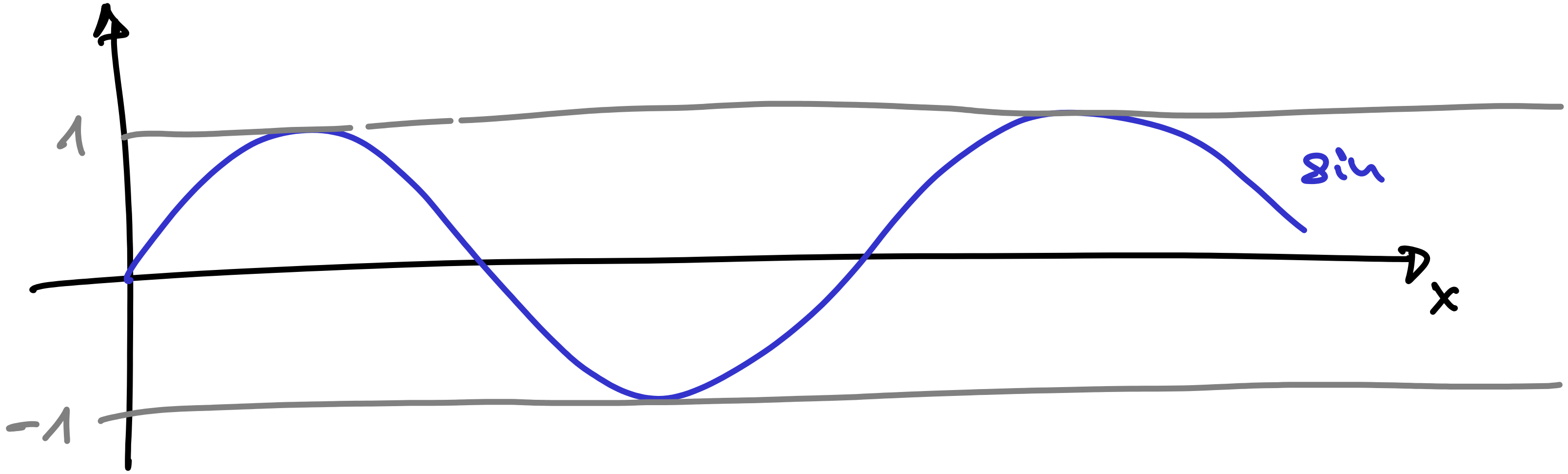
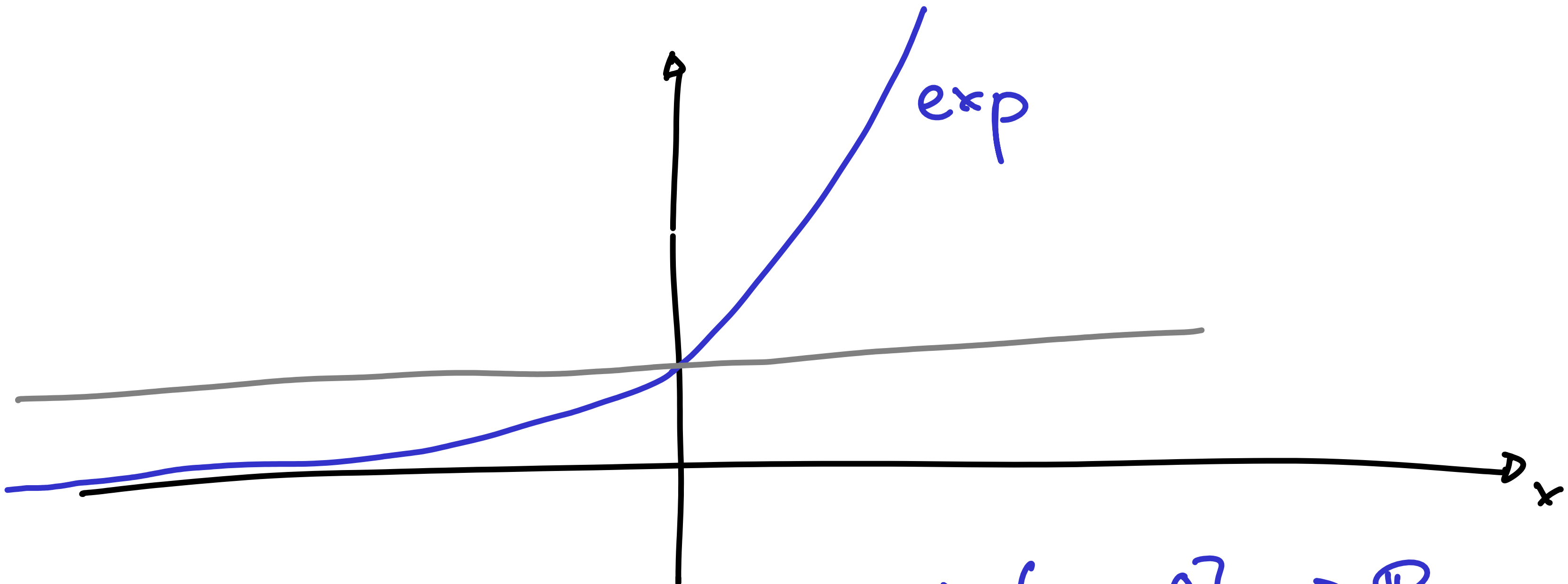
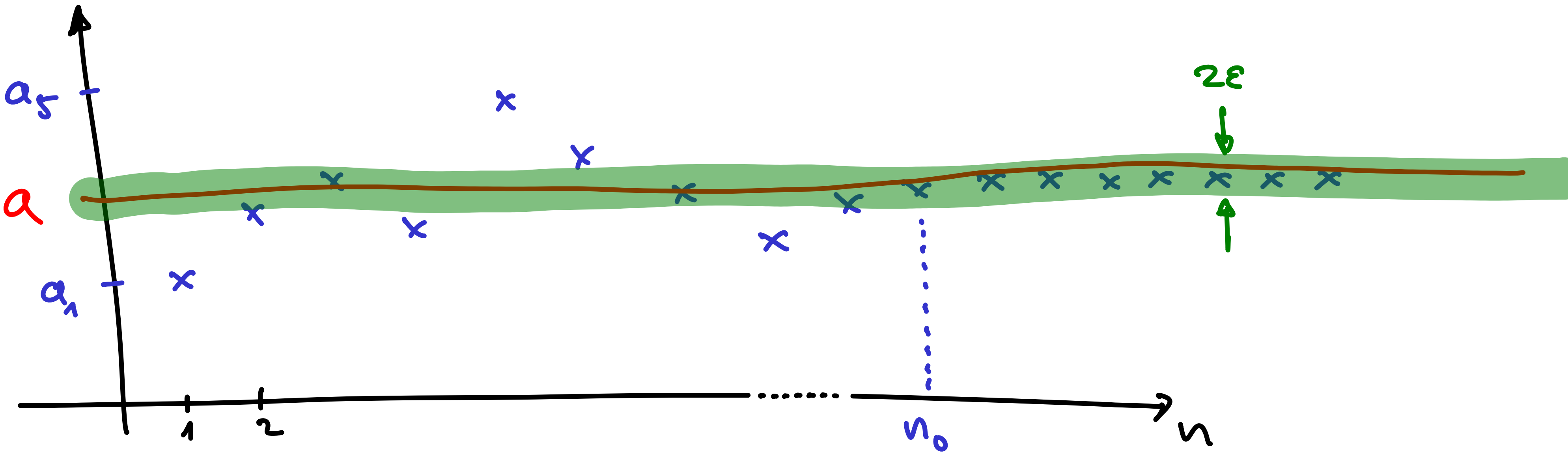


$r = 1$



$\exp : (-\infty, 0] \to \mathbb{R}$

beschränkt : $|\exp(x)| \leq 1 \quad \forall x \in (-\infty, 0]$



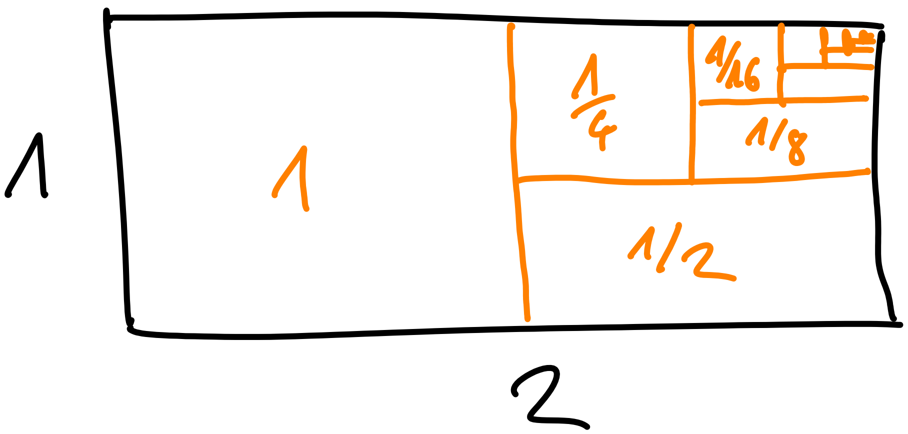
$a_n = \frac{1}{n}$, $n \in \mathbb{N}$

Behauptung: $\lim\limits_{n \to \infty} a_n = 0$

$|a_n - 0| = |\frac{1}{n} - 0| = |\frac{1}{n}| = \frac{1}{n} \overset{!}{<} \varepsilon$ hätte wir gerne

$\Leftrightarrow \quad n > \frac{1}{\varepsilon}$

Wähle $n_0 \geq \frac{1}{\varepsilon}$, dann klappt's ☺



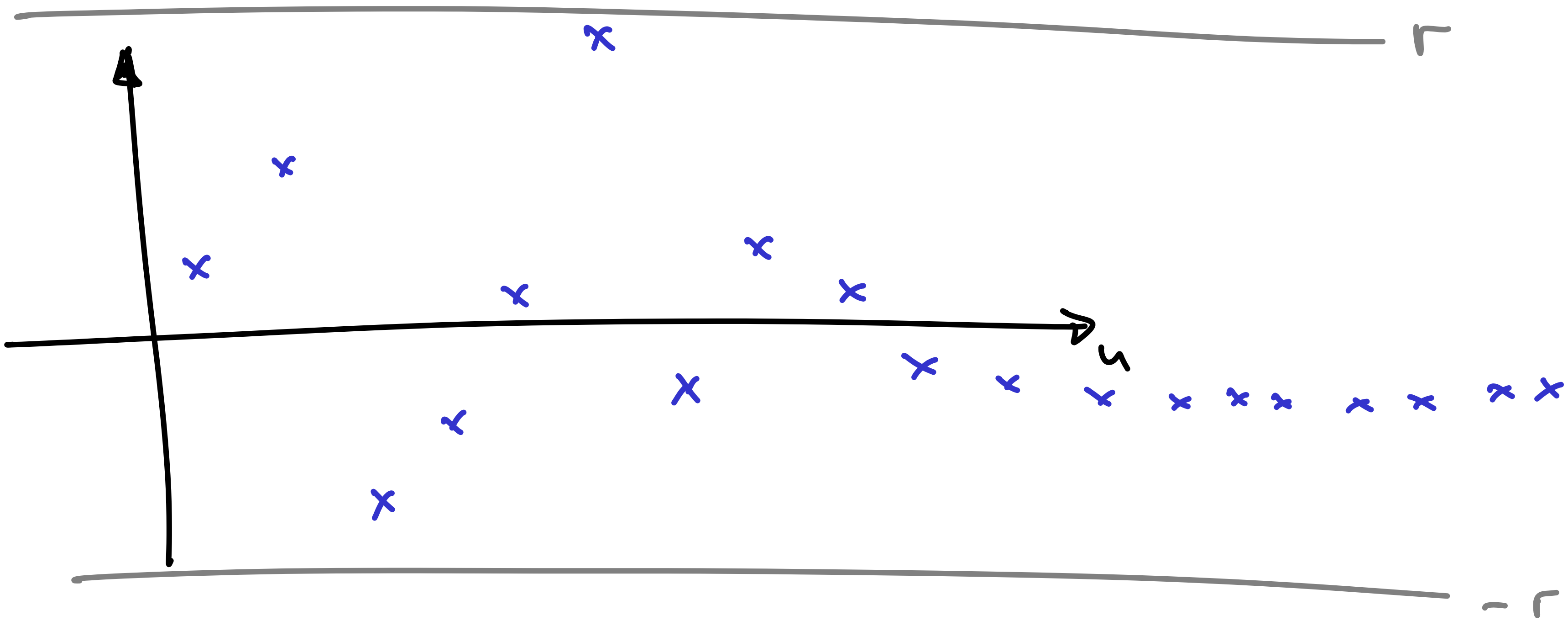
Fläche: $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \dots = 2$

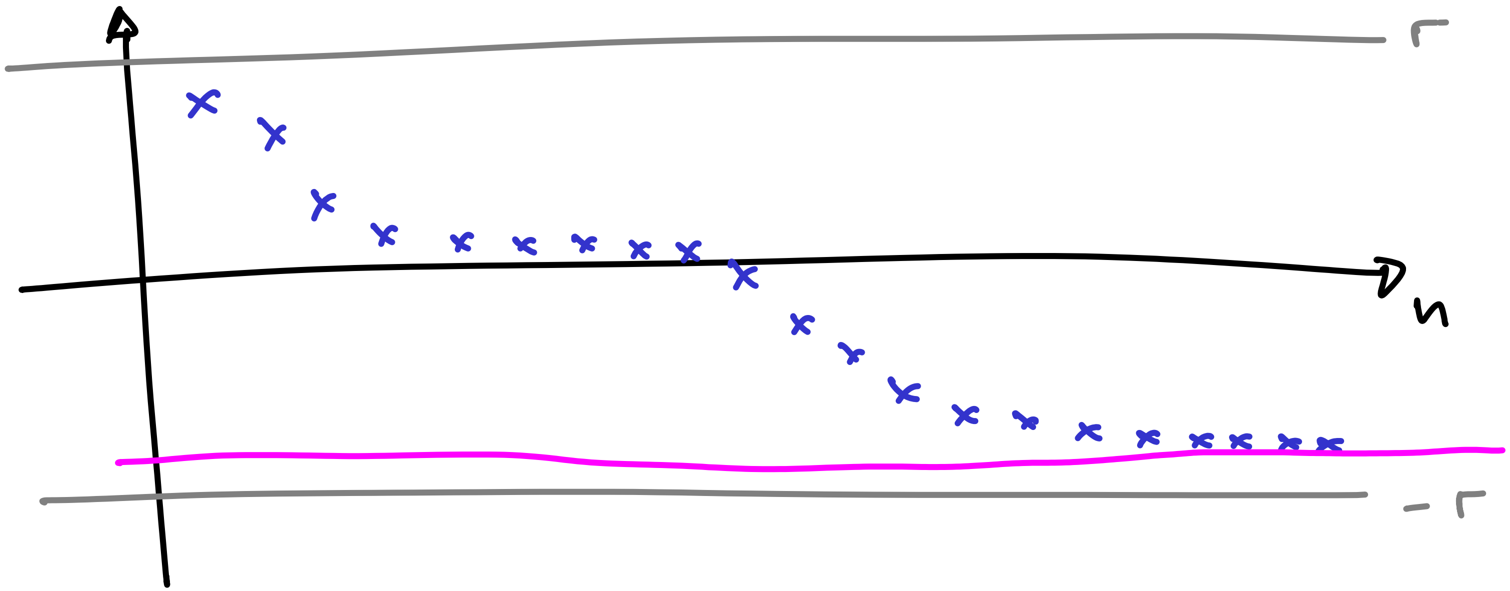
$$\sum_{k=0}^{3} \left(\frac{1}{2}\right)^k = \left(\frac{1}{2}\right)^0 + \left(\frac{1}{2}\right)^1 + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^3$$

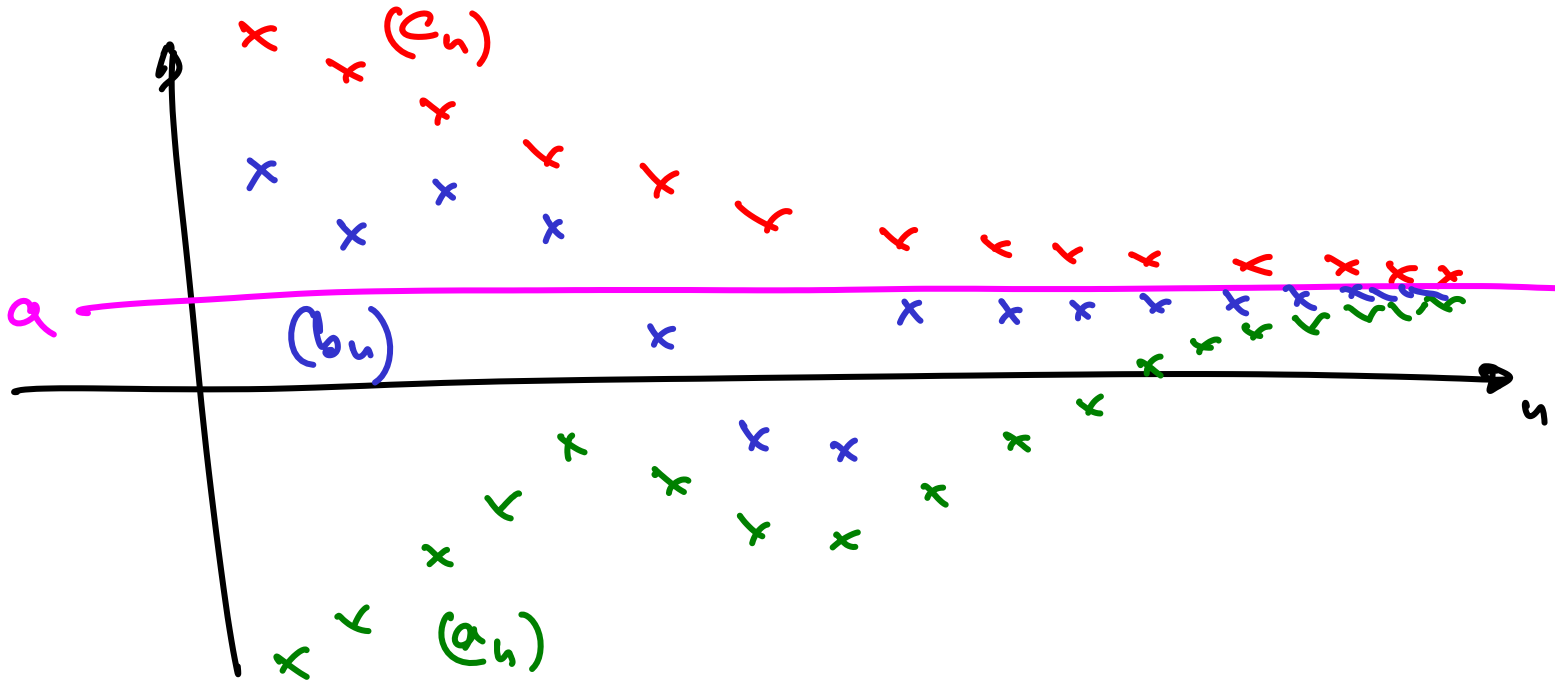
$$= 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8}$$

$$\sum_{k=0}^{\infty} \left(\frac{1}{2}\right)^k = 2$$

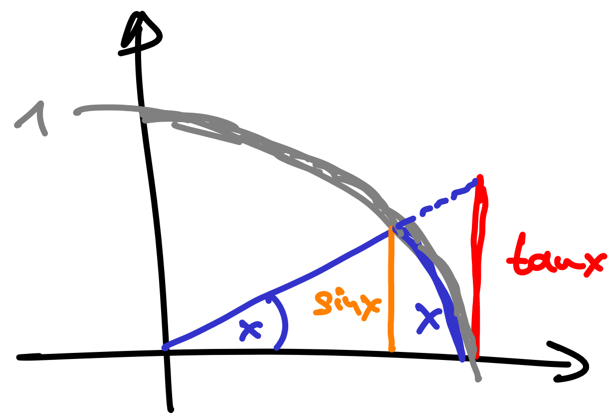
$$\vec{a}_n = \begin{pmatrix} 5 - 1/n \\ \sum_{k=0}^{n} \left(\frac{1}{2}\right)^k \end{pmatrix} \xrightarrow[n \to \infty]{} \begin{pmatrix} 5 \\ 2 \end{pmatrix}$$


r
n
-r


r
n
-r



$$\frac{\sin\left(\frac{1}{n}\right)}{1/n} = n \cdot \sin\left(\frac{1}{n}\right) \xrightarrow[n\to\infty]{} \; ?$$



for $0 < x < \frac{\pi}{2}$

$$\sin x < x < \tan x$$

$$\Leftrightarrow \quad \frac{1}{\sin x} > \frac{1}{x} > \frac{1}{\tan x} = \frac{\cos x}{\sin x} \quad \Big| \cdot \sin x$$

$$\Leftrightarrow \quad 1 > \frac{\sin x}{x} > \cos x$$

d.h.

$$1 > \frac{\sin\left(\frac{1}{n}\right)}{\frac{1}{n}} > \cos\left(\frac{1}{n}\right) \xrightarrow[n\to\infty]{} \cos(0) = 1$$

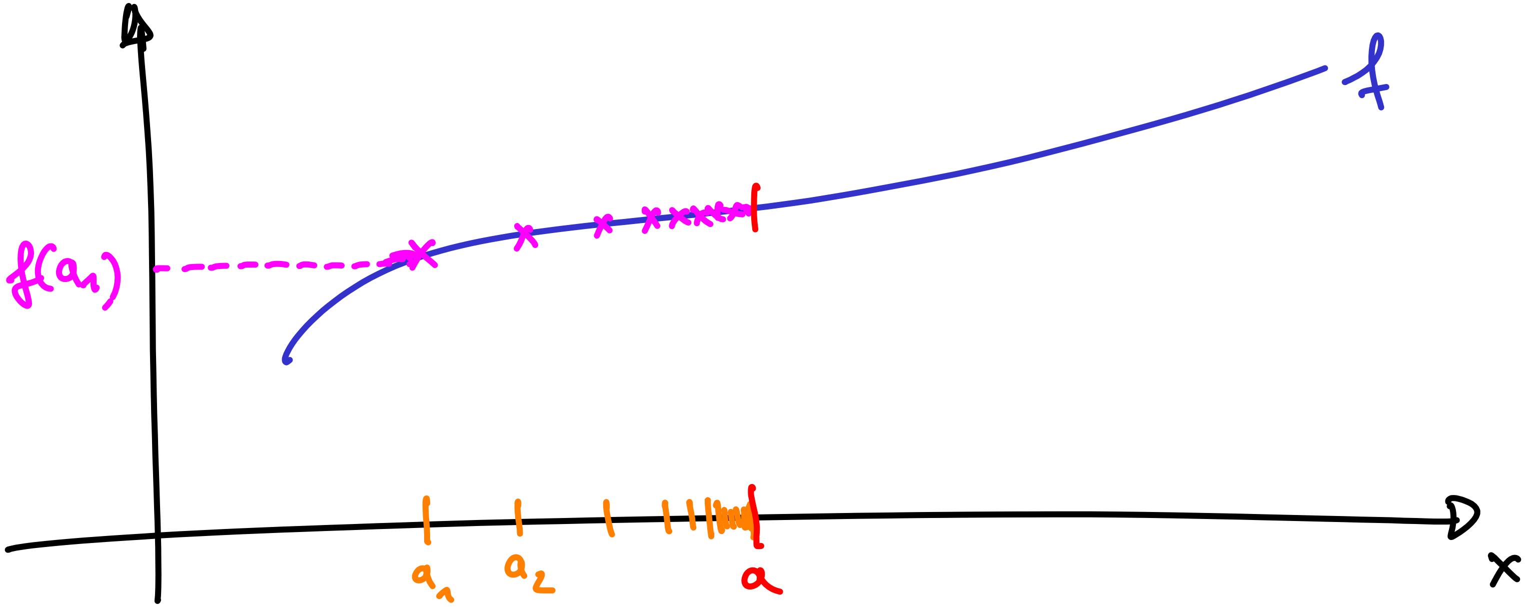
$$\Rightarrow \quad \lim_{n\to\infty} \frac{\sin\left(\frac{1}{n}\right)}{\frac{1}{n}} = 1$$

$$\lim_{n\to\infty} \frac{n^{42} - 7n^8}{8n^5 + 3n^{42}} = \lim_{n\to\infty} \frac{1 - 7\frac{1}{n^{34}}}{8\frac{1}{n^{37}} + 3}$$

$$= \frac{\left(\lim\limits_{n\to\infty} 1\right) - 7\left(\lim\limits_{n\to\infty}\left(\frac{1}{n^{34}}\right)\right)}{8\left(\lim\limits_{n\to\infty}\left(\frac{1}{n^{37}}\right)\right) + \left(\lim\limits_{n\to\infty} 3\right)} = \frac{1 - 7\cdot 0}{8\cdot 0 + 3} = \frac{1}{3}$$

aber nicht:

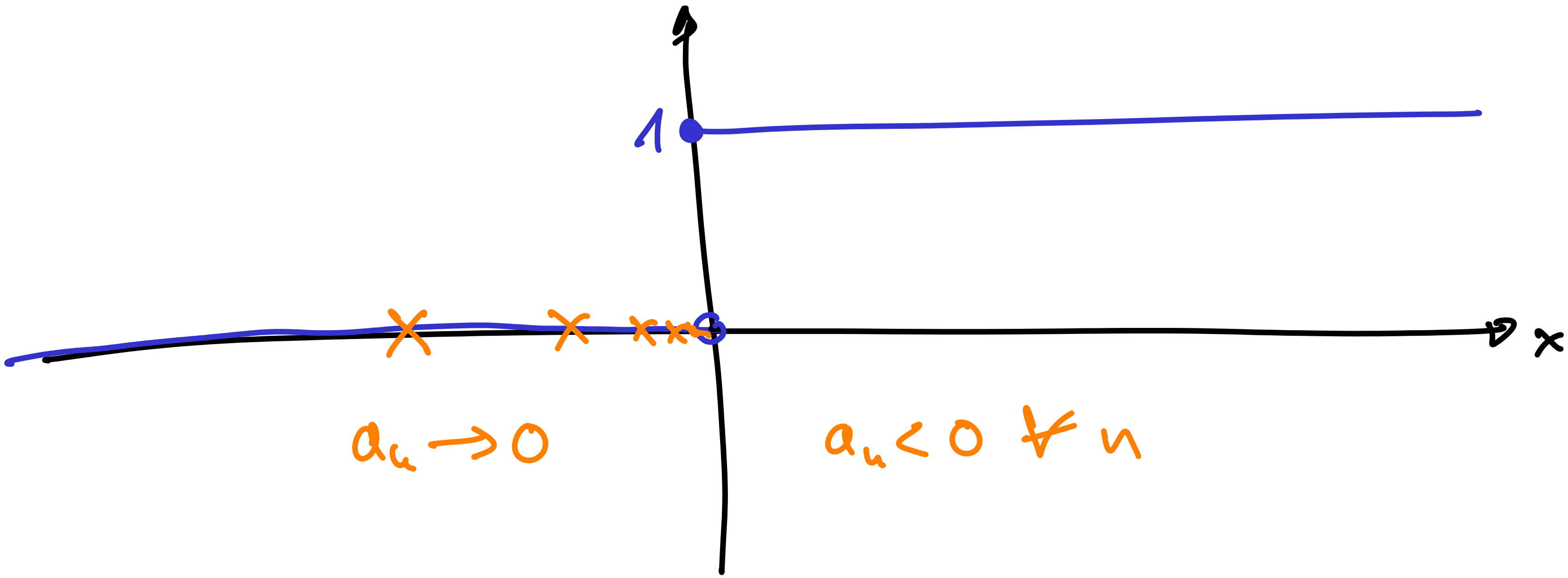
$$\lim_{n\to\infty} n\cdot\sin\left(\frac{1}{n}\right) \neq \underbrace{\left(\lim_{n\to\infty} n\right)}_{\infty}\cdot\underbrace{\left(\lim_{n\to\infty} \sin\left(\frac{1}{n}\right)\right)}_{0}$$



$$\lim_{n\to\infty} f(a_n) = f\left(\lim_{n\to\infty} a_n\right) = f(a)$$

Stetigkeit

$$\Theta(x) = \begin{cases} 1, & x \geq 0 \\ 0, & x < 0 \end{cases}$$



z.B. $a_n = -\frac{1}{n}$

$$\lim_{n\to\infty}\Theta(a_n) = \lim_{n\to\infty} 0 = 0$$

$$\neq$$

$$\Theta\left(\lim_{n\to\infty} a_n\right) = \Theta(0) = 1$$

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geom. Folge $(1), \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16} \ldots \to 0$

$(-\frac{1}{3}), \frac{1}{9}, -\frac{1}{27}, \frac{1}{81}, \ldots \to 0$

$$\lim_{n\to\infty} q^n = 0 \quad \forall\, |q| < 1$$

geom. Summe

$$S_n = \sum_{k=0}^{n} q^k = 1 + q + q^2 + \dots + q^n$$

$$q \cdot S_n = q + q^2 + q^3 + \dots + q^{n+1}$$

Differenz:

$$S_n - q \cdot S_n = 1 - q^{n+1} \quad \Big| \cdot \frac{1}{1-q} \qquad q \neq 1$$

$$\Longleftrightarrow \quad S_n = \frac{1-q^{n+1}}{1-q}$$

d.h.

$$\boxed{\sum_{k=0}^{n} q^k = \frac{1-q^{n+1}}{1-q} \quad \forall\, q \neq 1}$$

also für $|q| < 1$:

$$\sum_{k=0}^{\infty} q^k = \lim_{n\to\infty} \sum_{k=0}^{n} q^k = \lim_{n\to\infty} \frac{1-q^{n+1}}{1-q} = \frac{1}{1-q} \quad \text{geom. Reihe}$$