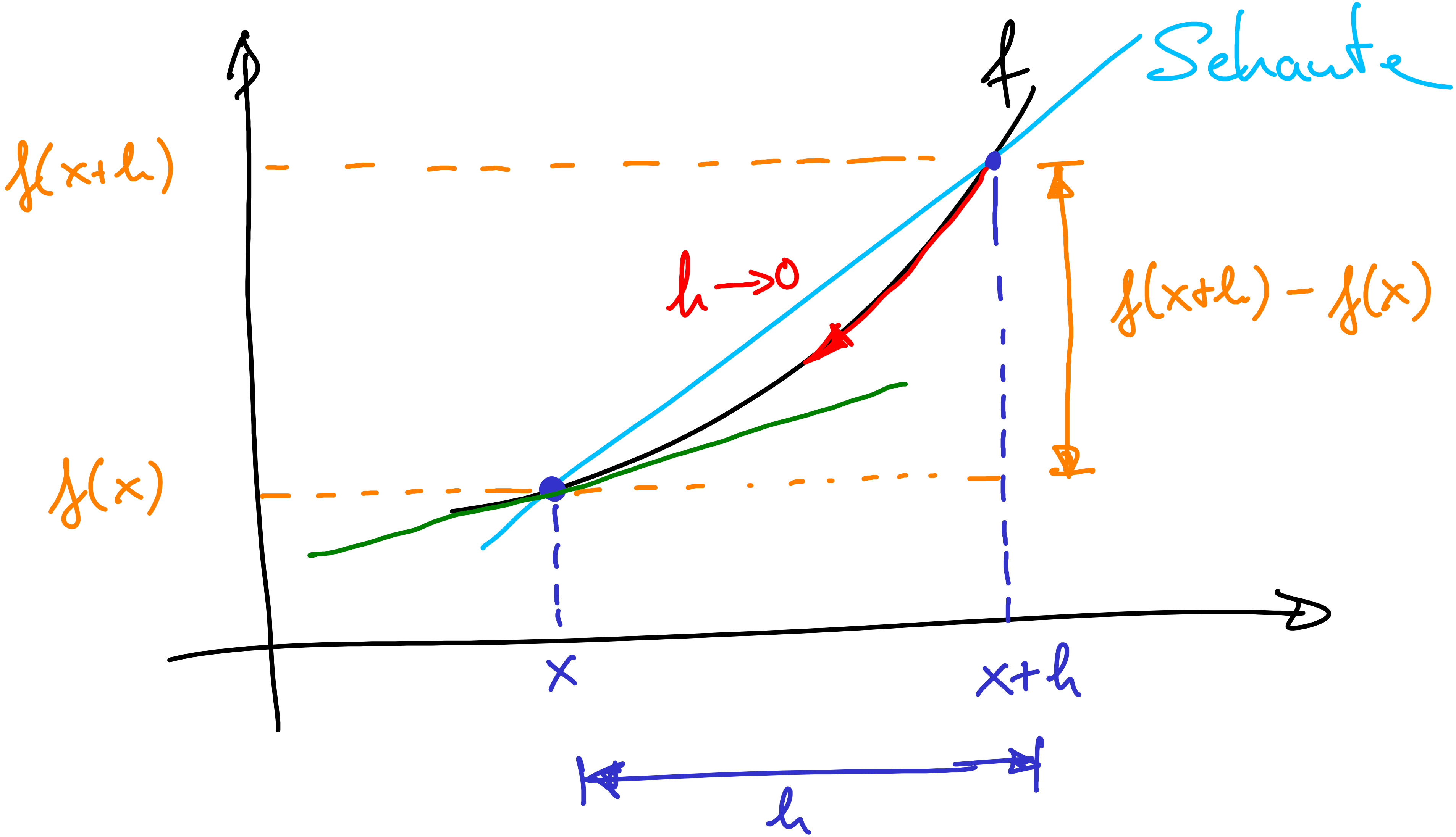


Sekante $\xrightarrow[h \to 0]{}$ Tangente

Ableitung $f'(x)$: Steigung der Tangente

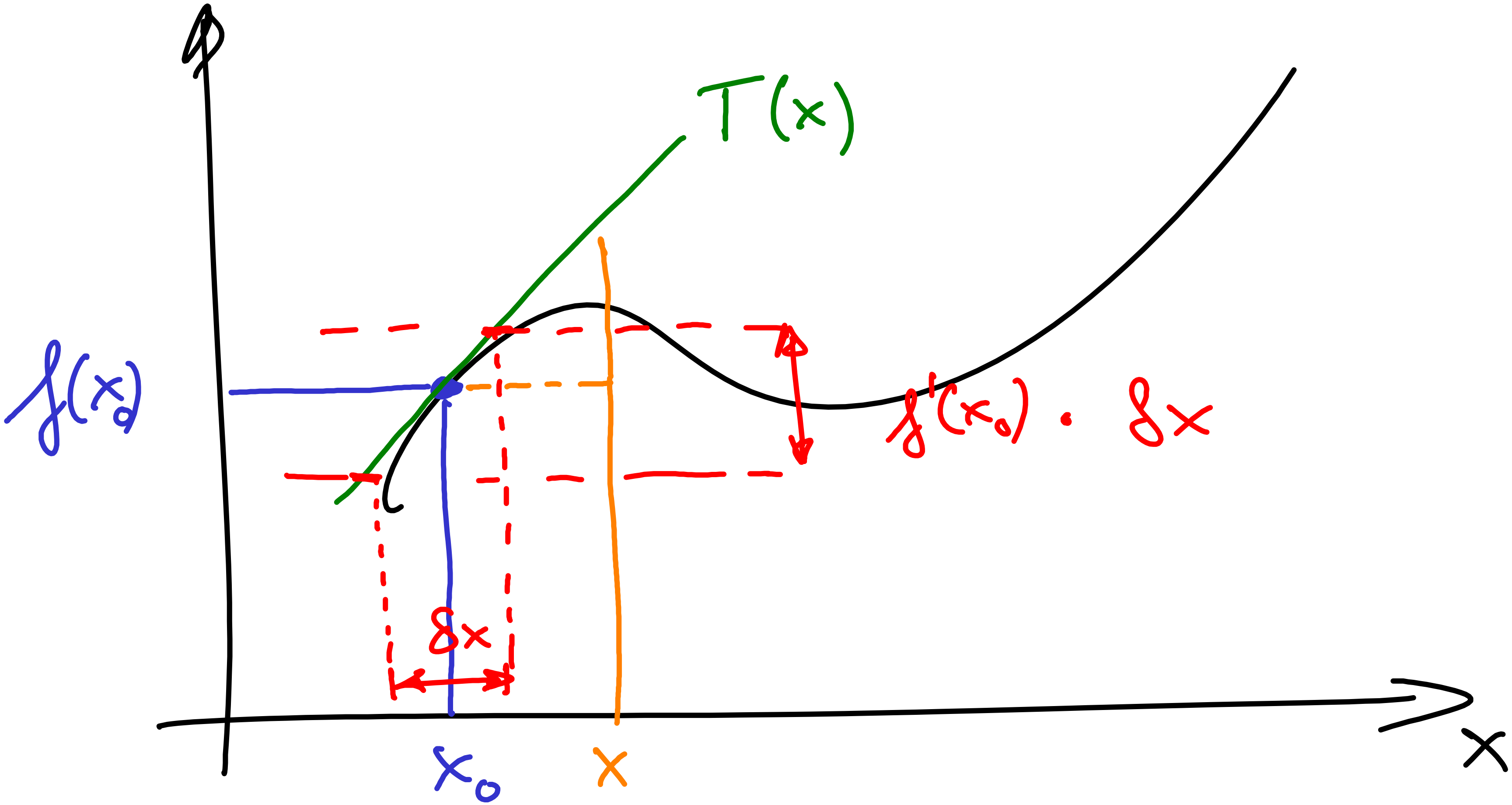
Notation

$$f^{(4)} = f''''$$

$$\left(f^{(0)} = f \right)$$

$$\frac{d}{dx} f(x) = f'(x)$$

$$\frac{d^2}{dx^2} f(x) = f''(x)$$

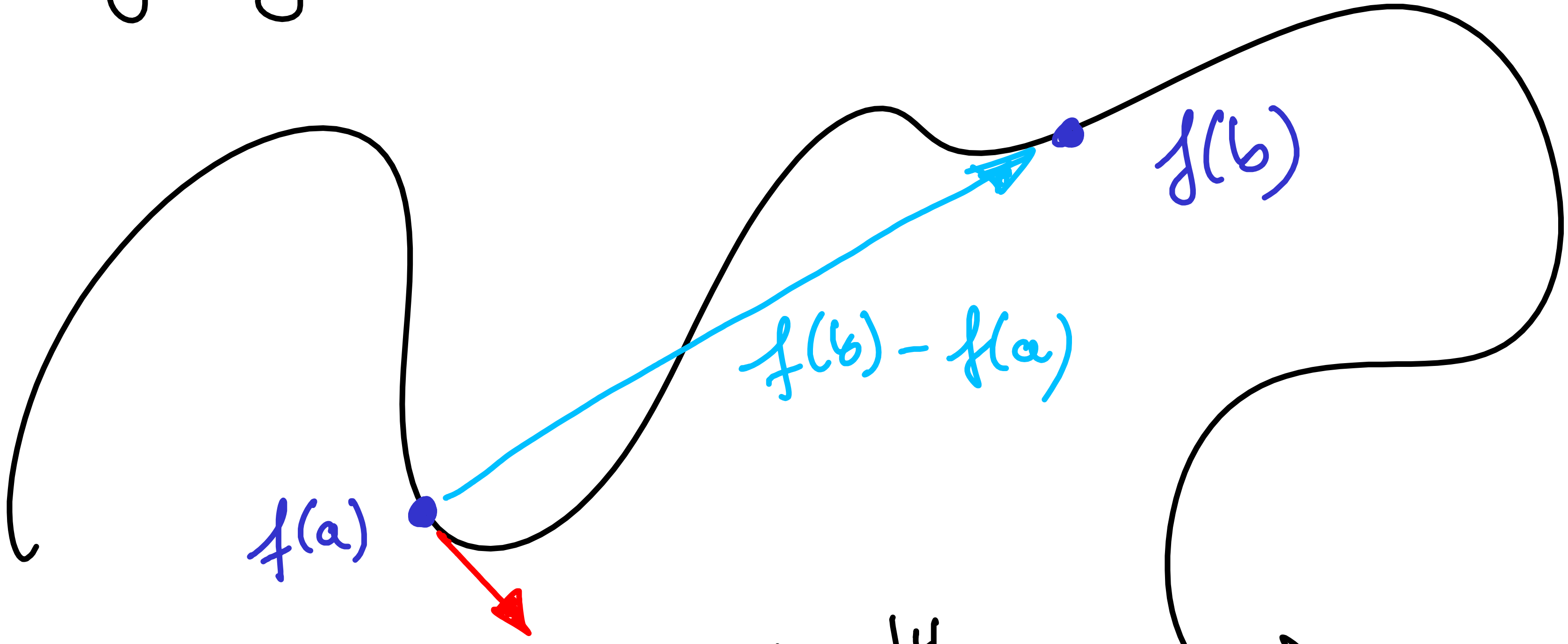


$$T(x) = f(x_0) + f'(x_0)\,(x - x_0)$$

nähere Fehler $\quad \delta y = \delta f(x) \approx \delta T(x) = |f'(x_0)|\,\delta x$

(Meßwert x_0)

Bewegung im $\mathbb{R}^2$



$$\bar{v} = \frac{f(b) - f(a)}{b - a}$$ (mittlere Geschwindigkeit)

$f'(a)$ Momentangeschwindigkeit

$$f(x) = x^n \quad , \quad n \in \mathbb{N}$$

$$\frac{f(x+h) - f(x)}{h} = \frac{(x+h)^n - x^n}{h}$$

$$= \frac{(x+h)(x+h)\cdots(x+h) - x^n}{h}$$

$$= \frac{\cancel{x^n} + \cancel{h}\, x^{n-1} n + h^{\cancel{2}}(\dots) + \dots + h^{\cancelto{n-1}{n}} - \cancel{x^n}}{\cancel{h}}$$

$$\xrightarrow[h \to 0]{} n\, x^{n-1}$$

$$f(x) = e^x = \exp(x)$$

$$\lim_{h\to 0} \frac{e^{x+h} - e^x}{h} = \lim_{h\to 0} \frac{e^x \cdot e^h - e^x}{h}$$

$$= \lim_{h\to 0} e^x \frac{e^h - 1}{h} = e^x \lim_{h\to 0} \frac{e^h - 1}{h}$$

(genauso für $f(x) = a^x$, $a \in \mathbb{R}$)

$$f'(x) = e^x f'(0)$$

$e = 2{,}718\ldots$ so gewählt, daß $\lim_{h\to 0} \frac{e^h - 1}{h} = 1$

damit $(e^x)' = e^x$

$$\lim_{h \to 0} \frac{f(x+h)\,g(x+h) - f(x)g(x)}{h}$$

$$= \lim_{h \to 0} \frac{f(x+h)\,g(x+h) - f(x+h)\,g(x) + f(x+h)\,g(x) - f(x)g(x)}{h}$$

$$= \lim_{h \to 0} f(x+h)\,\frac{g(x+h) - g(x)}{h} + \lim_{h \to 0} g(x)\,\frac{f(x+h) - f(x)}{h}$$

$$= f(x)\;g'(x) + g(x)\;f'(x)$$

$$(e^{2x})' = e^{2x} \cdot (2x)'$$
$$= e^{2x} \cdot 2$$

$$\left(\sin(x^2)\right)' = \cos(x^2) \cdot (2x)$$
$$= 2x \cos(x^2)$$

$$f(x) = e^x \quad , \quad f^{-1}(x) = \log x$$

$$f'(x) = e^x$$

$$f^{-1\,\prime}(y) = \frac{1}{f'(f^{-1}(y))} = \frac{1}{e^{f^{-1}(y)}} = \frac{1}{e^{\log y}}$$

$$= \frac{1}{y}$$

also $\quad (\log x)' = \frac{1}{x}$

$f(x) = \sin x$, $f^{-1}(x) = \arcsin x$, $f'(x) = \cos x$

$$f^{-1}(y) = \frac{1}{f'(f^{-1}(y))} = \frac{1}{\cos(f^{-1}(y))} = \frac{1}{\cos(\arcsin \cancel{y}\, x)}$$

$\sin^2 x = (\sin x)^2$

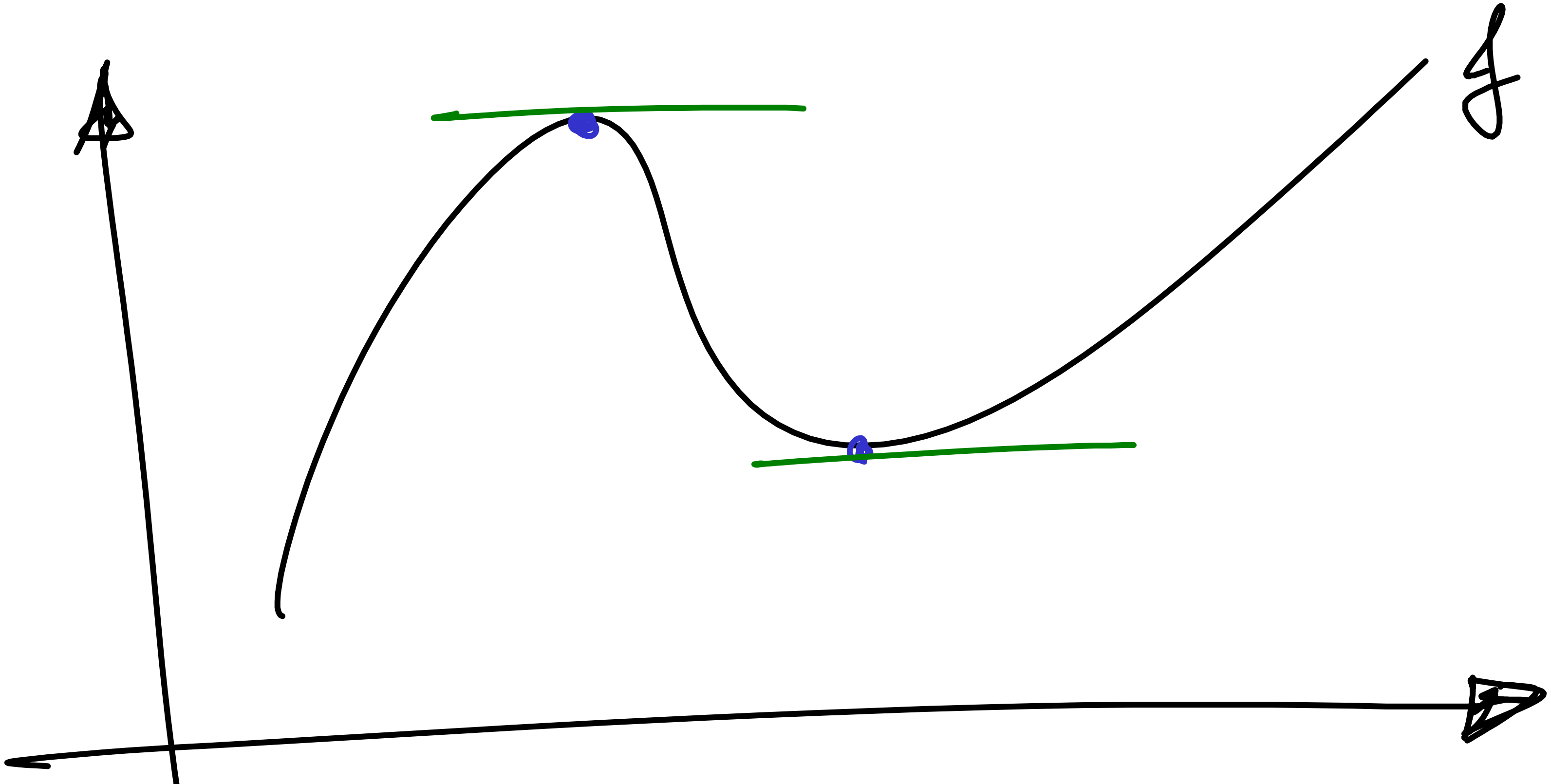
$$= \frac{1}{\sqrt{1 - \sin^2(\arcsin \cancel{y}\, x)}}$$

$\cos^2 x + \sin^2 x = 1$

$$= \frac{1}{\sqrt{1 - \cancel{y}\, x^2}}$$

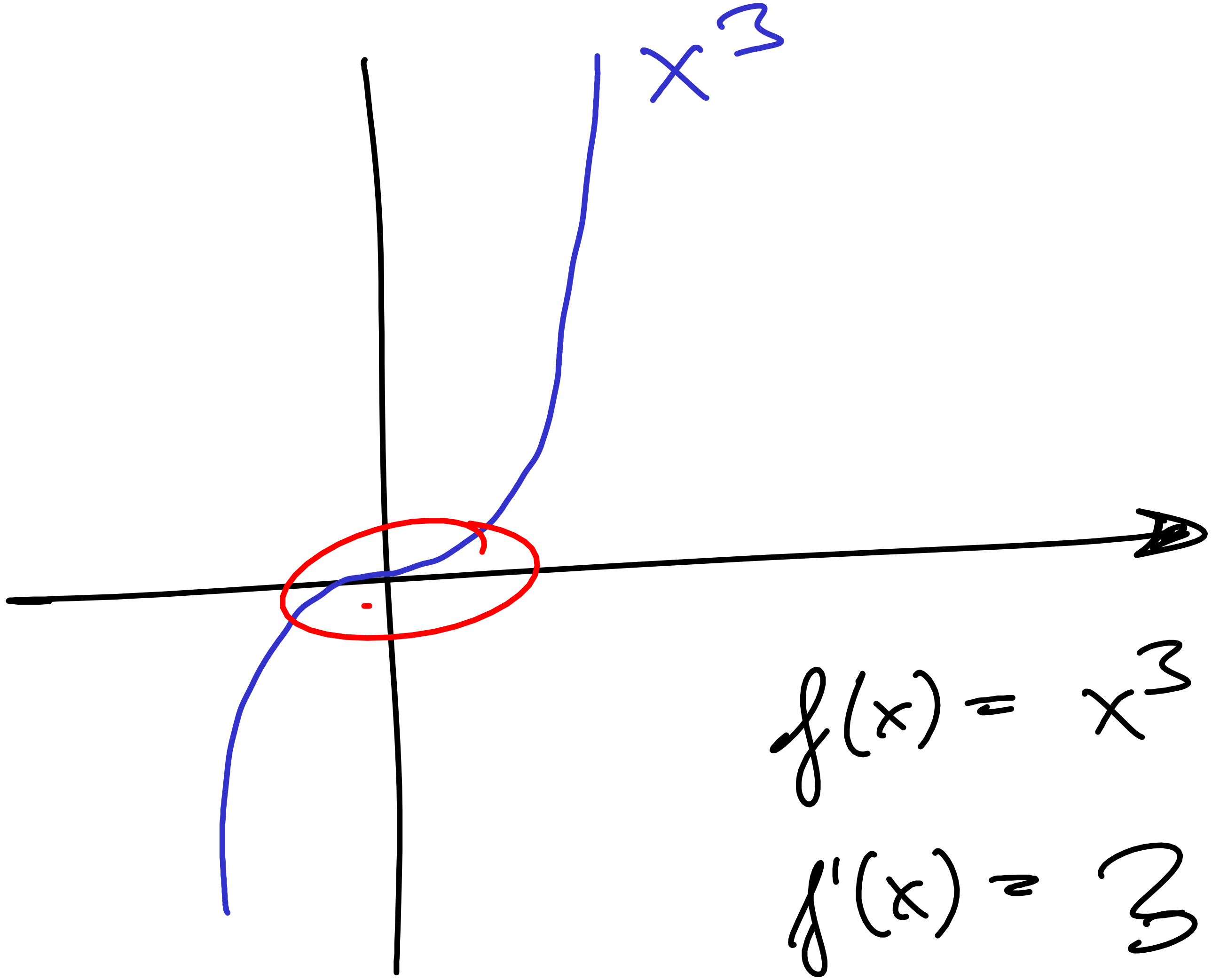
also

$$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}}$$



Extremum $\Longrightarrow f' = 0$

(an der Stelle)



$$f(x) = x^3$$

$$f'(x) = 3x^2$$

$$f'(0) = 0$$