Nochmal Definitheit

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \underset{c=b}{=} \begin{pmatrix} a & b \\ b & d \end{pmatrix}$$

$$f(x,y) = \begin{pmatrix} x \\ y \end{pmatrix} \cdot \left[A \begin{pmatrix} x \\ y \end{pmatrix} \right]$$

$$= a\,x^2 + 2b\,xy + d\,y^2$$

$$= a\left(x^2 + \frac{2b}{a}xy\right) + dy^2$$

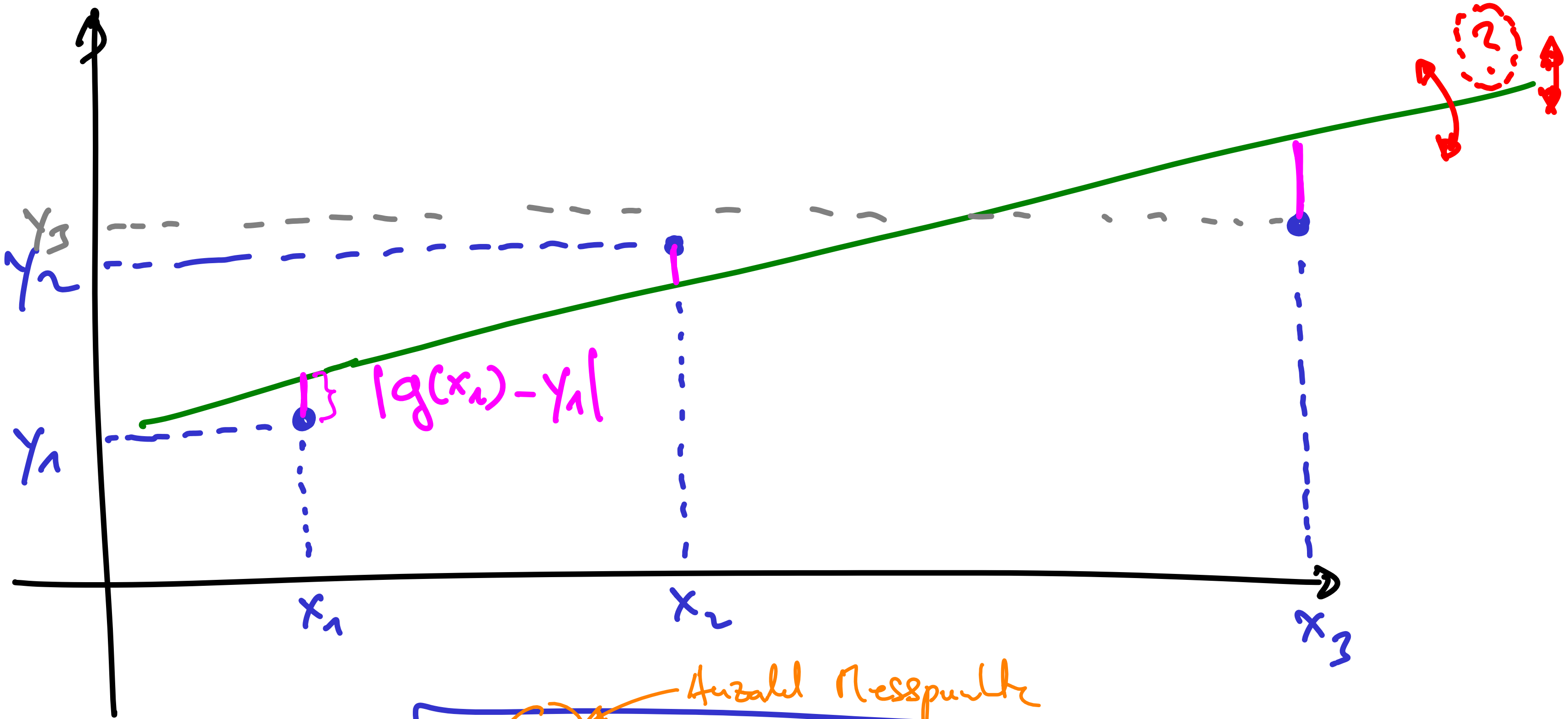
$$= a\left(\left(x + \frac{b}{a}y\right)^2 - \frac{b^2}{a^2}y^2\right) + dy^2$$

$$= a\left(x + \frac{b}{a}y\right)^2 - \frac{b^2}{a}y^2 + dy^2$$

$$= a\,\underbrace{\left(x + \frac{b}{a}y\right)^2}_{\geq 0} + \frac{ad-b^2}{a}\,\underbrace{y^2}_{\geq 0}$$

falls $a > 0$ <u>und</u> $ad - b^2 > 0$

$\Rightarrow f(x,y) \geq 0 \quad \forall x,y$



Anzahl Messpunkte

$$D(m,b) = \sqrt{\sum_{i=1}^{n} \left(g(x_i) - y_i\right)^2}$$

$$= \sqrt{\sum_{i=1}^{n} \left(m x_i + b - y_i\right)^2}$$

$$D \text{ minimal} \Leftrightarrow \left(D(m,b)\right)^2 = \underline{\sum_{i=1}^{n} (m x_i + b - y_i)^2} =: f(m,b) \text{ minimal}$$

$$f(m,b) = (mx_1+b-y_1)^2 + (mx_2+b-y_2)^2 + \dots + (mx_n+b-y_n)^2$$

$$\begin{cases} \frac{\partial f}{\partial m}(m,b) = \sum_{i=1}^{n} 2(mx_i+b-y_i)\cdot x_i \overset{!}{=} 0 \\ \frac{\partial f}{\partial b}(m,b) = \sum_{i=1}^{n} 2(mx_i+b-y_i) \overset{!}{=} 0 \end{cases}$$

$$\Longleftrightarrow \begin{cases} \left(\sum_{i=1}^{n} \cancel{2}x_i\right)\cdot b + \left(\sum_{i=1}^{n} \cancel{2}x_i^2\right)m = \sum_{i=1}^{n} \cancel{2}y_i\, x_i \\ \underbrace{\left(\sum_{i=1}^{n} \overset{1}{\cancel{2}}\right)}_{=n} b + \left(\sum_{i=1}^{n} \cancel{2}x_i\right)m = \sum_{i=1}^{n} \cancel{2}y_i \end{cases}$$

$$\sum_{i=1}^{n} (x_i-\bar{x})(y_i-\bar{y})$$

$$= \sum_{i=1}^{n} (x_i y_i - x_i \bar{y} - \bar{x} y_i + \bar{x}\bar{y})$$

$$= \sum_{i=1}^{n} x_i y_i \underbrace{- \bar{y} \underbrace{\sum_{i=1}^{n} x_i}_{=n\bar{x}} - \bar{x} \underbrace{\sum_{i=1}^{n} y_i}_{=n\bar{y}} + n\bar{x}\bar{y}}_{=-n\bar{x}\bar{y}}$$

$$= \sum_{i=1}^{n} x_i y_i - n\bar{x}\bar{y}$$

analog $\sum_{i=1}^{n} (x_i-\bar{x})^2 = \sum_{i=1}^{n} x_i^2 - n\bar{x}^2$

Dürften wir durch $\underbrace{\sum_{i=1}^{n} (x_i-\bar{x})^2}_{\geq 0}$ teilen?

ganzer Ausdruck $=0$ nur dann, wenn $x_i = \bar{x} \ \forall i$, also <u>kein</u> Problem

Positiv definit? (Minimum?)

$$H = \begin{pmatrix} 2n & 2n\bar{x} \\ 2n\bar{x} & 2\sum_{i=1}^{n} x_i^2 \end{pmatrix}$$

① $2n > 0$

② $$4n\sum_{i=1}^{n} x_i^2 - 4n^2\bar{x}^2 = 4n\underbrace{\left(\sum_{i=1}^{n} x_i^2 - n\bar{x}^2\right)}_{= \sum_{i=1}^{n}(x_i - \bar{x})^2 \geq 0}$$

☺

(bzw. > 0 wenn nicht alle x_i gleich)

Schl-Bsp.

i	1	2	3	4	5	6
x_i	20	16	15	16	13	10
y_i	0	3	7	4	6	10

$\bar{x} = 15$, $\bar{y} = 5$

$$m = \frac{\sum_{i=1}^{6} (x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{6} (x_i - \bar{x})^2}$$

$$= \frac{(20-15)\cdot(0-5) + (16-15)\cdot(3-5) + (15-15)(7-5) + \ldots}{(20-15)^2 + (16-15)^2 + (15-15)^2 + \ldots}$$

$\approx -0{,}982$ (MATLAB)

$b = \bar{y} - m\bar{x} \approx 19{,}7$