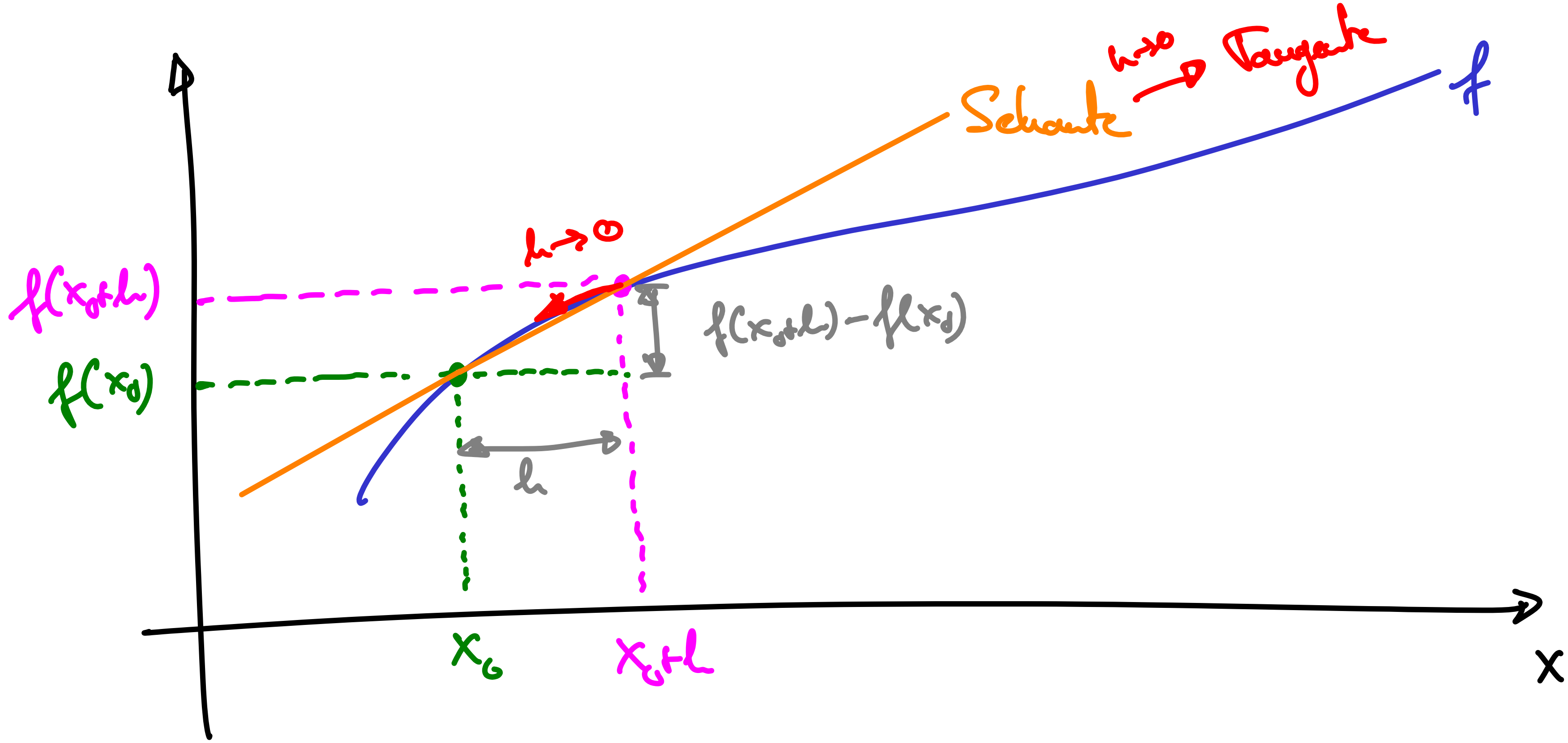


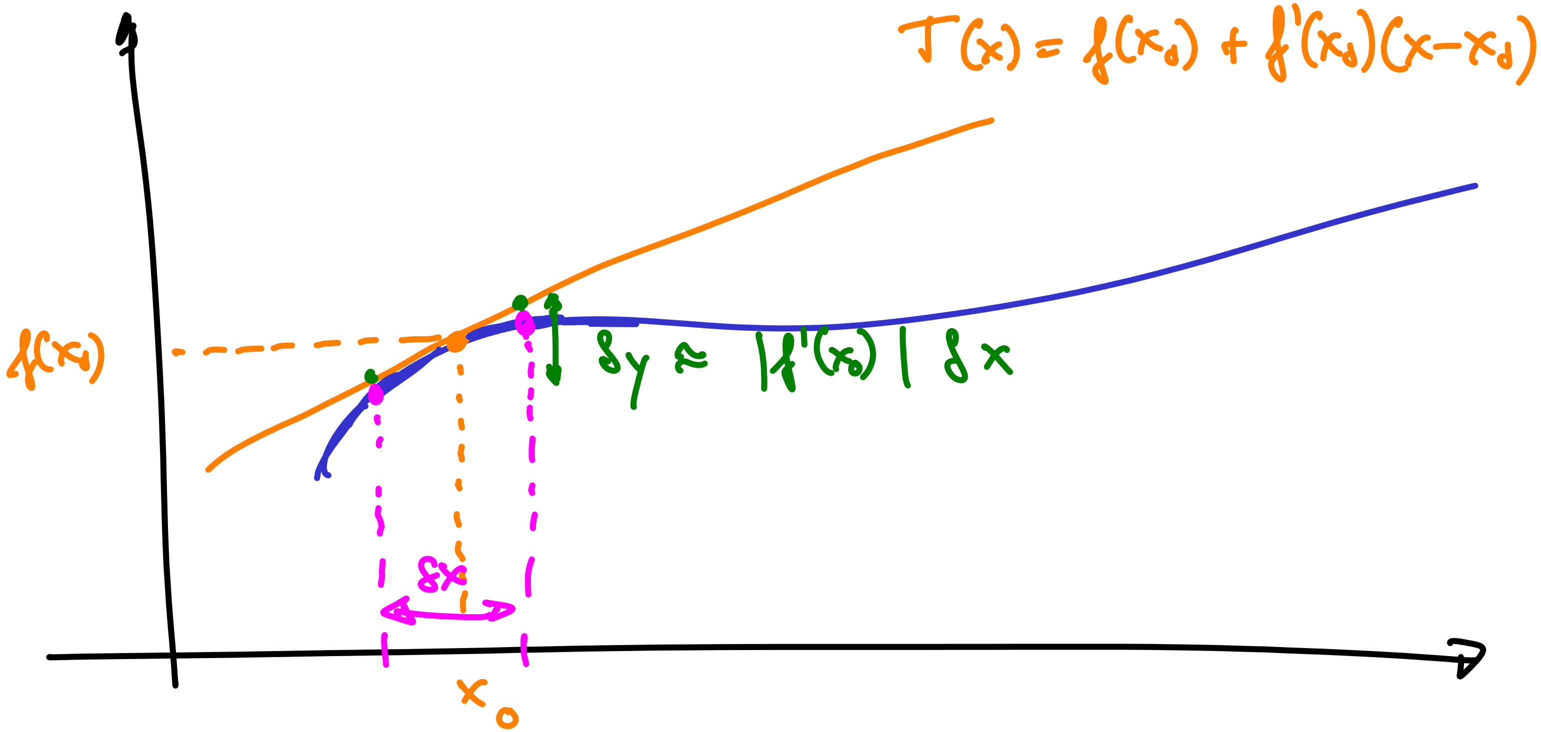
Notation:

$$f' = \frac{df}{dx} = \frac{d}{dx} f$$

$$f''(x) = f^{(2)}(x) = \frac{d^2 f}{dx^2}(x)$$

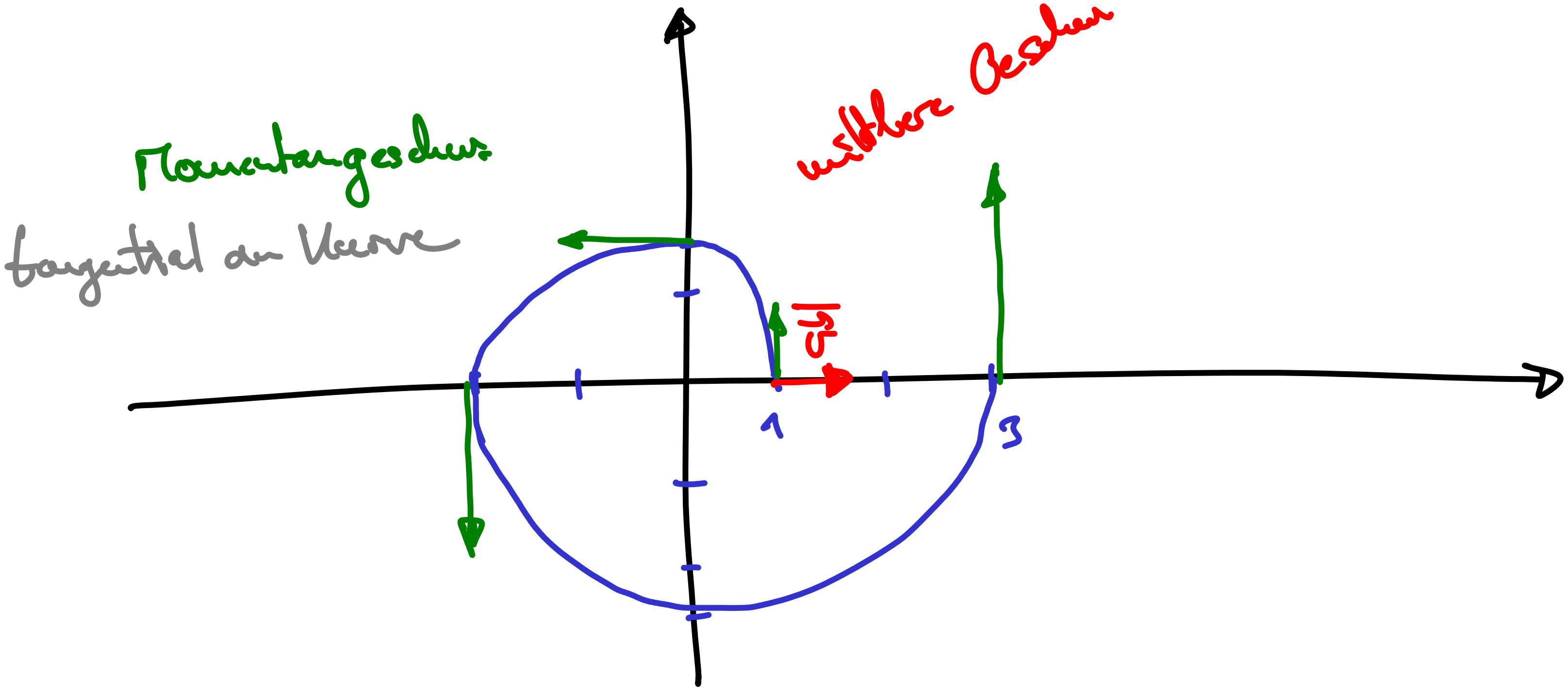
$$f'''''(x) = f^{(5)}(x)$$

$$\dot{x}(t) = \frac{dx}{dt}(t)$$



$$x \in \left[x_0 - \frac{\delta x}{2},\ x_0 + \frac{\delta x}{2}\right]$$

Kurve $\vec{x}: [0,2\pi] \to \mathbb{R}^2$



<u>als Formeln:</u> zunächst Kreis

$$\vec{x}(t) = \begin{pmatrix} r\cos t \\ r\sin t \end{pmatrix}, \quad 0 \le t \le 2\pi$$

Radius

<u>Spirale:</u>

$$\vec{x}(t) = \begin{pmatrix} \left(1+\frac{t}{\pi}\right)\cos t \\ \left(1+\frac{t}{\pi}\right)\sin t \end{pmatrix}, \quad 0 \le t \le 2\pi$$

$$\dot{\vec{x}} = \begin{pmatrix} \frac{1}{\pi}\cos t + \left(1+\frac{t}{\pi}\right)(-\sin t) \\ \frac{1}{\pi}\sin t + \left(1+\frac{t}{\pi}\right)\cos t \end{pmatrix}$$

$$\vec{v} = \frac{\vec{x}(2\pi) - \vec{x}(0)}{2\pi} = \frac{\begin{pmatrix}3\\0\end{pmatrix} - \begin{pmatrix}1\\0\end{pmatrix}}{2\pi} = \frac{1}{\pi}\begin{pmatrix}1\\0\end{pmatrix}$$

Bsp. für Ableitung

$$f(x) = \frac{1}{x}$$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{1}{h}\left(\frac{1}{x+h}\cdot\frac{x}{x} - \frac{1}{x}\cdot\frac{x+h}{x+h}\right)$$

$$= \lim_{h\to 0} \frac{1}{h}\left(\frac{x-(x+h)}{x(x+h)}\right) = \lim_{h\to 0} \frac{(-\cancel{h})}{\cancel{h}\,x(x+h)} = -\frac{1}{x^2}$$

$f(x) = a^x$

$$f'(x) = \lim_{h\to 0} \frac{a^{x+h} - a^x}{h} = \lim_{h\to 0} \frac{a^x \cdot a^h - a^x}{h}$$

$$= a^x \underbrace{\lim_{h\to 0} \frac{a^h - 1}{h}}_{\text{hängt nicht von } x \text{ ab}}$$

für $a = e = 2{,}718\ldots$ gerade $= 1$

d.h. $\lim_{h\to 0} \frac{e^h - 1}{h} = 1$

$f(x) = \sin x$

$$f'(x) = \lim_{h\to 0} \frac{\sin(x+h) - \sin x}{h} = \lim_{h\to 0} \frac{\sin x \cos h + \cos x \sin h - \sin x}{h}$$

$$= \sin x \lim_{h\to 0} \frac{\cos h - 1}{h} + \cos x \lim_{h\to 0} \frac{\sin h}{h}$$ $= 1$ (letzte Woche)

bleibt noch!

$$\lim_{h\to 0} \frac{\cos h - 1}{h} = \lim_{h\to 0} \frac{\sqrt{1 - \sin^2 h} - 1}{h} \cdot \frac{\sqrt{1-\sin^2 h} + 1}{\sqrt{1-\sin^2 h} + 1}$$

$$= \lim_{h\to 0} \frac{1-\sin^2 h - 1}{h\left(\sqrt{1-\sin^2 h}+1\right)} = \lim_{h\to 0} \underbrace{\frac{\sin h}{h}}_{\to 1} \underbrace{\frac{(-\sin h)}{\sqrt{1-\sin^2 h}+1}}_{\to 0}$$

ausgesagt $\quad (\sin x)' = \cos x$

Produktregel

$$(f\cdot g)'(x) = \lim_{h\to 0} \frac{f(x+h)g(x+h) - f(x)g(x)}{h}$$

$$= \lim_{h\to 0} \frac{[f(x+h) - f(x)]\, g(x+h) + f(x)g(x+h) - f(x)g(x)}{h}$$

$$= \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} g(x+h) + \lim_{h\to 0} f(x) \frac{g(x+h)-g(x)}{h}$$

$$= f'(x)\, g(x) + f(x)\, g'(x)$$

Schreibweise bei Kettenregel

$$(f \circ g)(x) = f(g(x))$$

"nach"

Bsp: $f(x) = e^x$, $g(x) = -\lambda x$ | $f'(x) = e^x$ | $g'(x) = -\lambda$

$$f(g(x)) = e^{-\lambda x} =: h(x)$$

$$h'(x) = f'(g(x)) \cdot g'(x)$$

$$= f'(-\lambda x) \cdot (-\lambda)$$

$$= e^{-\lambda x} \cdot (-\lambda)$$

$$f(x) = \cos(x^2)$$

$$f'(x) = \underbrace{-\sin}_{\text{äußere Abl.}}(x^2) \cdot \underbrace{2x}_{\text{innere Ableitung}}$$

Ableitung der Umkehrfkt.

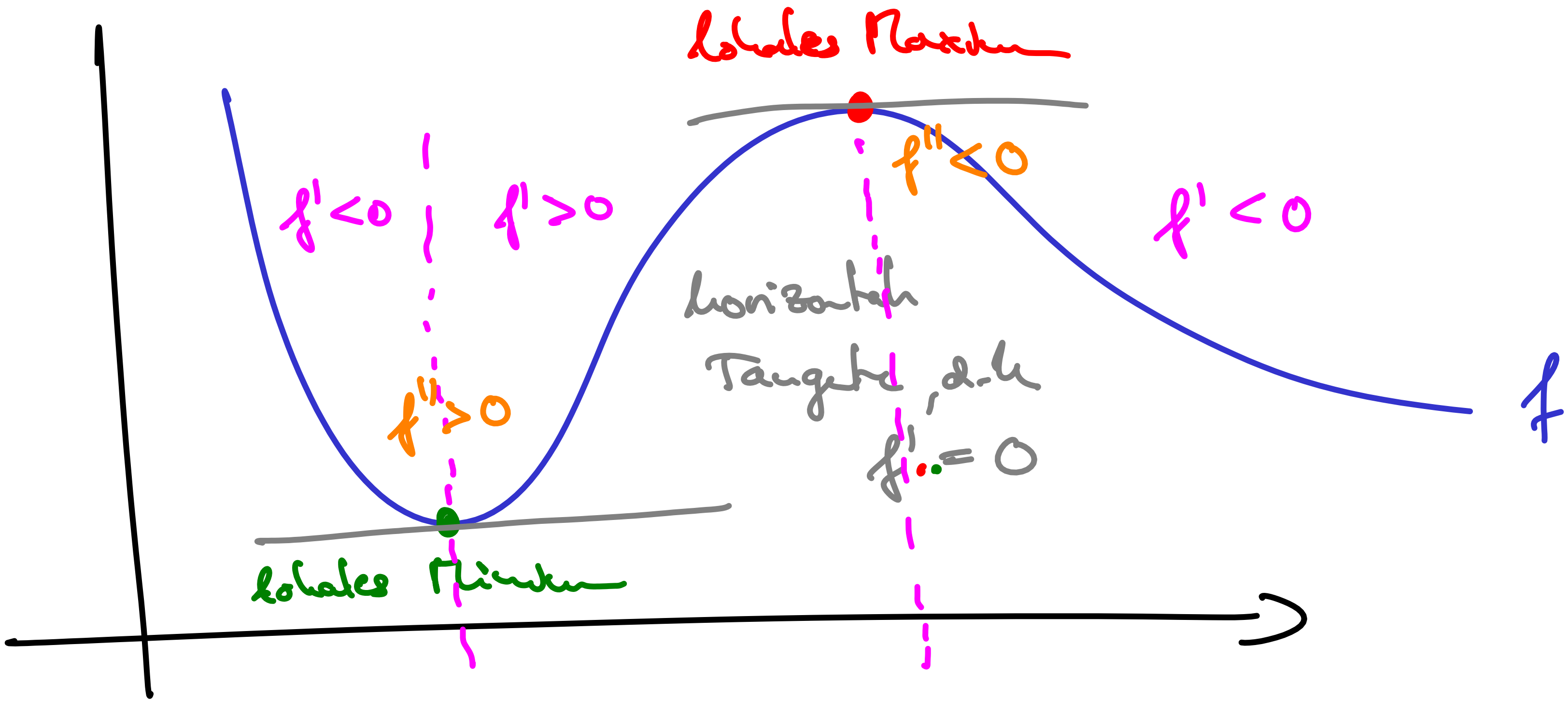
$$(f \circ f^{-1})'(x) = \frac{d}{dx} \underline{f(f^{-1}(x))} = \frac{d}{dx} \underline{x}$$

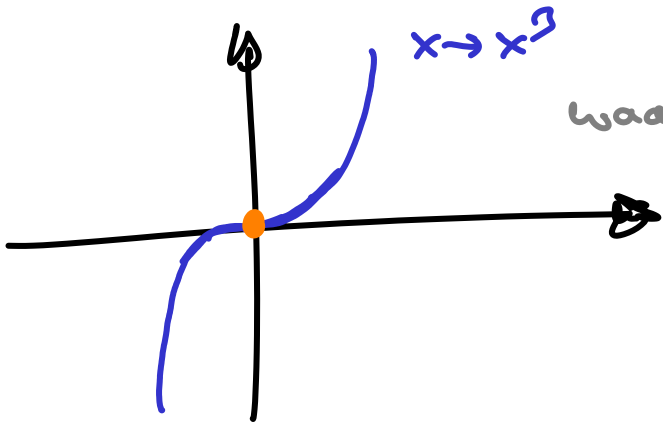
$$= f'(f^{-1}(x)) \cdot f^{-1\,\prime}(x) = 1$$

$$\Rightarrow \quad f^{-1\,\prime}(x) = \frac{1}{f'(f^{-1}(x))} \qquad \text{falls } f'(f^{-1}(x)) \neq 0$$

Bsp: $f(x) = \exp(x)$, $f^{-1}(x) = \log(x)$

$$\log'(x) = \frac{1}{\exp'(\log(x))} = \frac{1}{\exp(\log x)} = \frac{1}{x}$$





waagr. Tangente aber kein Min/Max