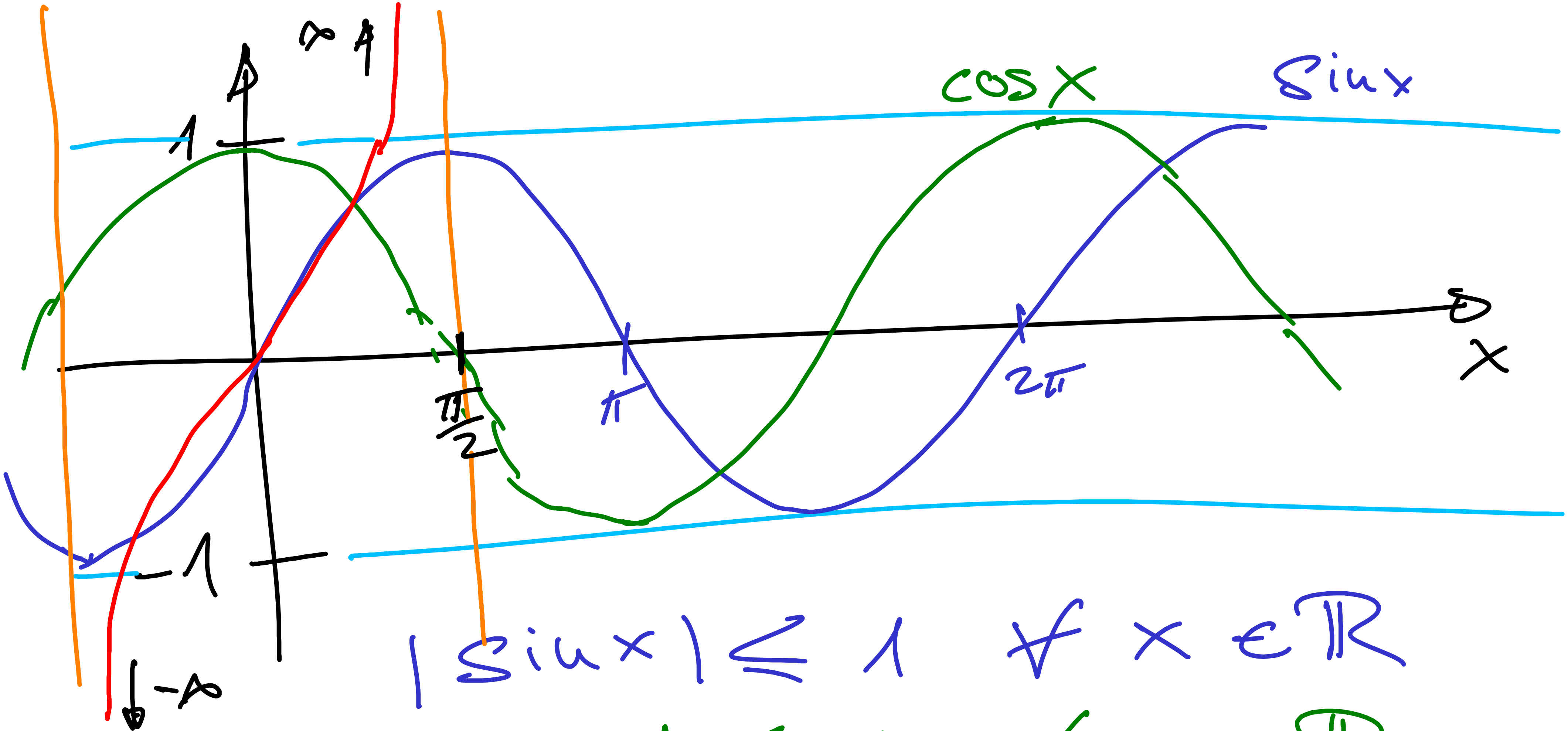
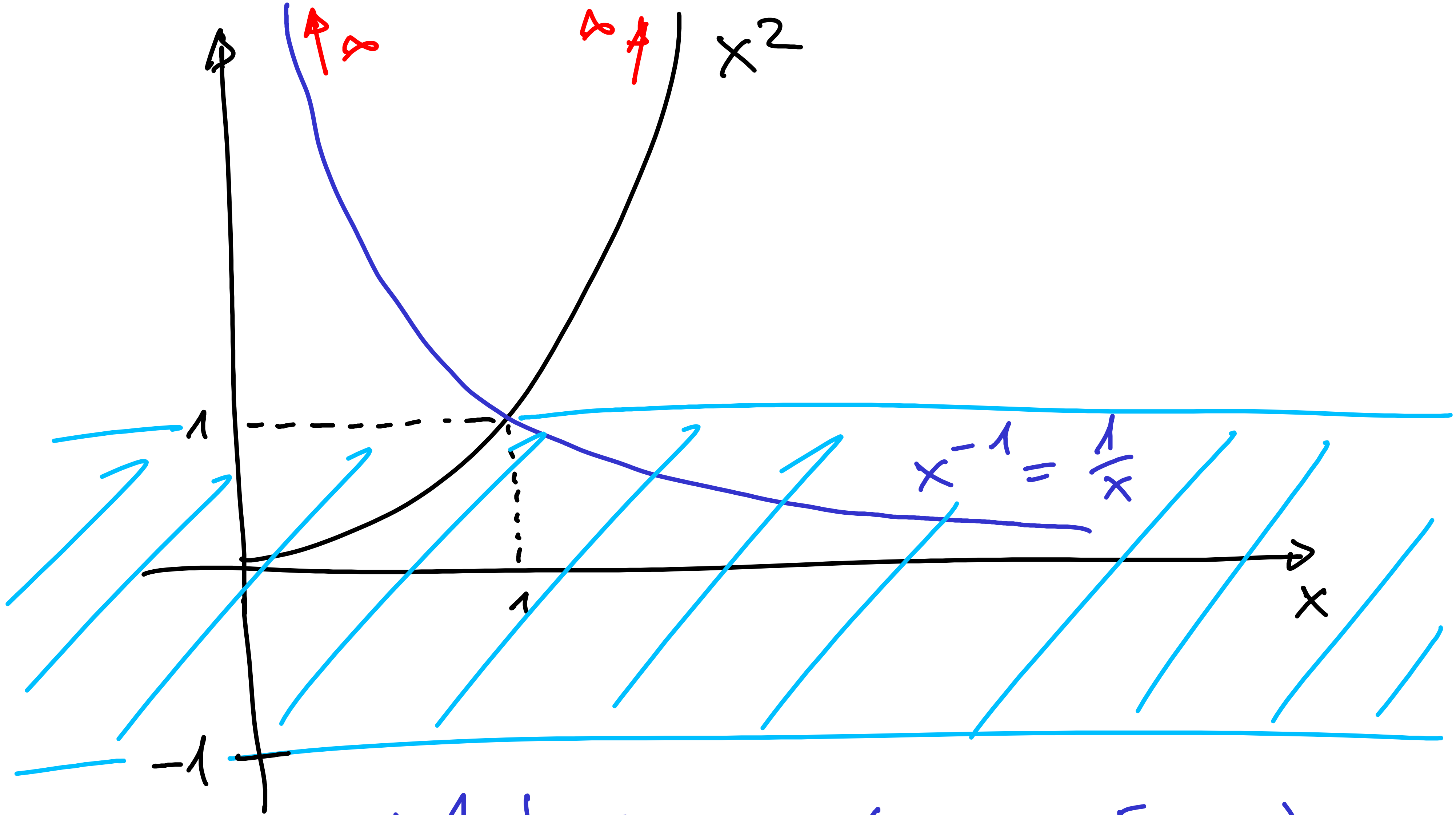


$|\sin x| \leq 1 \quad \forall x \in \mathbb{R}$

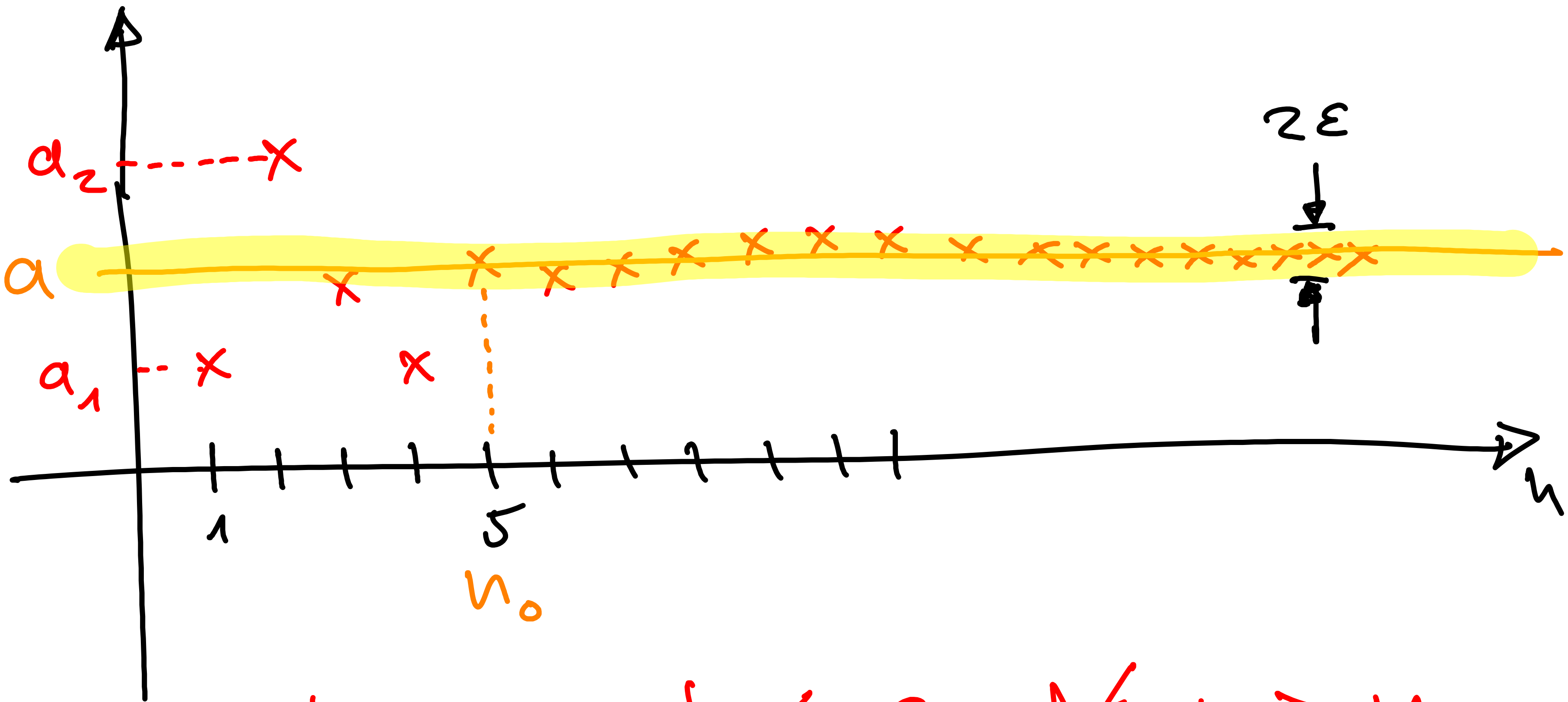
$|\cos x| \leq 1 \quad \forall x \in \mathbb{R}$

$(\sin^2 x + \cos^2 x = 1)$

tan x nicht beschränkt



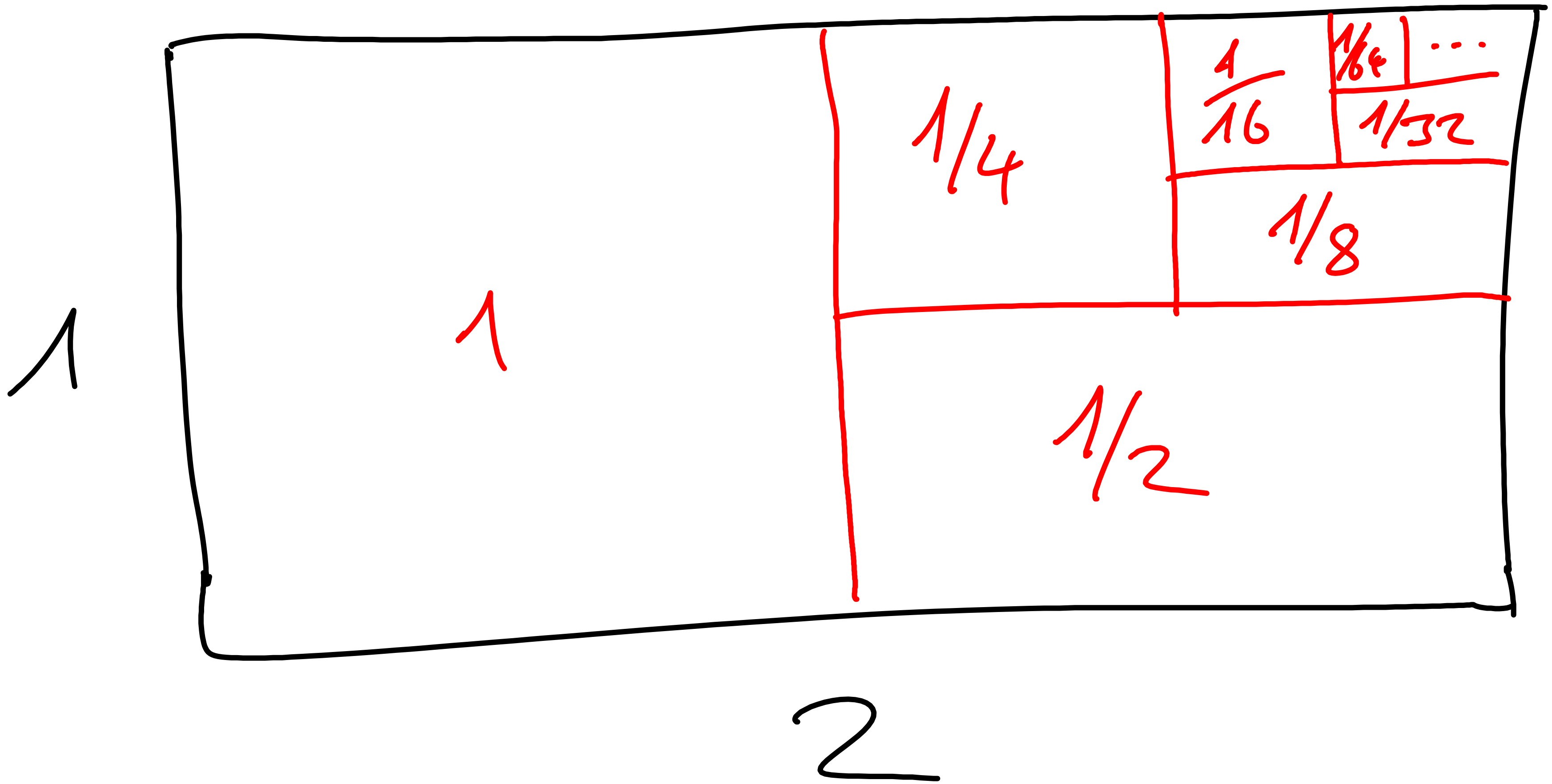
$$\left|\frac{1}{x}\right| \leq 1 \quad \forall \; x \in [1, \infty)$$



$$|a_n - a| < \varepsilon \quad \forall\, n \geq n_0$$

Bem: $n_0 = n_0(\varepsilon)$

Rechteck

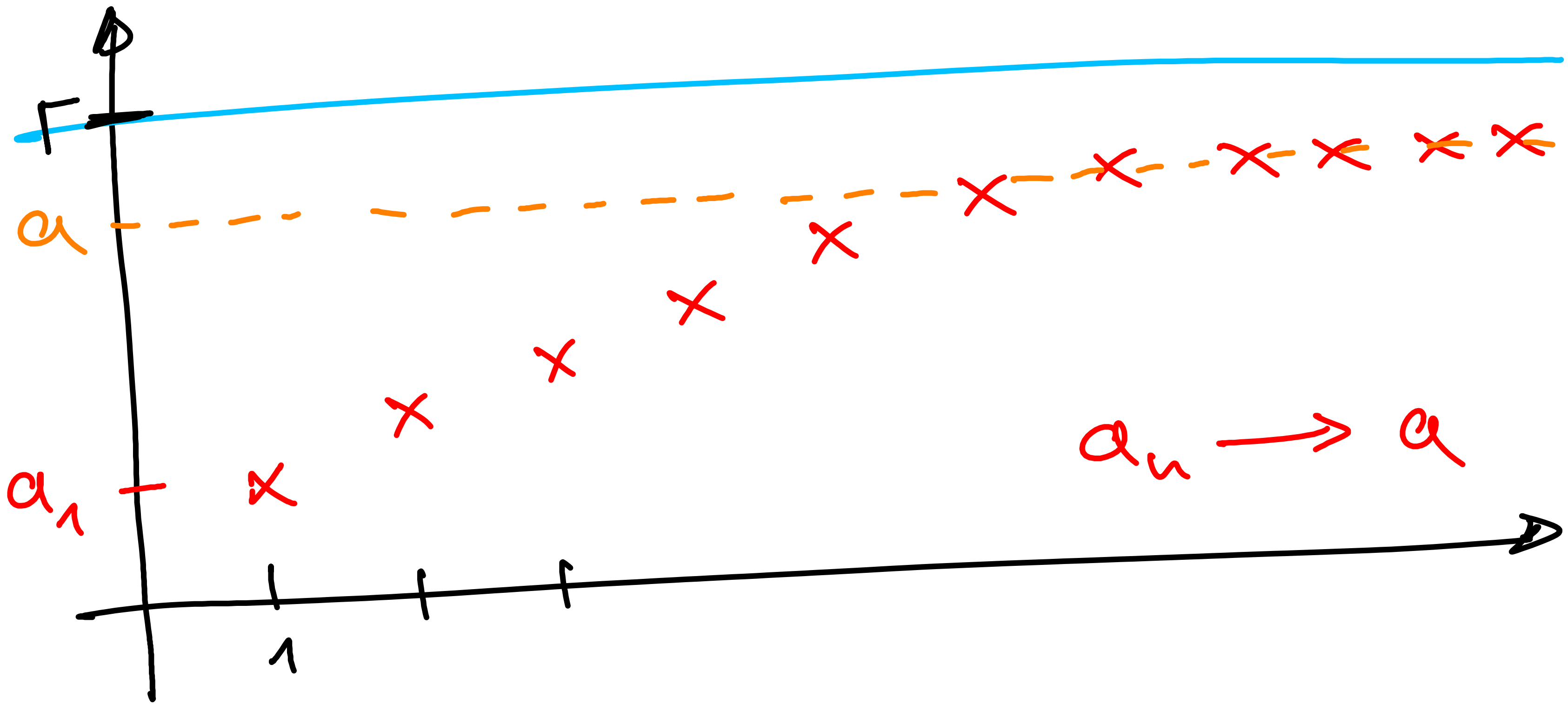


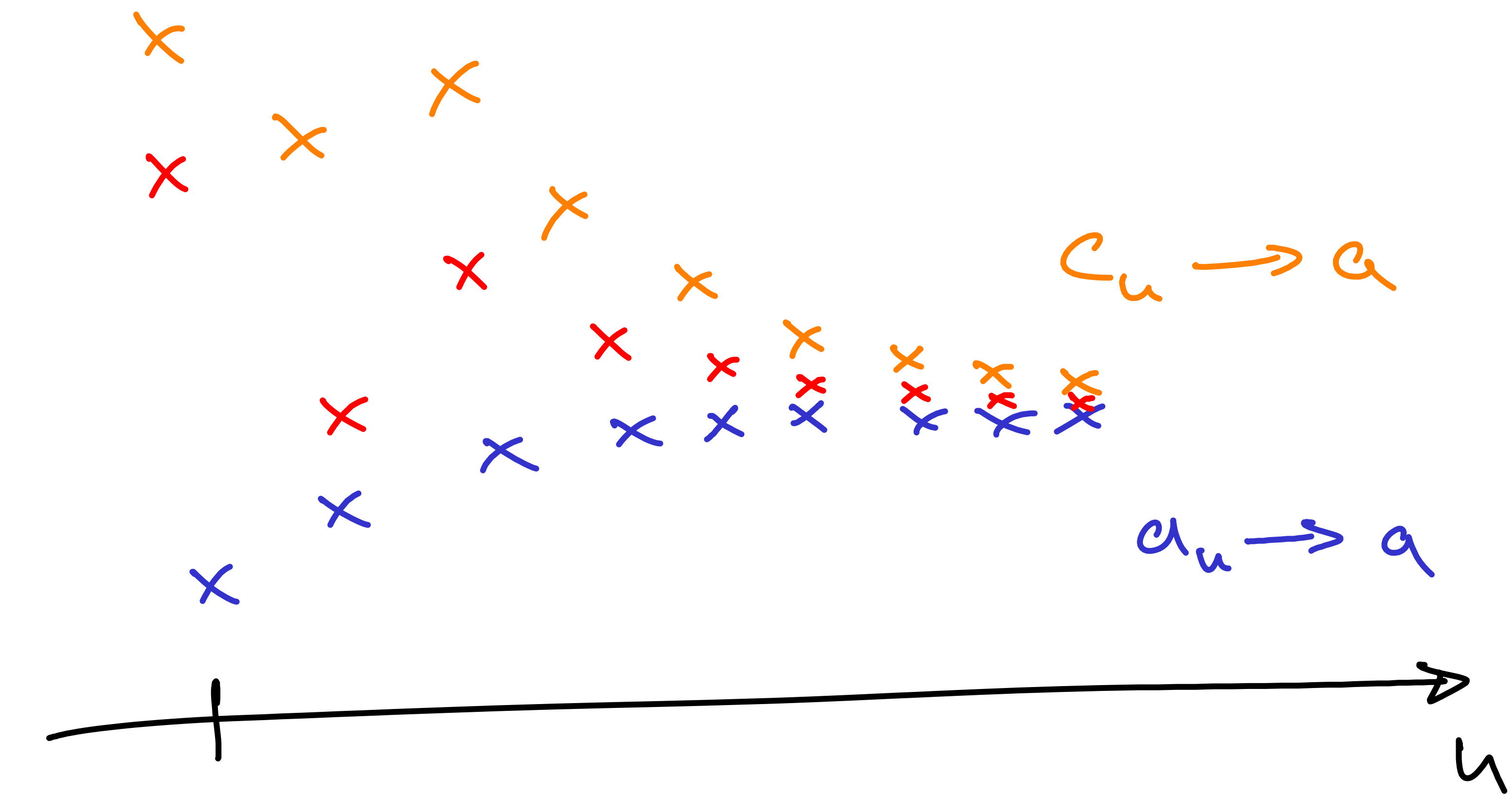
Fläche $= 2 = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots$

Folge a_n, monoton wachsend, d.h.

$$a_{n+1} \geq a_n \quad \forall \; n$$

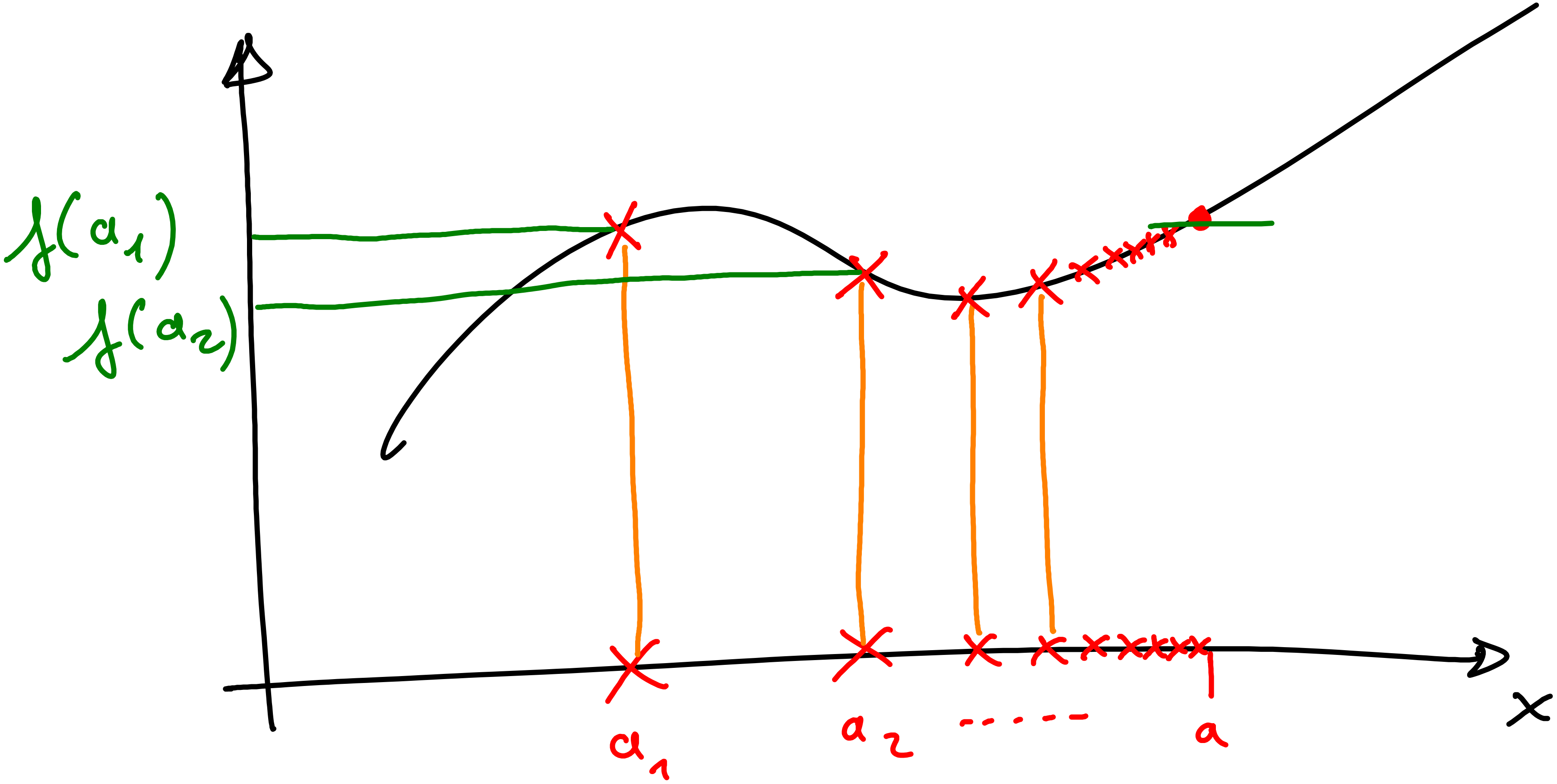
und beschränkt $|a_n| \leq r \;\; \forall n$





$$a_n \le b_n \le c_n$$

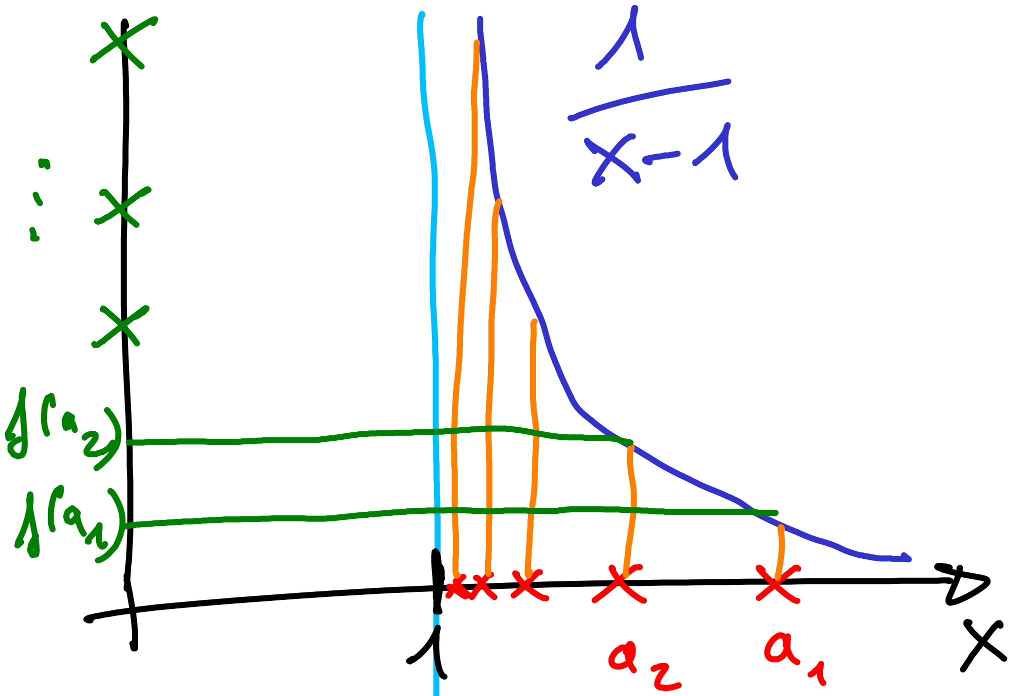
$$\Longrightarrow \quad b_n \to a$$



$a_n \longrightarrow a$

Stetighet $\lim\limits_{n\to\infty} f(a_n) = f(\lim\limits_{n\to\infty} a_n) = f(a)$

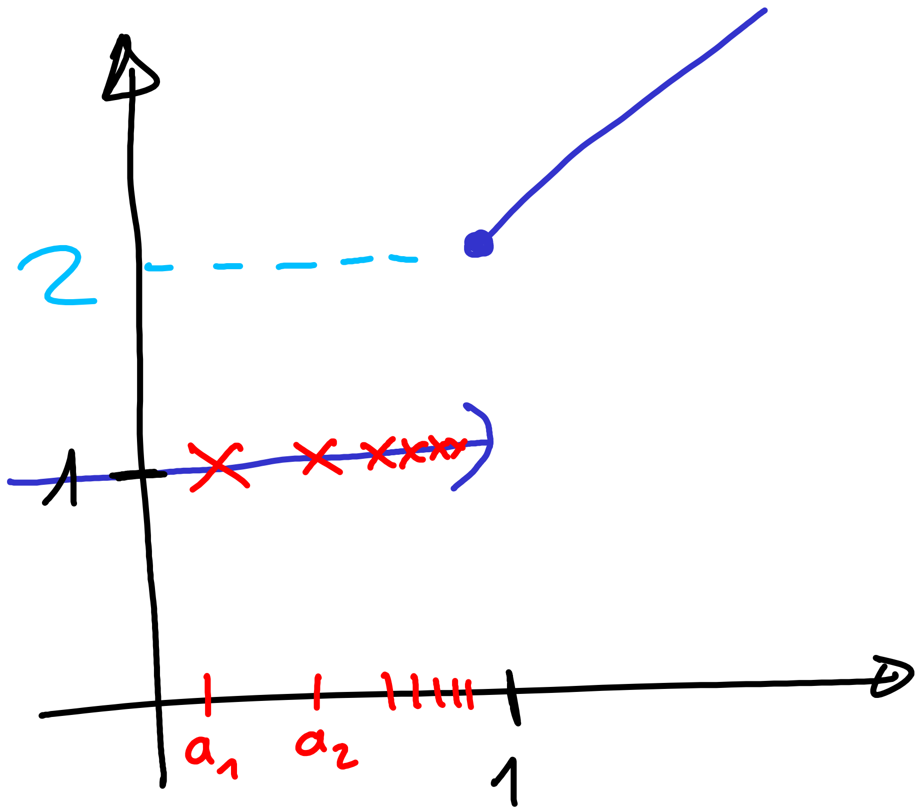
1) $f(a_n)$ divergiert



nicht stetig

$a_n \xrightarrow[n\to\infty]{} 1$

2) $f(a_n) \to b \neq f(a)$

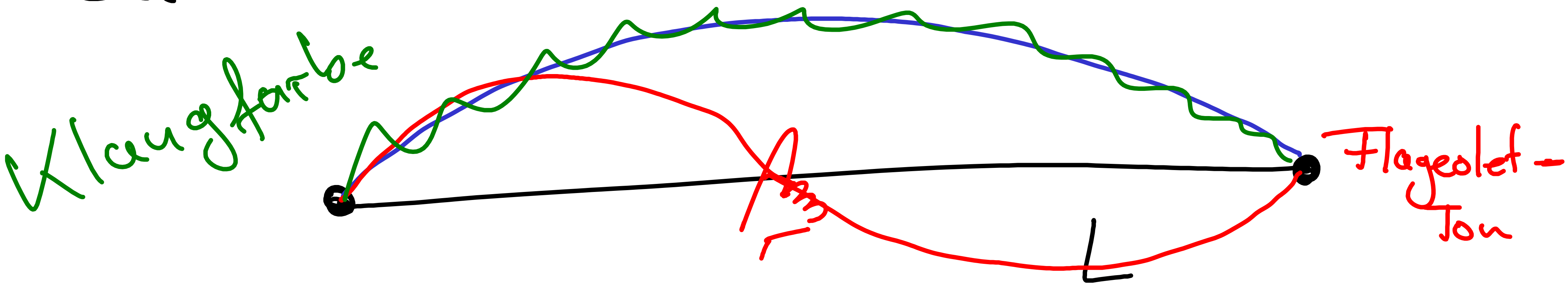


$a_n \to 1$, $a_n < 1$

$f(a_n) = 1 \quad \forall n$

$\lim\limits_{n\to\infty} f(a_n) = 1 \neq f(1) = 2$

Saite



Wellenlänge: $\lambda_0 = 2L$ $\qquad \lambda_1 = L$

Frequenz: $\nu_0 = \frac{c}{\lambda_0}$ (c: Ausbreitungsgeschw.)

$\nu_1 = 2\nu_0$

(Kreisfrequenz): $\omega_0 = 2\pi\nu_0$

Grundton $\qquad$ Oberton

$a_n = q^n$

1) $q > 1$: Annahme: beschränkt, d.h. $\exists r > 0$:

$$|a_n| < r \quad \forall n$$

<u>aber</u>: $a_n = q^n < r$ (log monoton wachsend)

$$n \log q < \log r \quad \Big| \frac{1}{\log q}$$

log q > 0 da q > 1

$$n < \frac{\log r}{\log q}$$

↯ Widerspruch zur Annahme dass a_n beschränkt

Beweis: geom. Reihe

zunächst: $S_n = \sum_{k=0}^{n} q^k = \dfrac{1-q^{n+1}}{1-q}$, $q \neq 1$

Beweis: $S_n = 1 + q + q^2 + \dots + q^n$

$q\,S_n = q + q^2 + q^3 + \dots + q^{n+1}$

Differenz: $S_n - q\,S_n = 1 - q^{n+1}$

$\Rightarrow S_n = \dfrac{1-q^{n+1}}{1-q}$

jetzt noch $n \to \infty$: $q^{n+1} \xrightarrow[n\to\infty]{} 0$ falls $|q|<1$

also $\lim\limits_{n\to\infty} S_n = \dfrac{1}{1-q}$ $\square$