Massenberechnung Würfel

$$m = \int_0^1 \int_0^1 \int_0^1 (1 - z + xy)\, dz\, dy\, dx$$

$$= \int_0^1 \int_0^1 \left[z - \frac{z^2}{2} + xyz \right]_{z=0}^{z=1} dy\, dx$$

$$= \int_0^1 \int_0^1 \left(\underbrace{1 - \frac{1}{2}}_{1/2} + xy \right) dy\, dx$$

$$= \int_0^1 \left[\frac{1}{2} y + \frac{x}{2} y^2 \right]_{y=0}^{y=1} dx$$

$$= \int_0^1 \left(\frac{1}{2} + \frac{x}{2} \right) dx = \left[\frac{x}{2} + \frac{x^2}{4} \right]_0^1 = \frac{1}{2} + \frac{1}{4} = \frac{3}{4}$$

Reihenfolge egal

- $$\int_0^1 \int_0^\pi x \sin y \, dy \, dx$$

$$= \int_0^1 \left[-x \cos y \right]_{y=0}^{y=\pi} dx = \int_0^1 (x + x) \, dx = \int_0^1 2x \, dx$$

$$= \left[x^2 \right]_{x=0}^{x=1} = 1$$

- $$\int_0^\pi \int_0^1 x \sin y \, dx \, dy = \int_0^\pi \left[\frac{x^2}{2} \sin y \right]_{x=0}^{x=1} dy$$

$$= \int_0^\pi \frac{1}{2} \sin y \, dy = \left[-\frac{1}{2} \cos y \right]_{y=0}^{y=\pi} = \frac{1}{2} + \frac{1}{2} = 1$$

"Parabeldach"

$$V = \int_{-1}^{1} \int_{-1}^{1} \int_{0}^{2-x^2-y^2} 1 \, dz \, dy \, dx$$

muß zuerst ausgeführt werden!

$$= \int_{-1}^{1} \int_{-1}^{1} \Big[z \Big]_{z=0}^{z=2-x^2-y^2} dy \, dx$$

$$= \int_{-1}^{1} \int_{-1}^{1} \left(2 - x^2 - y^2\right) dy \, dx$$

$$= \int_{-1}^{1} \left[2y - x^2 y - \frac{1}{3} y^3 \right]_{y=-1}^{y=1} dx$$

$$= \int_{-1}^{1} \left(2 - x^2 - \frac{1}{3} - \left(-2 + x^2 + \frac{1}{3}\right)\right) dx$$

$$= \int_{-1}^{1} \left(4 - 2x^2 - \frac{2}{3}\right) dx = \int_{-1}^{1} \left(\frac{10}{3} - 2x^2\right) dx$$

$$= \left[\frac{10}{3}x - \frac{2}{3}x^3\right]_{-1}^{1}$$

$$= \frac{10}{3} - \frac{2}{3} - \left(-\frac{10}{3} + \frac{2}{3}\right) = \frac{16}{3}$$