$$f: \mathbb{R} \to \mathbb{R}^2$$
$$x \mapsto \begin{pmatrix} \sin x \\ e^x - x \end{pmatrix}$$

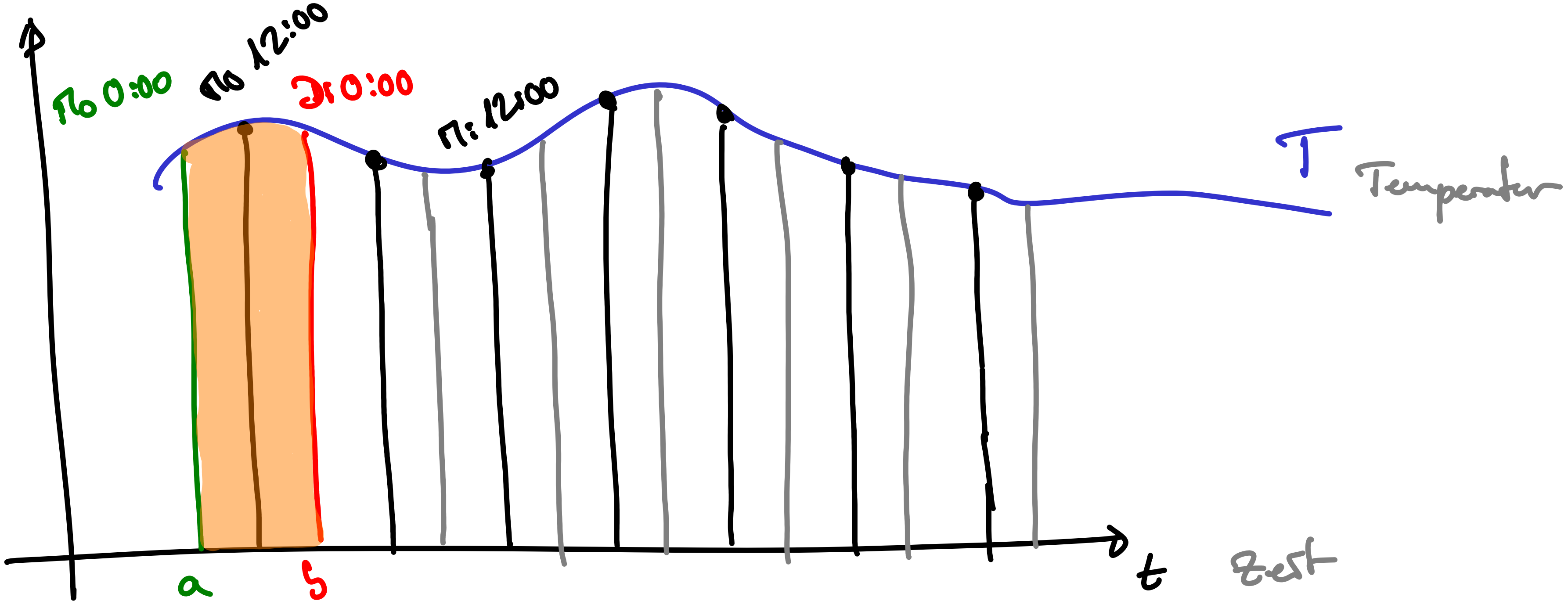
ges.: F mit $F' = f$

$$F(x) = \begin{pmatrix} -\cos x \\ e^x - \frac{1}{2}x^2 \end{pmatrix}$$

weitere Stammfkt. z.B.

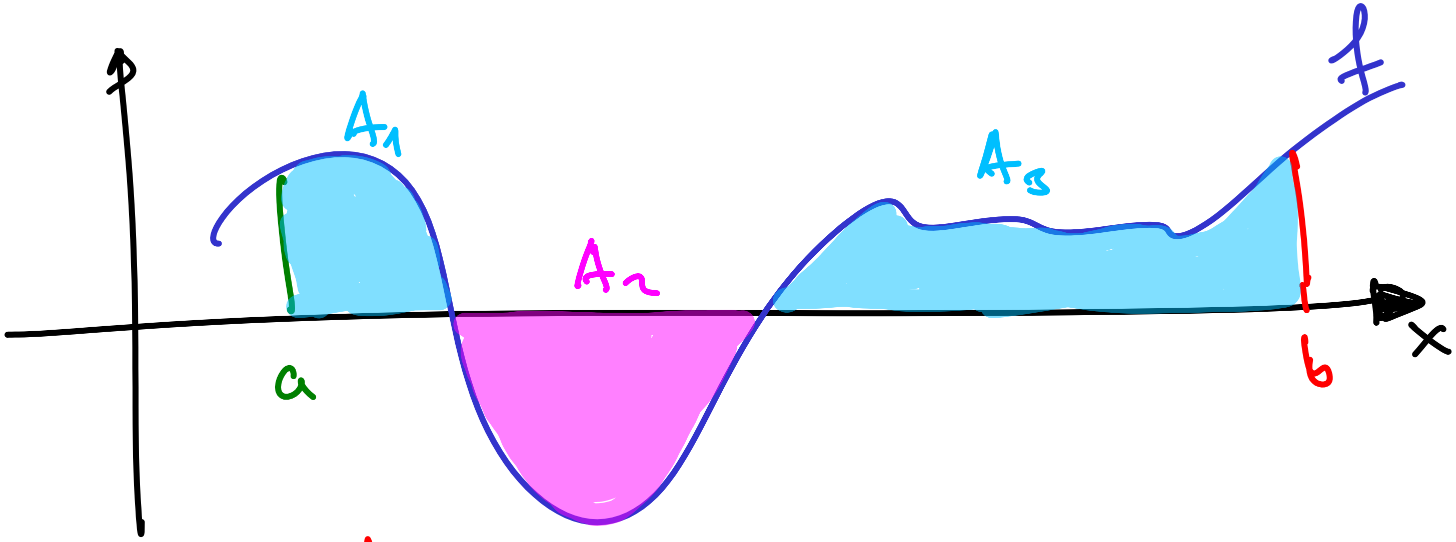
$$\tilde{F}(x) = \begin{pmatrix} \pi - \cos x \\ e^x - \frac{1}{2}x^2 + 2015 \end{pmatrix}$$

$$\tilde{F}'(x) = F'(x) = f(x)$$



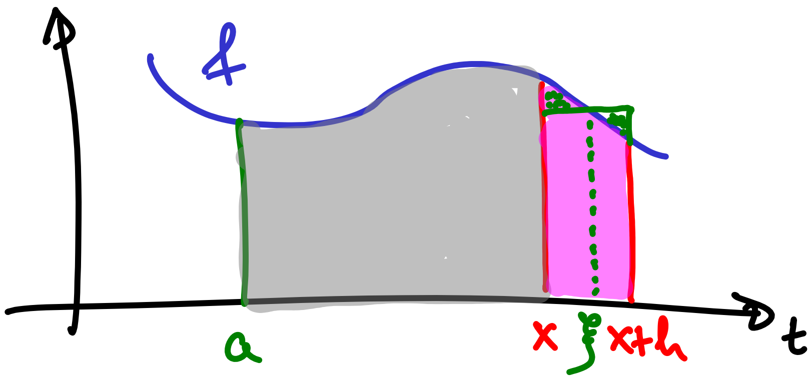
$$24h\text{-Mittel} = \frac{\text{Fläche}}{24h}$$

$$\text{Fläche} = \int_a^b T(t)\,dt$$



$$\int_a^b f(x)\,dx = A_1 - A_2 + A_3$$

Beweisidee HS



$$F(x) = \int_a^x f(t)\,dt$$

z.z.: F ist Stammfkt. von f

$$F'(x) = \lim_{h\to 0} \frac{F(x+h) - F(x)}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h} \cdot f(\xi)}{\cancel{h}}$$

ξ liegt zwischen x und x+h

$$= f(x)$$

da für $h \to 0$ auch $\xi \to x$

- $$\int_0^1 \left(2x^7 - 3x^2 + 5\right) dx$$

$$= \left[\frac{1}{4}x^8 - x^3 + 5x\right]_0^1$$ Schreibweise für

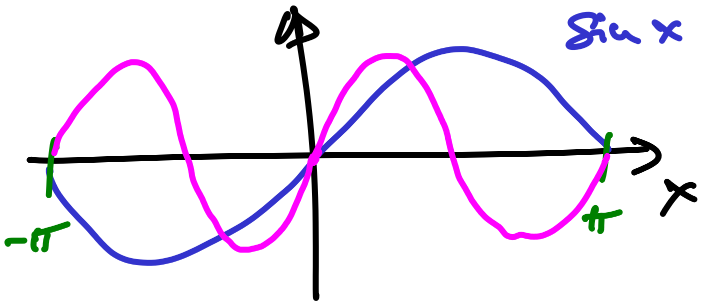
$$= \left(\frac{1}{4}\cdot 1^8 - 1^3 + 5\cdot 1\right) - \left(\frac{1}{4}\cdot 0^8 - 0^3 + 5\cdot 0\right)$$

$$= \frac{1}{4} - 1 + 5 - 0 = \frac{17}{4}$$

- $$\int_1^3 \frac{dx}{x^2} = \int_1^3 \frac{1}{x^2}\, dx = \int_1^3 x^{-2}\, dx = \left[-x^{-1}\right]_1^3 = \left[-\frac{1}{x}\right]_1^3$$

$$= -\frac{1}{3} + 1 = +\frac{2}{3}$$

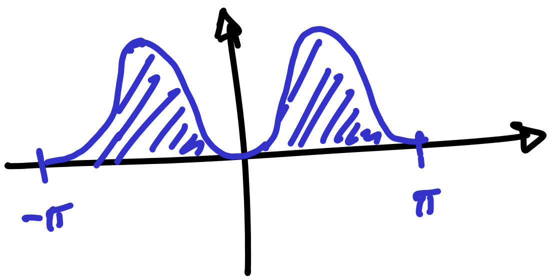
- $$\int_1^3 \frac{dx}{x} = \left[\log x\right]_1^3 = \log 3 - \log 1 = \log 3$$

- 

erwarte also $\int_{-\pi}^{\pi} \sin(nx)\,dx = 0 \quad \forall n \in \mathbb{N}$

$$\int_{-\pi}^{\pi} \sin(nx)\,dx = \left[-\cos(nx)\cdot\frac{1}{n}\right]_{-\pi}^{\pi} = -\frac{1}{n}\left[\cos(nx)\right]_{-\pi}^{\pi}$$

$$= -\frac{1}{n}\left((-1)^n - (-1)^n\right) = 0$$

- 

$$\cos(2x) = \cos^2 x - \sin^2 x$$ Add. Thm. cos

$$= 1 - \sin^2 x - \sin^2 x$$ Pythagoras

$$= 1 - 2\sin^2 x$$

$$\Rightarrow \quad \sin^2 x = \frac{1}{2}\left(1 - \cos(2x)\right)$$

$$\int_{-\pi}^{\pi} \sin^2(nx)\,dx = \frac{1}{2}\int_{-\pi}^{\pi}\left(1-\cos(2nx)\right)dx$$

$$= \frac{1}{2}\left[x - \sin(2nx)\cdot\frac{1}{2n}\right]_{-\pi}^{\pi}$$

$$= \frac{1}{2}\left(\left(\pi - \underbrace{\sin(2n\pi)}_{=0}\frac{1}{2n}\right) - \left(-\pi - \underbrace{\sin(-2n\pi)}_{=0}\frac{1}{2n}\right)\right)$$

$$= \pi$$

Produktregel: $\left(f(x)\,g(x)\right)' = f'(x)\,g(x) + f(x)\,g'(x)$

$$\int_a^b \ldots\, dx \implies$$

$$\left[f(x)\,g(x)\right]_a^b = \underline{\int_a^b f'(x)\,g(x)\,dx} + \int_a^b f(x)\,g'(x)\,dx$$

$$\iff \underline{\int_a^b f'(x)\,g(x)\,dx} = \left[f(x)\,g(x)\right]_a^b - \int_a^b f(x)\,g'(x)\,dx$$

- $$\int_0^{\pi/2} \underset{g}{x}\, \underset{f'}{\cos x}\,dx = \Big[\underset{g}{x}\, \underset{f}{\sin x}\Big]_0^{\pi/2} - \int_0^{\pi/2} \underset{g'}{1}\cdot \underset{f}{\sin x}\,dx$$

$$= \frac{\pi}{2}\cdot 1 - 0\cdot 0 - \Big[-\cos x\Big]_0^{\pi/2}$$

$$= \frac{\pi}{2} + \Big[\cos x\Big]_0^{\pi/2} = \frac{\pi}{2} + 0 - 1 = \frac{\pi}{2} - 1$$

- $$\int \log x\,dx = \int \underset{f'}{1}\cdot \underset{g}{\log x}\,dx$$

$$= x\log x - \int x\cdot\frac{1}{x}\,dx = x\log x - \overbrace{\int 1\,dx}^{=\int dx}$$

$$= x\log x - x$$

Test: $(x\log x - x)' = \log x + x\cdot\frac{1}{x} - 1 = \log x$ ☺

- $n \neq m \quad , \quad n, m \in \mathbb{N}$

$$\int_{-\pi}^{\pi} \underset{f'}{\sin(nx)}\, \underset{g}{\sin(mx)}\, dx = \underbrace{\left[-\frac{\cos(nx)}{n} \sin(mx) \right]_{-\pi}^{\pi}}_{=0} + \int_{-\pi}^{\pi} \frac{\cos(nx)}{n} \cos(mx) \cdot m \, dx$$

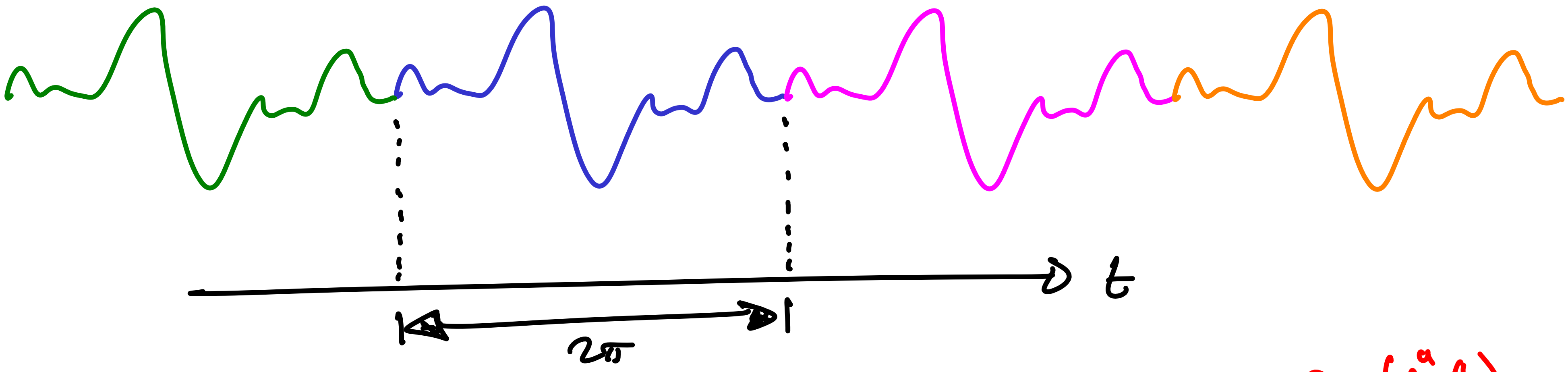
$$= \frac{m}{n} \int_{-\pi}^{\pi} \underset{f'}{\cos(nx)}\, \underset{g}{\cos(mx)}\, dx$$

$$= \underbrace{\frac{m}{n} \left[\frac{\sin(nx)}{n} \cos(mx) \right]_{-\pi}^{\pi}}_{=0} + \frac{m}{n} \int_{-\pi}^{\pi} \frac{\sin(nx)}{n} \sin(mx) \cdot m \, dx$$

$$= \frac{m^2}{n^2} \int_{-\pi}^{\pi} \sin(nx) \sin(mx)\, dx$$

$$\Leftrightarrow \underbrace{\left(1 - \frac{m^2}{n^2}\right)}_{\neq 0} \int_{-\pi}^{\pi} \sin(nx) \sin(mx)\, dx = 0$$

$$\Rightarrow \int_{-\pi}^{\pi} \sin(nx) \sin(mx)\, dx = 0$$



$$\int_{-\pi}^{\pi} f(t)\sin(mt)\,dt = \underbrace{\int_{-\pi}^{\pi} a_0 \sin(mt)\,dt}_{=0} + \sum_n \left(\overbrace{\int_{-\pi}^{\pi} a_n \cos(nt)\sin(mt)\,dt}^{=0\ (\text{ÜA})} + \underbrace{\int_{-\pi}^{\pi} b_n \sin(nt)\sin(mt)\,dt}_{=\begin{cases}0, & n\neq m\\ \pi b_m, & n=m\end{cases}} \right)$$

$$\Rightarrow b_m = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t)\sin(mt)\,dt$$