NK 12/13

$\boxed{5}$ a)

(i) $x + y \geq 2{,}5$

(ii) $\dfrac{0{,}4x}{x+y} \leq 0{,}2$

(iii) $y \leq 2$

K 09/10

$\boxed{6}$ b) $\log n = \underline{17 - 2M}$ (aus MATLAB-Code)

$\Rightarrow n = e^{17-2M} = e^{17} \cdot e^{-2M}$

(Exponentialgesetz)

c) $W = 10^{\frac{3}{2}M - 2}$

$n(W) = ?$

$n(M) = e^{17-2M} \quad (*)$

$$\Rightarrow \log W = \left(\tfrac{3}{2}M - 2\right) \log 10$$

$$\Leftrightarrow M = \frac{2}{3}\left(\frac{\log W}{\log 10} + 2 \right)$$

in $(*)$

$$n(W) = e^{17 - \frac{4}{3}\left(\frac{\log W}{\log 10} + 2 \right)}$$

$$= e^{17 - \frac{8}{3}} \, e^{-\frac{4}{3\log 10} \cdot \log W}$$

$$= e^{43/3} \, W^{-\frac{4}{3 \log 10}}$$

d) Potenzgesetz

NK 13/14

$\boxed{2}$

$$e + s = 120 \qquad (i)$$

$$\underbrace{e - 20}_{\text{Alter von Eva vor 20 Jahren}} = 3\underbrace{(s - 20)}_{\text{Alter von Simon vor 20 Jahren}} \qquad (ii)$$

$$(i) \Rightarrow s = 120 - e$$

in (ii) $$e - 20 = 3(100 - e)$$

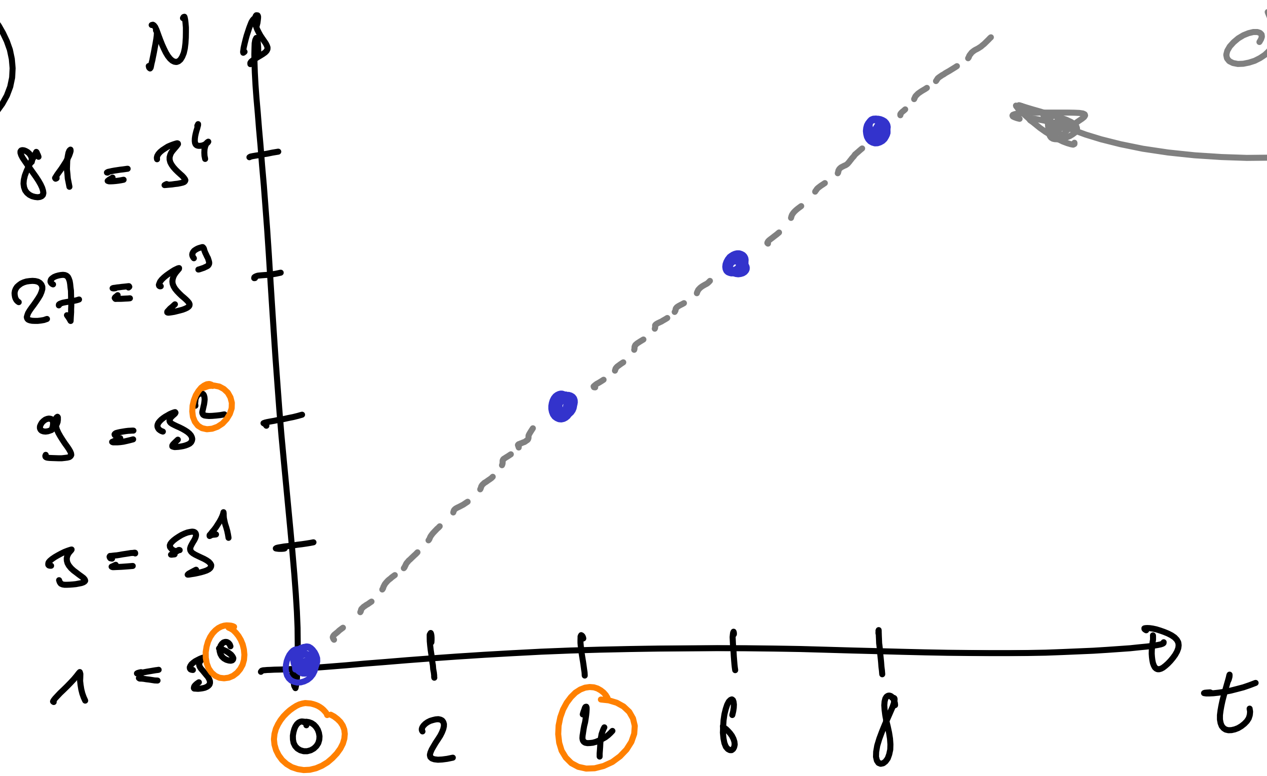
$$\Leftrightarrow 4e = 320 \Leftrightarrow e = 80$$

$$y = C \cdot x^{\alpha} \Rightarrow \underline{\underline{\log y}} = \log C + \alpha \cdot \underline{\underline{\log x}}$$

NK 11/12

$\boxed{2}$ a) exponentielles Wachstum

b)



c) Gerade

d) Achsenabschnitt ~~1~~ 0, Steigung ~~$\frac{5}{2}$~~ 1/2

$$\log_3 N = \frac{1}{2} \cdot t \qquad | \; 3^\wedge(\ldots)$$

$$\Rightarrow \quad N = 3^{t/2}$$

$$N(t) = C \cdot e^{\lambda t}$$ $(A = C = D)$

$$= A \cdot 10^{\beta t}$$

$$= D \cdot 3^{\gamma t}$$

$N(0) = 1 \Rightarrow D = 1$

$N(4) = 9 \Rightarrow 9 = 3^{\gamma \cdot 4} \Rightarrow \gamma = \frac{1}{2}$

$\Rightarrow N = 3^{\frac{1}{2}t} = 3^{t/2} = \sqrt{3^t}$

e) $88\,000 = \sqrt{3^t}$

$= (\sqrt{3})^t$

$(x^a)^b = (x^b)^a = x^{a \cdot b}$

$\Rightarrow \log(88\,000) = t \log\sqrt{3} \Leftrightarrow t = \dfrac{\log(88\,000)}{\log\sqrt{3}}$

K 14/15

$\boxed{4}$ f)

$$N(t) = \frac{1000}{1+Be^{-ct}}$$

$$\dot{N}(t) = \underbrace{(-1)\frac{1000}{(1+Be^{-ct})^2}}_{\text{"Ableitung von } \frac{1}{x}\text{"}} \underbrace{B \cdot e^{-ct}}_{\text{Kettenregel 1}} \underbrace{\cdot(-c)}_{\text{Kettenregel 2}}$$

$$= \frac{1000\,B\,c\,e^{-ct}}{(1+Be^{-ct})^2} \quad (*)$$

$$\frac{\gamma}{1000} N(t)\left(1000 - N(t)\right) = \frac{\gamma}{1+Be^{-ct}}\left(1000 - \frac{1000}{1+Be^{-ct}}\right)$$

$$= \frac{1000\,\gamma}{1+Be^{-ct}} \cdot \frac{1+Be^{-ct}-1}{1+Be^{-ct}} = \frac{1000\,\gamma\,B\,e^{-ct}}{(1+Be^{-ct})^2} \quad (+)$$

$(*) \overset{!}{=} (+) \Rightarrow$ d.h. für $C = \gamma$ löst die Fkt. die DGL

"momentan 100 Bakterien" d.h. $N(0) = 100$

$$N(0) = \frac{1000}{1+Be^{-C\cdot 0}} = \frac{1000}{1+B} \overset{!}{=} 100$$

$$\Leftrightarrow B = 9$$

NK 13/14

$\boxed{4}$ $\lim\limits_{x\to 0} \frac{\tan x}{x} = 1$

a) $\lim\limits_{x\to 0} \frac{\tan x}{x^3} = \lim\limits_{x\to 0} \left(\frac{1}{x^2} \cdot \frac{\tan x}{x} \right) = \infty$

Grenzw. v. Prod = Prod. v. Grenzw.

$\frac{1}{x^2} \to \infty$, $\frac{\tan x}{x} \to 1$

b) $$\lim_{x\to 0} \frac{x}{\tan x} = \lim_{x\to 0} \left(\frac{\tan x}{x}\right)^{-1}$$

$$= \left(\lim_{x\to 0} \frac{\tan x}{x}\right)^{-1} = \frac{1}{1} = 1$$

$y \mapsto \frac{1}{y}$ stetig (für $y \neq 0$)

c) $$\lim_{x\to 0} \frac{x^2}{\tan x} = \lim_{x\to 0} \left(x \cdot \frac{x}{\tan x}\right)$$

$$= \left(\lim_{x\to 0} x\right) \cdot \left(\lim_{x\to 0} \frac{x}{\tan x}\right) = 0 \cdot \underset{(b)}{1} = 0$$

wie in (a)

d) $$\lim_{x\to 0} \frac{x}{x+\tan x} = \lim_{x\to 0} \left(\frac{x+\tan x}{x}\right)^{-1}$$

$$= \left(\lim_{x\to 0} \left(\frac{x}{x} + \frac{\tan x}{x}\right)\right)^{-1} = \left(\lim_{x\to 0} \left(1 + \frac{\tan x}{x}\right)\right)^{-1}$$

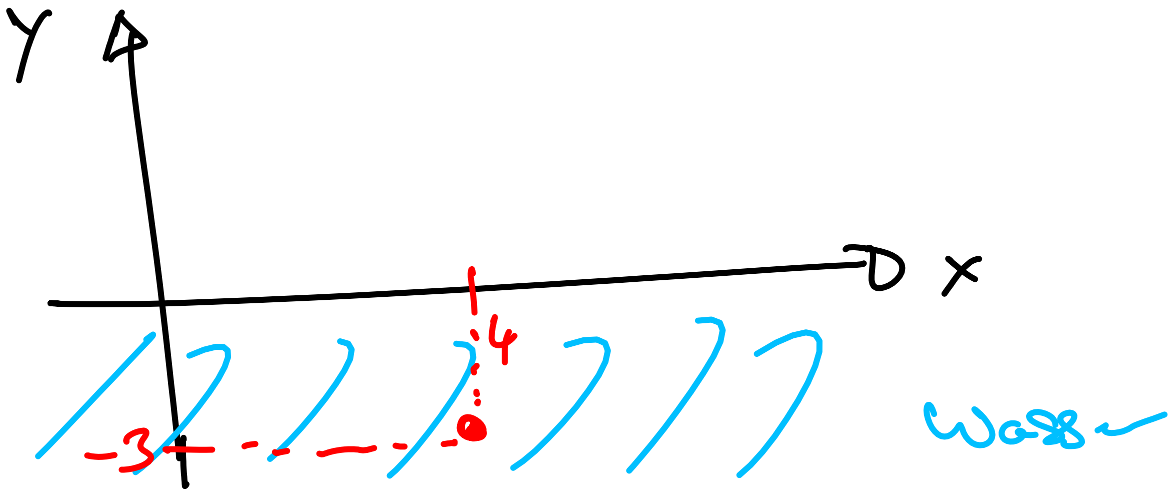
$$= (1+1)^{-1} = \frac{1}{2}$$

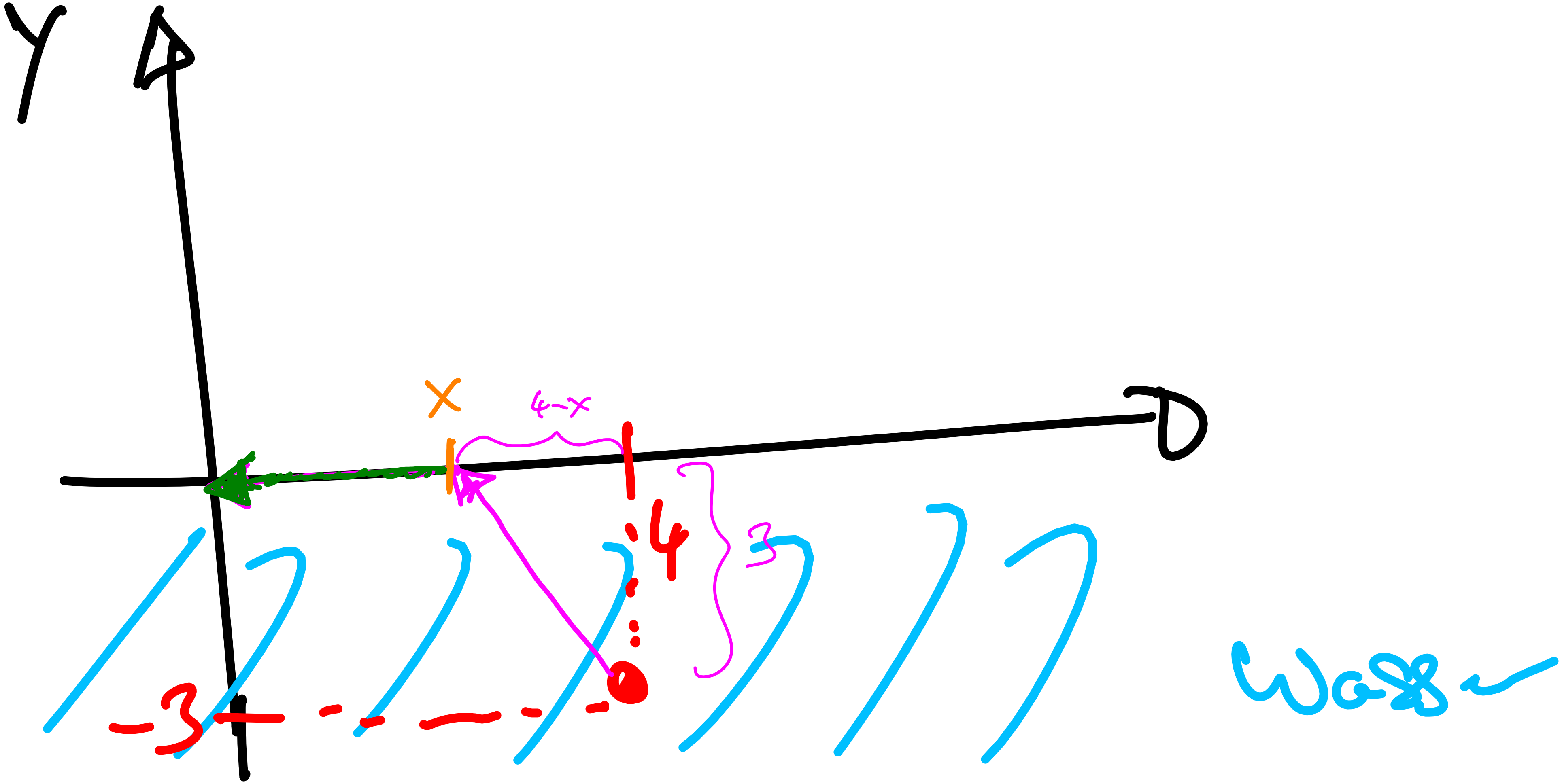
$$\lim_{x\to 0} \frac{x}{x+\tan x} = \lim_{x\to 0} \frac{1}{1+\frac{\tan x}{x}}$$

$$= \frac{1}{1+\lim_{x\to 0}\frac{\tan x}{x}} = \frac{1}{1+1} = \frac{1}{2}$$

$$\frac{a}{b+c} \neq \frac{a}{b} + \frac{a}{c} = \frac{a\cdot c + ab}{bc}$$

$\boxed{8}$





Energieverbrauch:

$$E(x) = 3x + 5\sqrt{9+(4-x)^2}$$

$$E'(x) = 3 + \frac{5}{\cancel{2}} \frac{1}{\sqrt{9+(4-x)^2}} \cancel{2}(x-4) \stackrel{!}{=} 0$$

$$\Leftrightarrow\; 3 = \frac{5}{\sqrt{9+(4-x)^2}}(4-x) \qquad \Big|\cdot\sqrt{\ldots}\;,\; \Big|\uparrow 2$$

$$\Rightarrow\; 9\cdot\left(9+(4-x)^2\right) = 25\cdot(4-x)^2 \qquad \Big|-9(4-x)^2$$

$$\Leftrightarrow\; 81 = 16(4-x)^2 \qquad \Big|\cdot\frac{1}{16}\;,\; \Big|\sqrt{\ldots}$$

$$\Leftrightarrow\; \pm\frac{9}{4} = 4-x \;\Leftrightarrow\; x = 4 \mp \frac{9}{4} = \begin{cases} 7/4 \\ 25/4 \end{cases}$$

Welche? $x = \dfrac{7}{4}$ (da $\frac{25}{4} > 4$)

hier muss der Graph an Land

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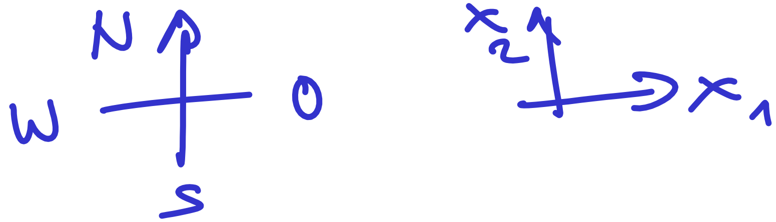

$\boxed{8}$ a)

$$\vec f = \frac{5}{\sqrt{2}}\begin{pmatrix}-1\\-1\end{pmatrix}$$

b)

$$\vec p = \frac{1}{\sqrt{2}}\begin{pmatrix}-1\\1\end{pmatrix}$$

c)

$$\vec m = \vec f + \vec p = \underline{\frac{1}{\sqrt{2}}\begin{pmatrix}-6\\-4\end{pmatrix}}$$

d)

$$\frac{\vec w}{|\vec w|} = \begin{pmatrix}0\\1\end{pmatrix} \Leftrightarrow \underline{\vec w = \begin{pmatrix}0\\1\end{pmatrix}\cdot|\vec w|}$$

e)

$$|\vec u| = 6$$

f)

$$\vec m = \vec u + \vec w \Leftrightarrow \vec u = \vec m - \vec w$$

$$\Rightarrow |\hat{u}|^2 = |\hat{m} - \hat{w}|^2 = |\hat{m}|^2 + |\hat{w}|^2 - 2\,\hat{m}\cdot\hat{w}$$

$$\Leftrightarrow \quad 36 = \frac{1}{2}(36+16) + |\hat{w}|^2 - 2\left(-4/\sqrt{2}\right)|\hat{w}|$$

$$|\hat{m} - \hat{w}|^2 = (\hat{m} - \hat{w})\cdot(\hat{m} - \hat{w})$$

$$\Leftrightarrow |\hat{w}|^2 + 4\sqrt{2}\,|\hat{w}| - 10 = 0$$

$$|\hat{w}| = \frac{-4\sqrt{2} \pm \sqrt{32+40}}{2} = \frac{-4\sqrt{2} \pm \sqrt{72}}{2}$$

$$= \frac{-4\sqrt{2} \pm \sqrt{36\cdot 2}}{2} = \frac{-4\sqrt{2} \pm 6\sqrt{2}}{2}$$

$$= \frac{2\sqrt{2}}{2} = \sqrt{2}$$

$+$ da $|\hat{w}| > 0$

$$\int \underset{g}{x} \cdot \underset{f'}{\sin x}\,dx = \underset{g}{x}\,(\underset{f}{-\cos x}) - \int \underset{g'}{1} \cdot (\underset{f}{-\cos x})\,dx$$