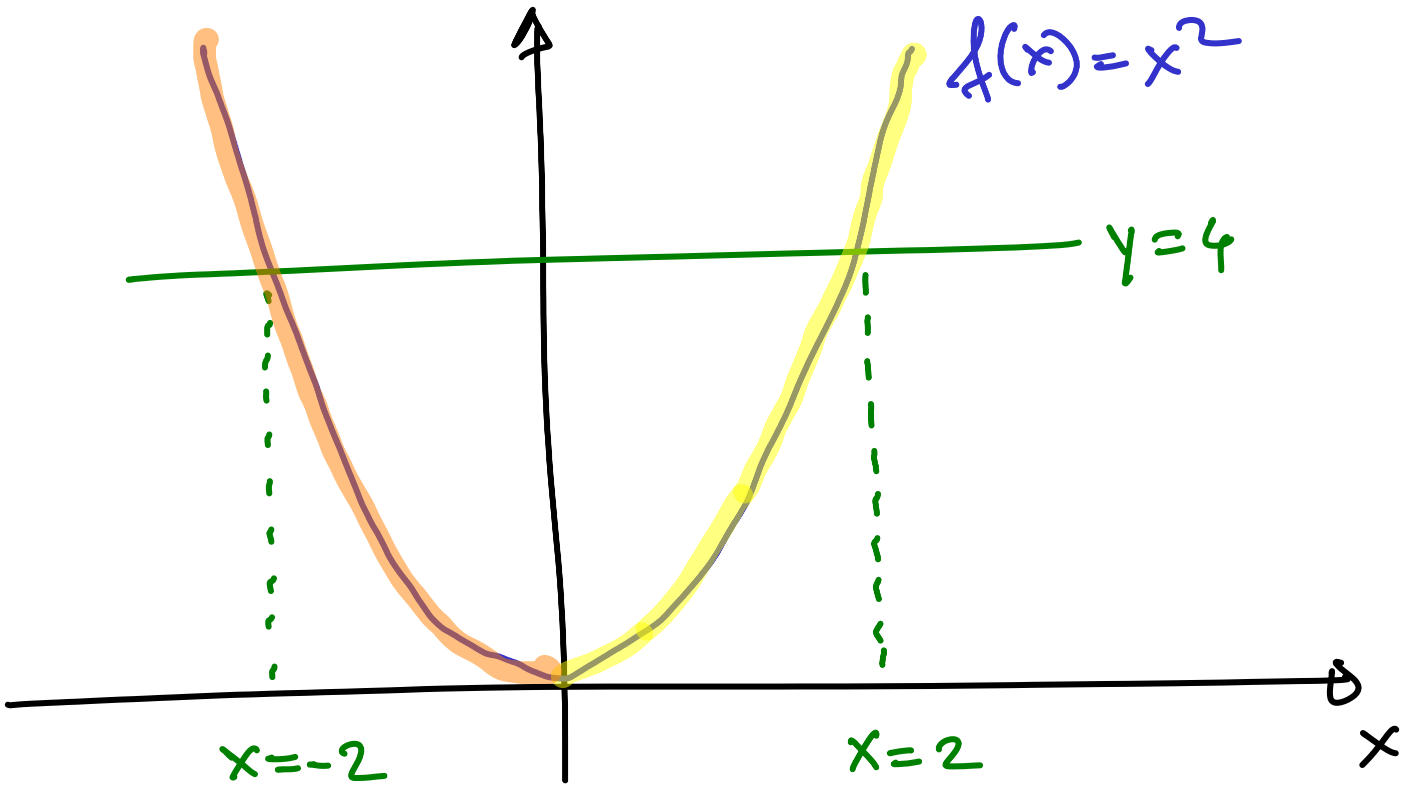
$$\log(x^\alpha) = \log((e^\gamma)^\alpha) \overset{P.R.}{=} \log(e^{\gamma \cdot \alpha}) = \gamma \cdot \alpha$$

$= \alpha \cdot \log x$

3 $\log\left(\frac{1}{x}\right) = -\log x$ 2 mit $\alpha = -1$

4 $\log(1) = \log(e^0) = 0$

$\log(e) = \log(e^1) = 1$



$f: \mathbb{R} \to \mathbb{R}_0^+$

$x \mapsto x^2$

nicht injektiv also nicht umkehrbar

$\tilde{f}: \mathbb{R}_0^+ \to \mathbb{R}_0^+$

$x \mapsto x^2$

ist bijektiv und damit umkehrbar $\tilde{f}^{-1}(x) = \sqrt{x}$

$\tilde{f} : \mathbb{R}_0^- \to \mathbb{R}_0^+$

$x \mapsto x^2$

auch umkehrbar

$\tilde{f}^{-1}(x) = -\sqrt{x}$

$\mathbb{R}_0^- = \{ x \in \mathbb{R} \mid x \leq 0 \}$

$\mathbb{R}^- = \{ x \in \mathbb{R} \mid x < 0 \}$