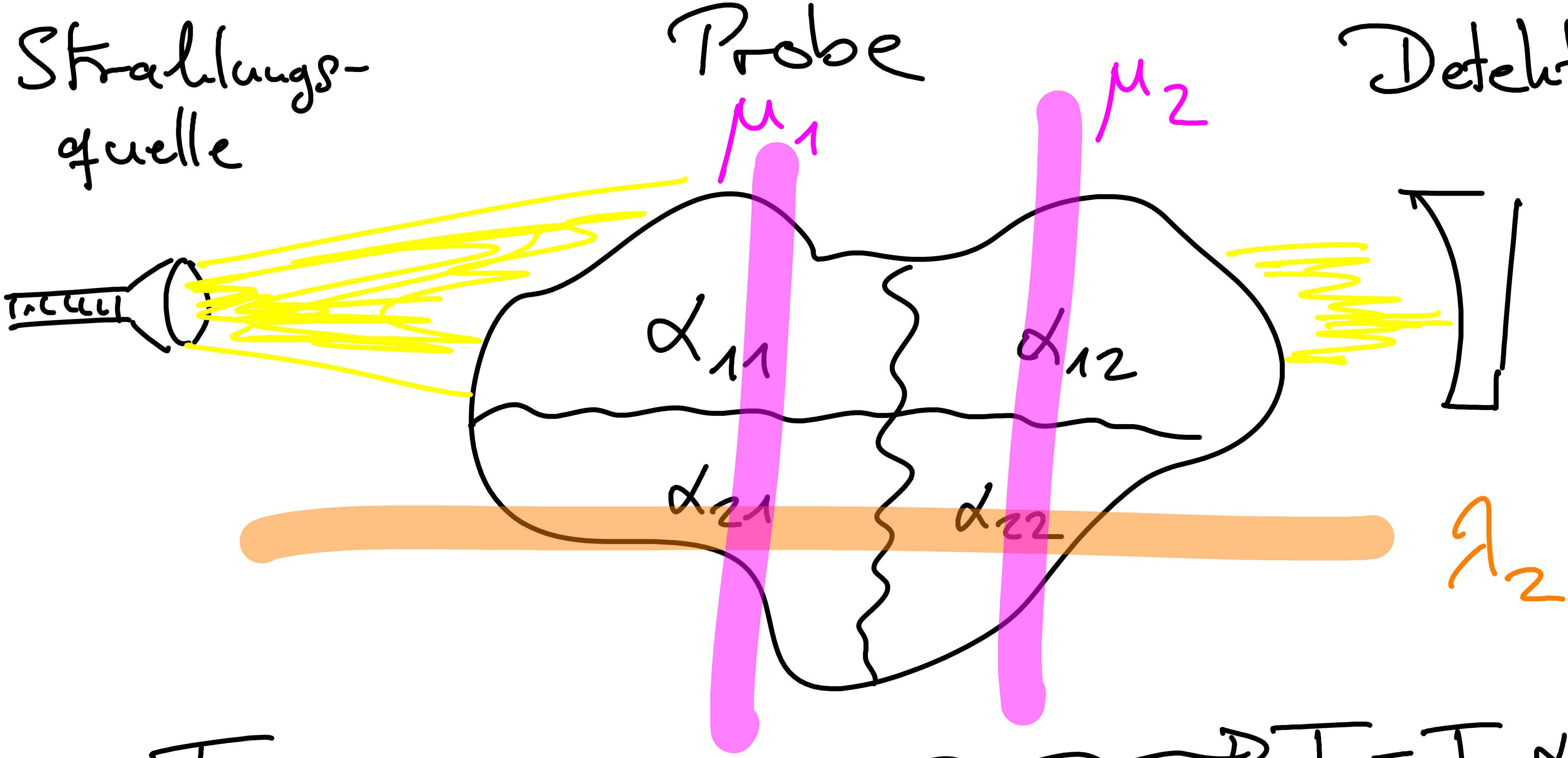


$$I_0 \longrightarrow I = I_0\,\alpha_{11}\alpha_{12}$$

$$\lambda_1 = \frac{I}{I_0} = \alpha_{11}\,\alpha_{12}$$

messen  gesucht

$$\begin{aligned} x_1 + x_2 &= 5 \\ x_1 + x_3 &= 4 \\ x_2 - x_3 &= 7 \end{aligned}$$

$$\underbrace{\begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & -1 \end{pmatrix}}_{A} \underbrace{\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}}_{x} = \underbrace{\begin{pmatrix} 5 \\ 4 \\ 7 \end{pmatrix}}_{b}$$

$$(A|b) = \left(\begin{array}{ccc|c} 1 & 1 & 0 & 5 \\ 1 & 0 & 1 & 4 \\ 0 & 1 & -1 & 7 \end{array}\right)$$

Kurz-schreibweise für

$$\left(\begin{array}{cc|c} 1 & 2 & 3 \\ 4 & 5 & 6 \end{array}\right) \quad \text{„lies"}$$

$$\begin{aligned} x_1 + 2x_2 &= 3 \quad \big|\,(-4) \\ 4x_1 + 5x_2 &= 6 \\ \hline x_1 + 2x_2 &= 3 \\ -3x_2 &= -6 \end{aligned}$$

$$
\left(\begin{array}{cccc|c}
4 & 4 & 3 & -2 & 16 \\
2 & 2 & 3 & -4 & 14 \\
-5 & -5 & -2/3 & -11/3 & -23/3
\end{array}\right)
\begin{array}{l}
\mid \cdot \frac{1}{4} \\
\\
\\
\end{array}
$$

$$
\left(\begin{array}{cccc|c}
1 & 1 & 3/4 & -1/2 & 4 \\
2 & 2 & 3 & -4 & 14 \\
-5 & -5 & -2/3 & -11/3 & -23/3
\end{array}\right)
\begin{array}{l}
(-2) \quad 5 \\
\\
\\
\end{array}
$$

$$
\left(\begin{array}{cccc|c}
1 & 1 & 3/4 & -1/2 & 4 \\
0 & 0 & 3/2 & -3 & 6 \\
0 & 0 & 37/12 & -37/6 & 37/3
\end{array}\right)
\begin{array}{l}
\\
\mid \cdot \frac{2}{3} \\
\mid \cdot 12/37
\end{array}
$$

$$
\left(\begin{array}{cccc|c}
1 & 1 & 3/4 & -1/2 & 4 \\
0 & 0 & 1 & -2 & 4 \\
0 & 0 & 1 & -2 & 4
\end{array}\right)
\begin{array}{l}
\\
-1 \\
\\
\end{array}
$$

$$\left(\begin{array}{cccc|c} 1 & 1 & 3/4 & -1/2 & 4 \\ 0 & 0 & 1 & -2 & 4 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

Zeilenstufenform

zweite Zeile: $x_3 - 2x_4 = 4$

wähle $x_4 = t \in \mathbb{R}$ beliebig

$\Rightarrow x_3 = 4 + 2t$

erste Zeile: $x_1 + x_2 + 3/4\, x_3 - 1/2\, x_4 = 4$

wähle $x_2 = s \in \mathbb{R}$ beliebig

$\Rightarrow x_1 = 4 - s + \frac{1}{2}t - \frac{3}{4}(4 + 2t)$

$= 1 - s - t$

Lösung

allgemeine Lösung:

$$x = \begin{pmatrix} 1 \\ 0 \\ 4 \\ 0 \end{pmatrix} + \begin{pmatrix} -1 \\ 1 \\ 0 \\ 0 \end{pmatrix} s + \begin{pmatrix} -1 \\ 0 \\ 2 \\ 1 \end{pmatrix} t$$

z.z. $L_0 = \{x \in \mathbb{R}^m \mid Ax = 0\}$ ist Unterraum

1) Nullvektor $0 \in L_0$, denn $A \cdot 0 = 0$

also $L_0 \neq \emptyset$

2) $L_0 \subseteq \mathbb{R}^m$ (offensichtlich)

3) $y_1, y_2 \in L_0$ ($\Leftrightarrow Ay_j = 0$, $j = 1,2$), $\lambda \in \mathbb{R}$

- Ist $y_1 + y_2$ auch in $L_0$?

$A(y_1 + y_2) = Ay_1 + Ay_2 = 0 + 0 = 0$

also $y_1 + y_2 \in L_0$ ✓

- Ebenso für $\lambda y_1$?

$A(\lambda y_1) = \lambda(Ay_1) = \lambda \cdot 0 = 0$

also $\lambda y_1 \in L_0$ ✓

4) Rechenregeln erbt $L_0$ von $\mathbb{R}^m$

$\Longrightarrow$ $L_0$ ist Unterraum

---

z.z.: $y \in L_b \iff y = u + x$ mit $x \in L_0$

"$\Leftarrow$": $y = u + x$

$$Ay = A(u+x) = Au + Ax = b + 0 = b$$

also $y \in L_b$

"$\Rightarrow$": $y \in L_b \iff Ay = b$

$x := y - u$

$$Ax = A(y-u) = Ay - Au = b - b = 0$$

also $x \in L_0$ und damit $y = u + x$ wie gewünscht $\square$

$A_B$ : Brüder von Anton

$A_S$ : Schwestern von Anton

$B_B$ : Brüder von Berta

$B_S$ : Schwestern

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| | | |
|---|---|---|
| (i) | $A_B = B_B - 1$ | $A_B \quad - B_B = -1$ |
| (ii) | $A_S = B_S + 1$ | $A_S \quad - B_S = 1$ |
| (iii) | $A_S = 2 A_B$ | $2A_B - A_S = 0$ |
| (iv) | $B_S = B_B$ | $B_B - B_S = 0$ |

$$\left(\begin{array}{cccc|c} 1 & 0 & -1 & 0 & -1 \\ 0 & 1 & 0 & -1 & 1 \\ 2 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \end{array}\right) \quad (-2)$$

---

$$\left(\begin{array}{cccc|c} 1 & 0 & -1 & 0 & -1 \\ 0 & 1 & 0 & -1 & 1 \\ 0 & -1 & 2 & 0 & 2 \\ 0 & 0 & 1 & -1 & 0 \end{array}\right)$$

$$\left(\begin{array}{cccc|c} 1 & 0 & -1 & 0 & -1 \\ 0 & 1 & 0 & -1 & 1 \\ 0 & 0 & 2 & -1 & 3 \\ 0 & 0 & 1 & -1 & 0 \end{array}\right) \quad -2$$

$$\left(\begin{array}{cccc|c} 1 & 0 & -1 & 0 & -1 \\ 0 & 1 & 0 & -1 & 1 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 & 3 \end{array}\right)$$

$B_S = 3$, $B_B = B_S = 3$

$A_S = 1 + B_S = 4$, $A_B = -1 + B_B = 2$

$\longrightarrow$ 7 Kinder