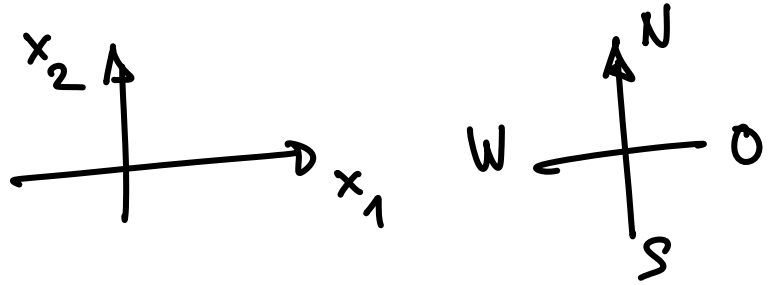
**52**




$\vec{w}_1 = 95 \cdot \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ (alles in km/h)

$\vec{v}_1 = 180 \cdot \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

Länge 1

Länge von $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$: $\left| \begin{pmatrix} 1 \\ -1 \end{pmatrix} \right| = \sqrt{1^2 + (-1)^2} = \sqrt{2}$

— u — $\frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$: $\left| \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix} \right| = \left| \begin{pmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{pmatrix} \right| = \sqrt{\left(\frac{1}{\sqrt{2}}\right)^2 + \left(-\frac{1}{\sqrt{2}}\right)^2}$

$= \sqrt{\frac{1}{2} + \frac{1}{2}} = \sqrt{1} = 1$

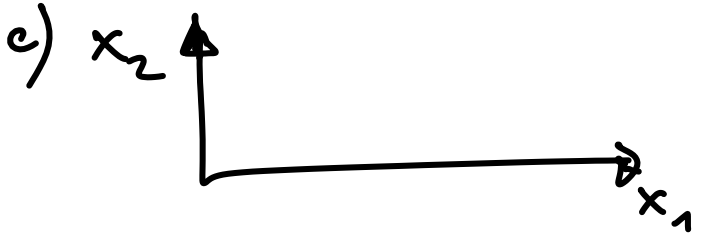
$\vec{v}_1 = \vec{u}_1 + \vec{w}_1$

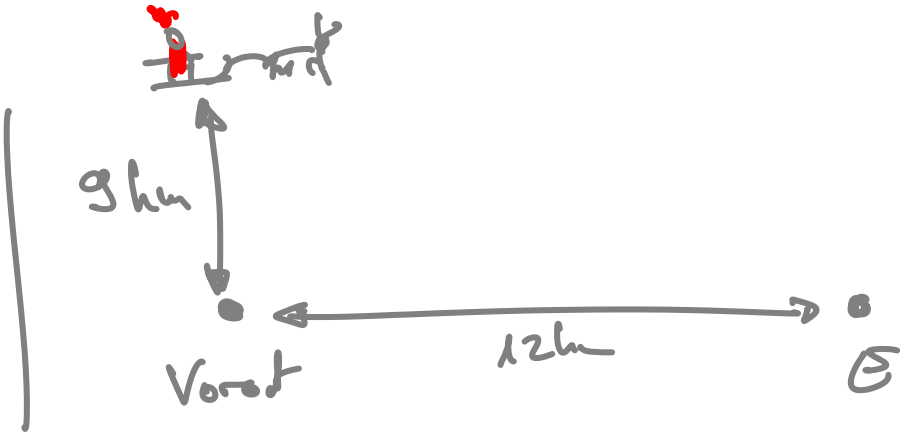
a) $\vec{u}_1 = \vec{v}_1 - \vec{w}_1 \Rightarrow |\vec{u}_1| = |\vec{v}_1 - \vec{w}_1|$

$$|\vec{u}_1| = \left|180 \cdot \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix} - 95\begin{pmatrix}0\\1\end{pmatrix}\right| = \left|\begin{pmatrix}180/\sqrt{2}\\-180/\sqrt{2} - 95\end{pmatrix}\right| \approx \underline{256}$$

b) $$-\frac{\vec{u}_1}{|\vec{u}_1|} \approx \begin{pmatrix}-0{,}50\\+0{,}87\end{pmatrix}$$

c)




$$\frac{\vec{v}_2}{|\vec{v}_2|} = \begin{pmatrix}12\\-9\end{pmatrix}\frac{1}{\sqrt{144+81}} = \begin{pmatrix}4\\-3\end{pmatrix}\frac{1}{\sqrt{16+9}} = \frac{1}{5}\begin{pmatrix}4\\-3\end{pmatrix} = \vec{e}$$

$$\vec{v}_2 = \vec{u}_2 + \vec{w}_2 \quad , \quad \vec{w}_2 = \begin{pmatrix}20\\0\end{pmatrix}$$

$$\Leftrightarrow \vec{u}_2 = \vec{v}_2 - \vec{w}_2 \Rightarrow |\vec{u}_2| = |\vec{v}_2 - \vec{w}_2| \quad (*)$$

$$|\vec{v}_2 - \vec{w}_2| = |\,|\vec{v}_2|\vec{e} - \vec{w}_2\,| = \sqrt{|\vec{v}_2|^2 + |\vec{w}_2|^2 - 2|\vec{v}_2|\,\vec{e}\cdot\vec{w}_2}$$

$$= \sqrt{|\vec{v}_2|^2 + 400 - 2|\vec{v}_2| \cdot 16}$$

in (*):

$$256^2 \approx |\vec{u}_2|^2 = |\vec{v}_2|^2 - 32|\vec{v}_2| + 400$$

quadratische Gl. für $|\vec{v}_2|$

$$\left| |\vec{v}_2|\vec{e} - \vec{w}_2 \right| = \sqrt{(|\vec{v}_2|\vec{e} - \vec{w}_2) \cdot (|\vec{v}_2|\vec{e} - \vec{w}_2)}$$

$$= \sqrt{|\vec{v}_2|^2 \underbrace{\vec{e} \cdot \vec{e}}_{=1} + \underbrace{(-\vec{w}_2)(-\vec{w}_2)}_{=400} - 2|\vec{v}_2|\vec{e} \cdot \vec{w}_2}$$

Nachklausur 13/14

7

$$L = \begin{pmatrix} 1 & 2 \\ \frac{1}{2} & 0 \end{pmatrix}, \quad \vec{N}^{(0)} = \begin{pmatrix} 100 \\ 50 \end{pmatrix}$$

a) $\vec{N}^{(-1)} = ?$

$L \vec{N}^{(-1)} = \vec{N}^{(0)}$ $\qquad \vec{N}^{(-1)} = \begin{pmatrix} a \\ b \end{pmatrix}$

$$L \vec{N}^{(-1)} = L \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} a + 2b \\ a/2 \end{pmatrix} = \begin{pmatrix} 100 \\ 50 \end{pmatrix}$$

zweite Zeile: $\frac{a}{2} = 50 \Rightarrow a = 100$

erste Zeile: $100 + 2b = 100 \Rightarrow b = 0$

also $\vec{N}^{(-1)} = \begin{pmatrix} 100 \\ 0 \end{pmatrix}$

Klausur 13/14, 6

40% · 15

a) $$L = \begin{pmatrix} 0 & 2 & 6 & 4 \\ \frac{1}{5} & 0 & 0 & 0 \\ 0 & \frac{4}{5} & 0 & 0 \\ 0 & 0 & \frac{2}{5} & 0 \end{pmatrix}$$

Nachklausur 11/12, 13

$$L = \begin{pmatrix} 0 & 1 & \frac{1}{2} \\ \frac{4}{5} & 0 & 0 \\ 0 & \frac{1}{2} & 0 \end{pmatrix}, \quad \vec{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

c) $L\vec{x} = \vec{x}$, LGS,

$$\begin{aligned} x_2 + \tfrac{1}{2}x_3 &= x_1 \\ \tfrac{4}{5}x_1 &= x_2 \\ \tfrac{1}{2}x_2 &= x_3 \end{aligned}$$

$$L = \begin{pmatrix} 0 & 1 & \frac{1}{2} \\ \frac{4}{5} & 0 & 0 \\ 0 & \frac{1}{2} & 0 \end{pmatrix}$$

$$\begin{aligned} -x_1 + x_2 + \tfrac{1}{2}x_3 &= 0 \\ \tfrac{4}{5}x_1 - x_2 &= 0 \\ \tfrac{1}{2}x_2 - x_3 &= 0 \end{aligned}$$

$$\left(\begin{array}{ccc|c} -1 & 1 & \frac{1}{2} & 0 \\ \frac{4}{5} & -1 & 0 & 0 \\ 0 & \frac{1}{2} & -1 & 0 \end{array}\right) \quad \cdot\, 4/5$$

$$\left(\begin{array}{ccc|c} -1 & 1 & \frac{1}{2} & 0 \\ 0 & -\frac{1}{5} & \frac{2}{5} & 0 \\ 0 & \frac{1}{2} & -1 & 0 \end{array}\right) \quad \cdot\, 5 \ldots \cdot\, 1/2$$

$$\left(\begin{array}{ccc|c} -1 & 1 & \frac{1}{2} & 0 \\ 0 & -1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

also $x_3 = t \in \mathbb{R}$ beliebig

$x_2 = 2x_3 = 2t$

$x_1 = x_2 + \frac{1}{2}x_3$

$= 2t + \frac{1}{2}t = \frac{5}{2}t$

d.h. $\vec{x} = \begin{pmatrix} 5/2 \\ 2 \\ 1 \end{pmatrix} t$ löst LGS (für bel. $t$)

c) (ii)

$x_2 = 5000 \iff t = 2500$

$$\vec{x} = \begin{pmatrix} 6250 \\ 5000 \\ 2500 \end{pmatrix}$$

Klausur 13/14, [8]

Einzelpreis von Reis (in €): $2x$

Kilopreis von Reis (in €): $25x$

analog für Bohnen

von Reis → $2x$ + $6y$ ← von Bohnen

Einzelpreis insgesamt:

(i) $2x+6y \geq 12$

(ii) $2x+6y \leq 36$

analog für Kilopreis