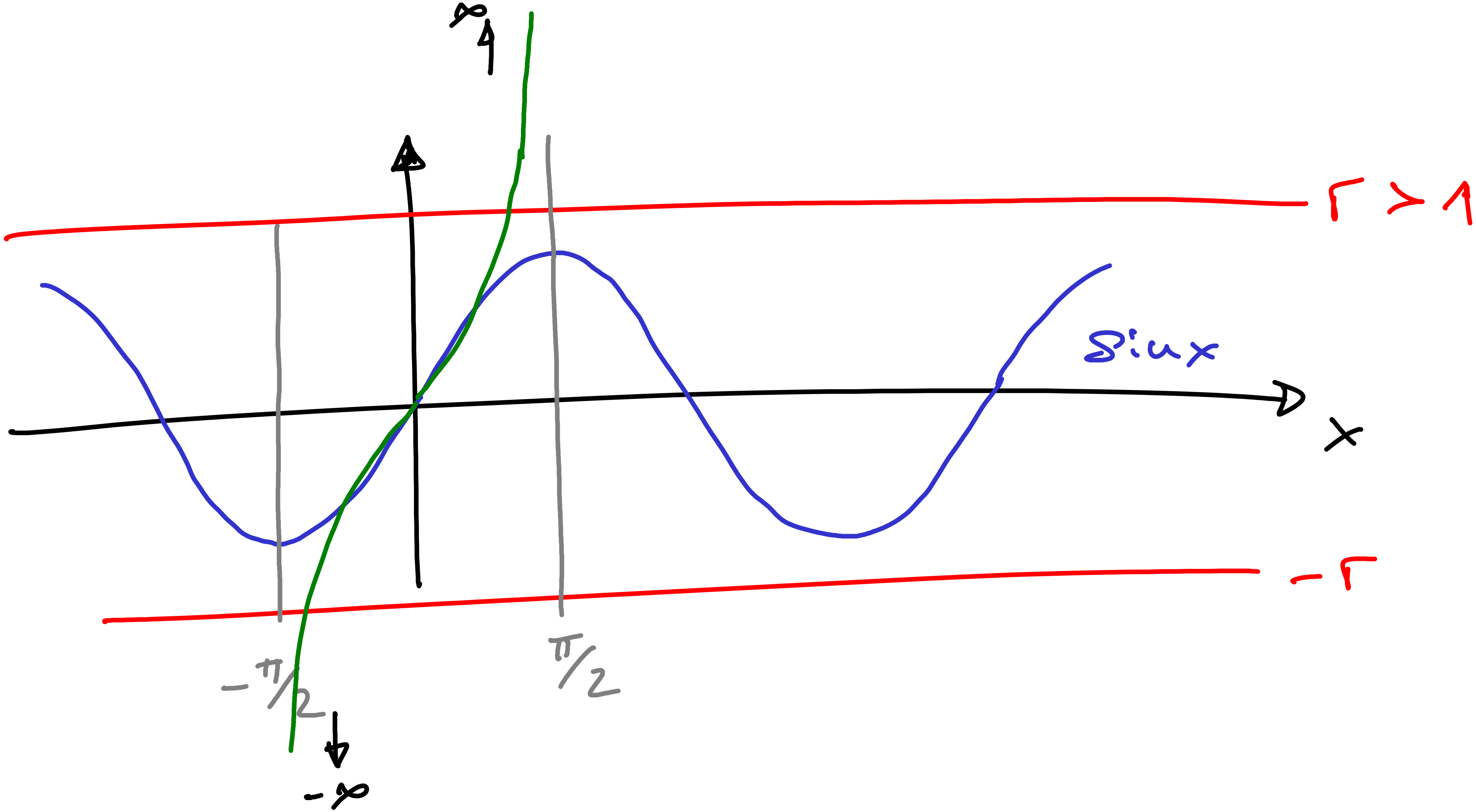
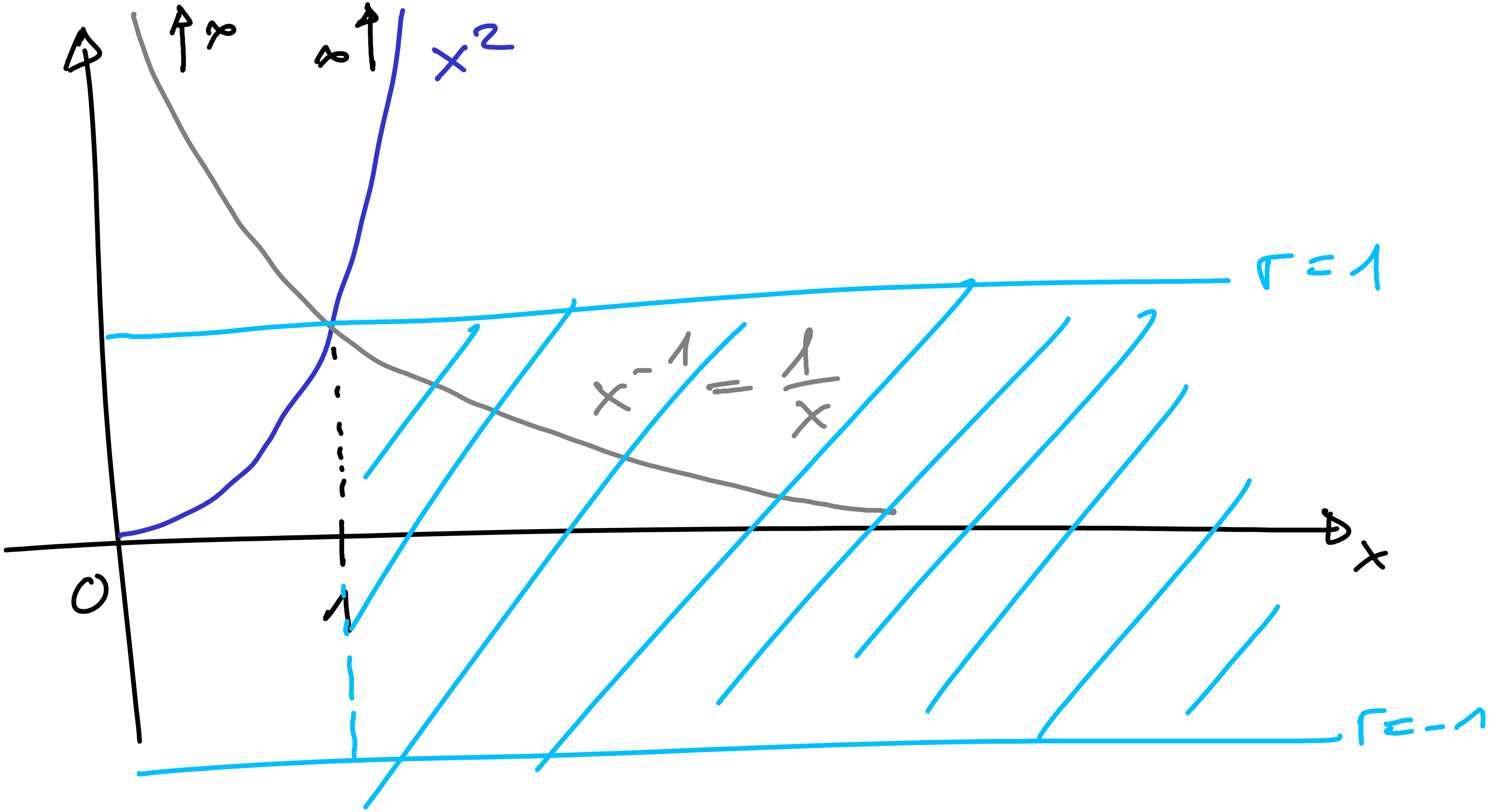
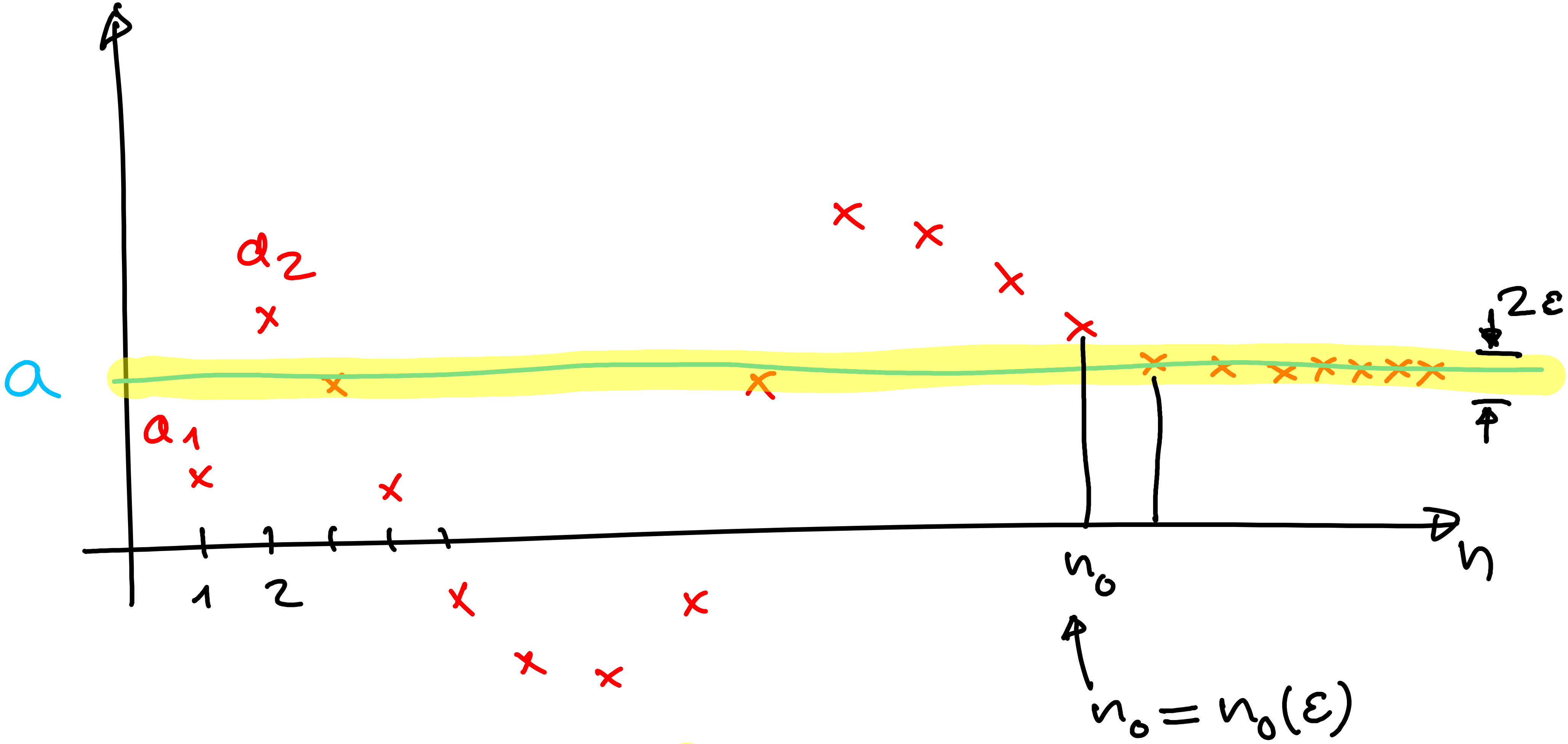


∞
$r > 1$
$\sin x$
x
$-r$
$-\pi/2$
$\pi/2$
$-\infty$



∞
∞
x^2
$r = 1$
$x^{-1} = \frac{1}{x}$
x
0
1
$r = -1$



$$|a_n - a| < \varepsilon \quad \forall \; n > n_0$$

$a_n = \frac{1}{n}$, $n \in \mathbb{N}$

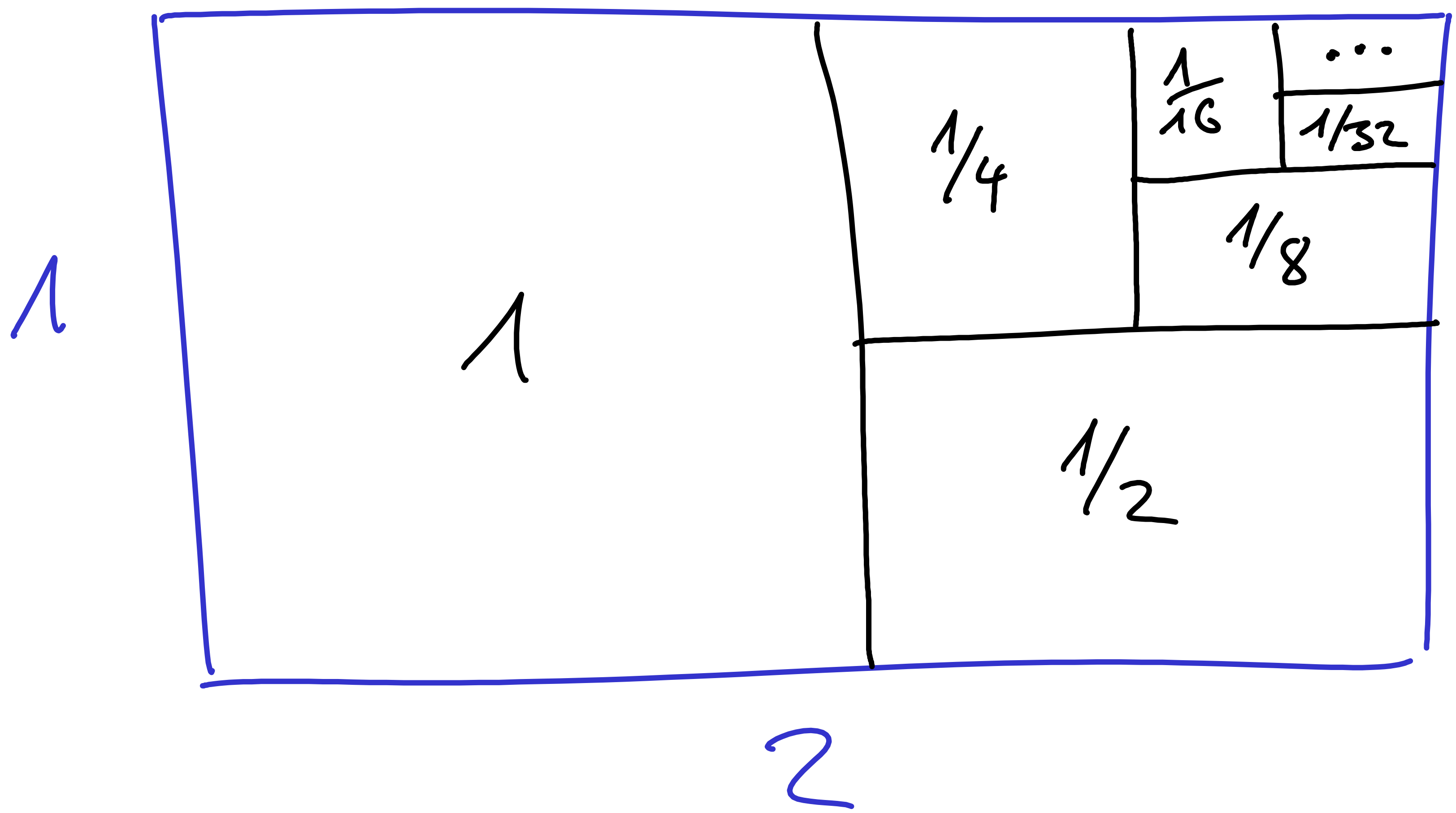
Behauptung: Limes 0

$|a_n - 0| = \left|\frac{1}{n}\right| = \frac{1}{n} < \varepsilon$

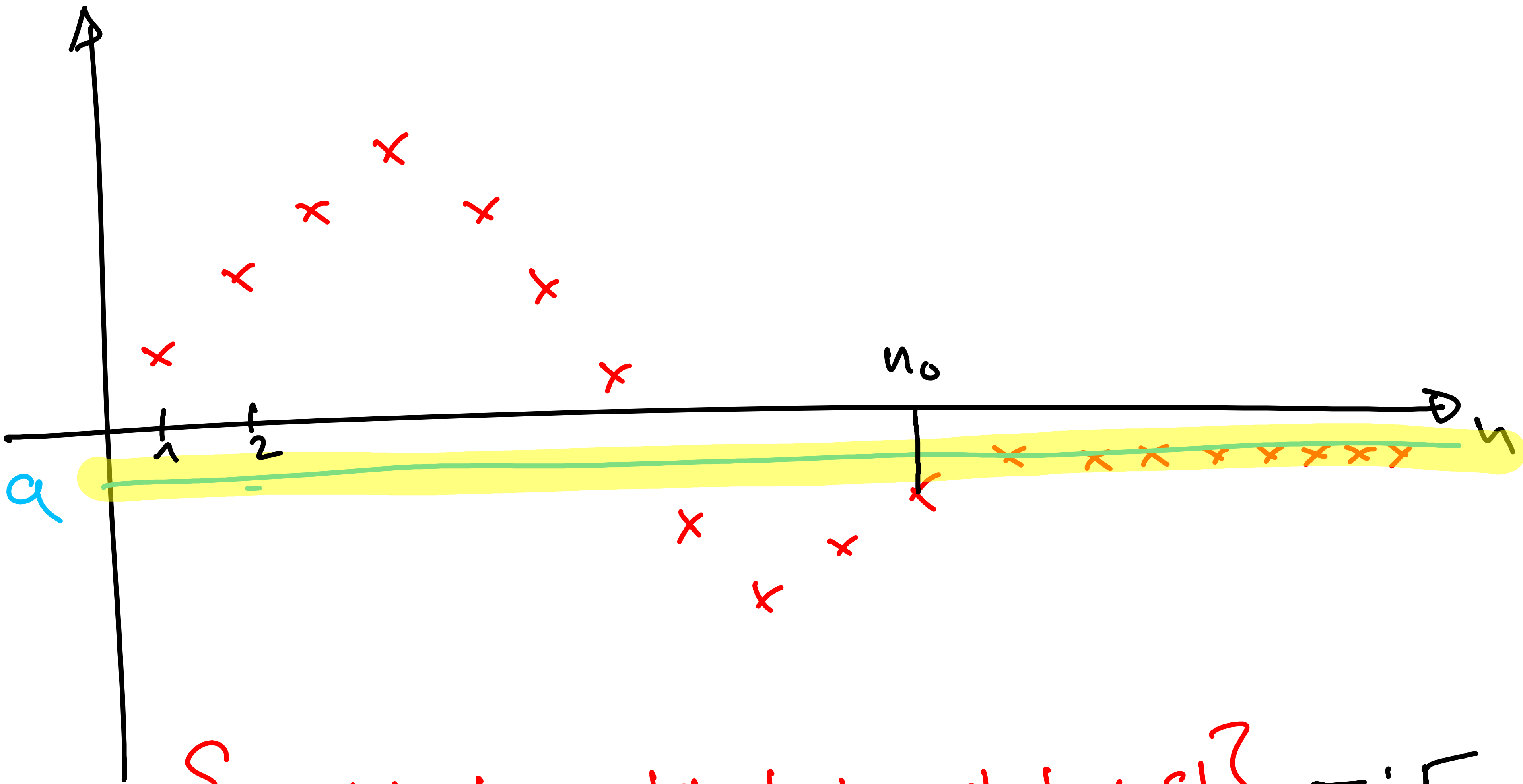
$\Leftrightarrow \quad n > \frac{1}{\varepsilon}$

Wähle $n_0 > \frac{1}{\varepsilon}$ $\square$

Rechteck



Fläche = 2 = $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots$



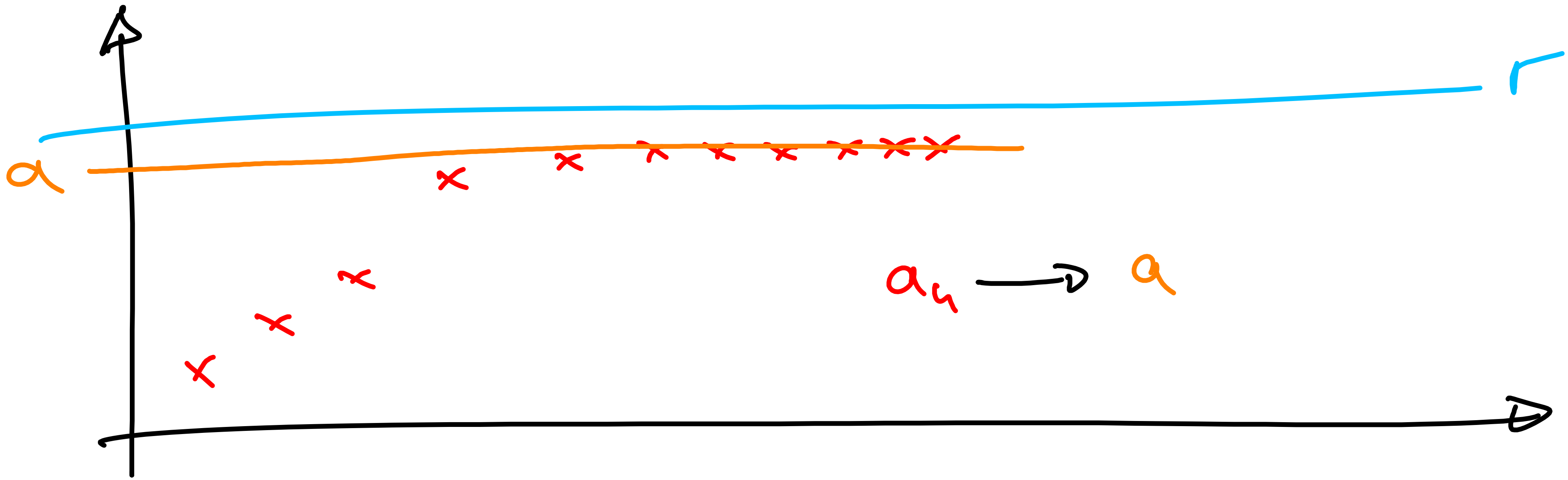
$$\max\{\underbrace{|a_1|, |a_2|, \ldots, |a_{n_0}|,}_{\text{nur endlich viele}} |a-\varepsilon|, |a+\varepsilon|\} =: r$$

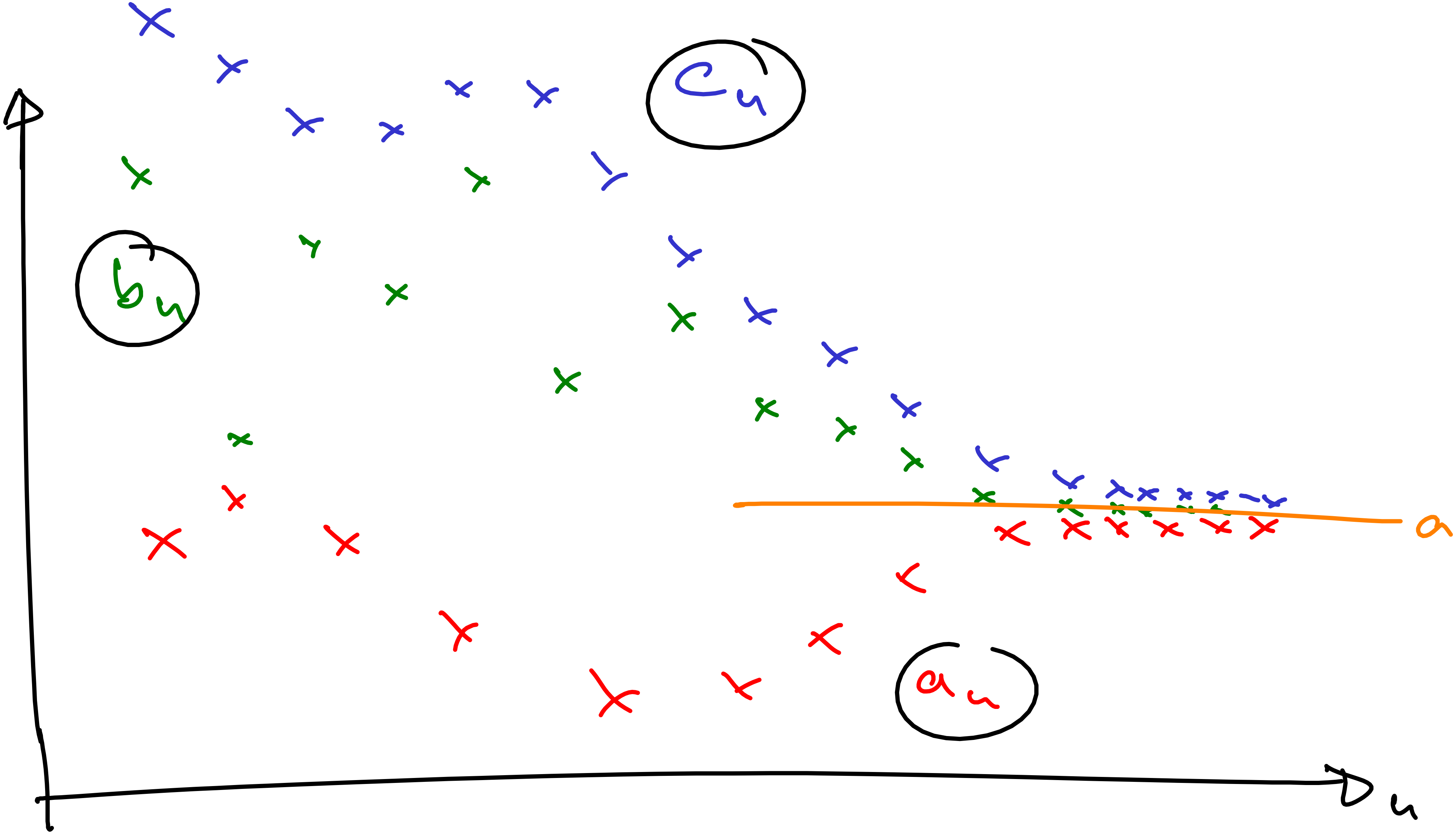
Dann ist r eine Schranke. $\square$

Folge a_n sei monoton wachsend, d.h.

$$a_{n+1} \geq a_n$$

und beschränkt $|a_n| \leq r \quad \forall n$

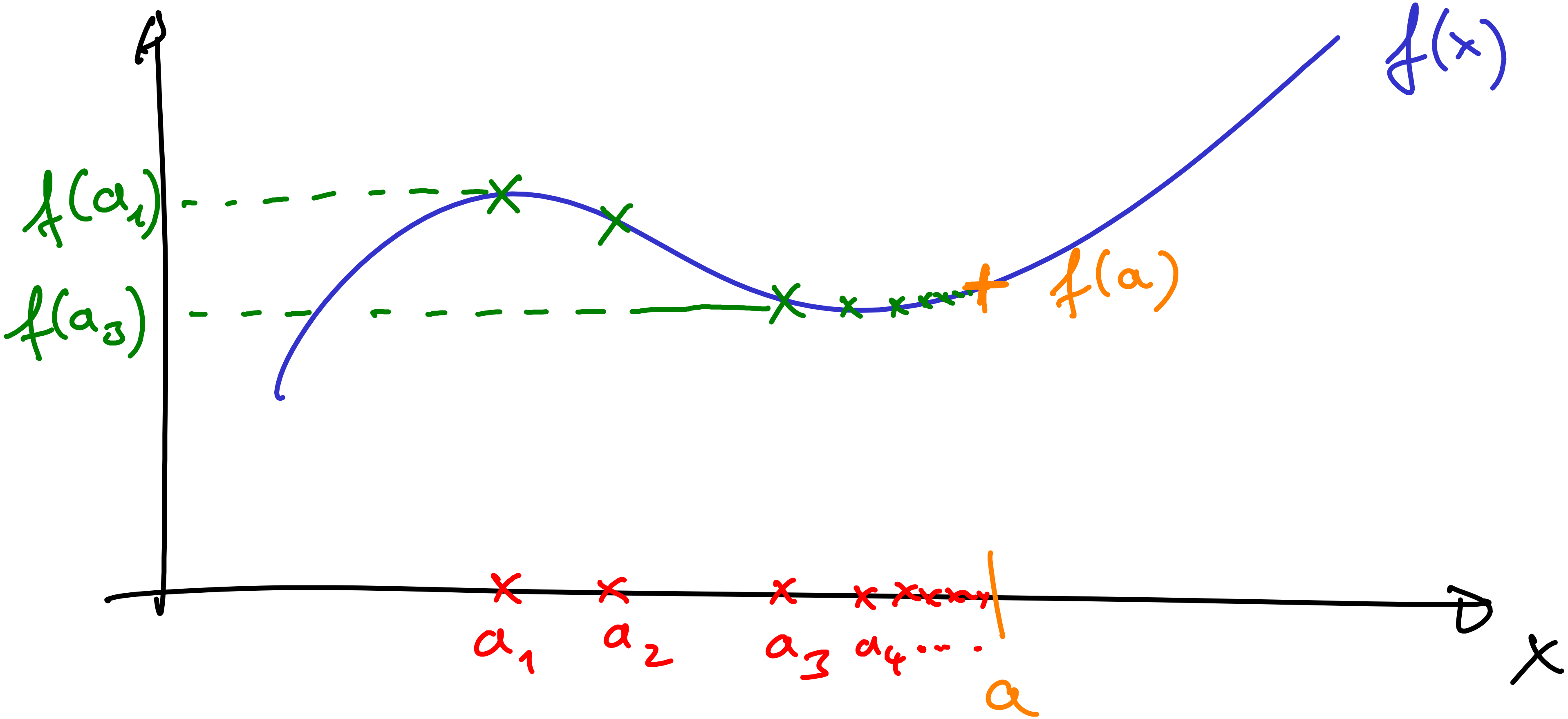




c_n
b_n
a_n
a
n

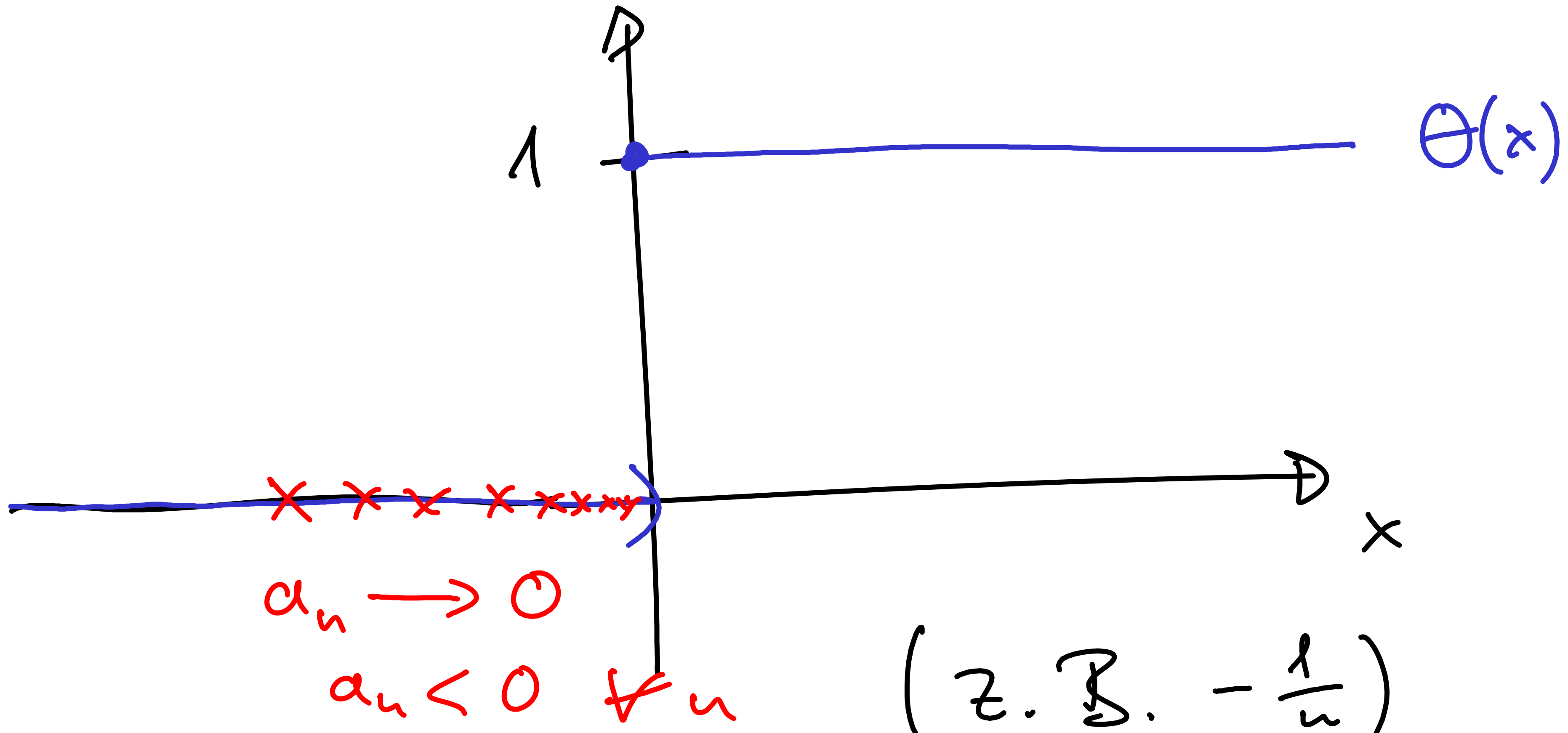
$$\lim_{n \to \infty} \frac{n^7 - 1}{2n^7 + n} = \lim_{n \to 0} \frac{1 - \frac{1}{n^7}}{2 + \frac{1}{n^6}}$$

$$= \frac{\lim_{n \to \infty} 1 - \lim_{n \to \infty} \frac{1}{n^7}}{\lim_{n \to \infty} 2 + \lim_{n \to \infty} \frac{1}{n^6}} = \frac{1 - 0}{2 - 0} = \frac{1}{2}$$



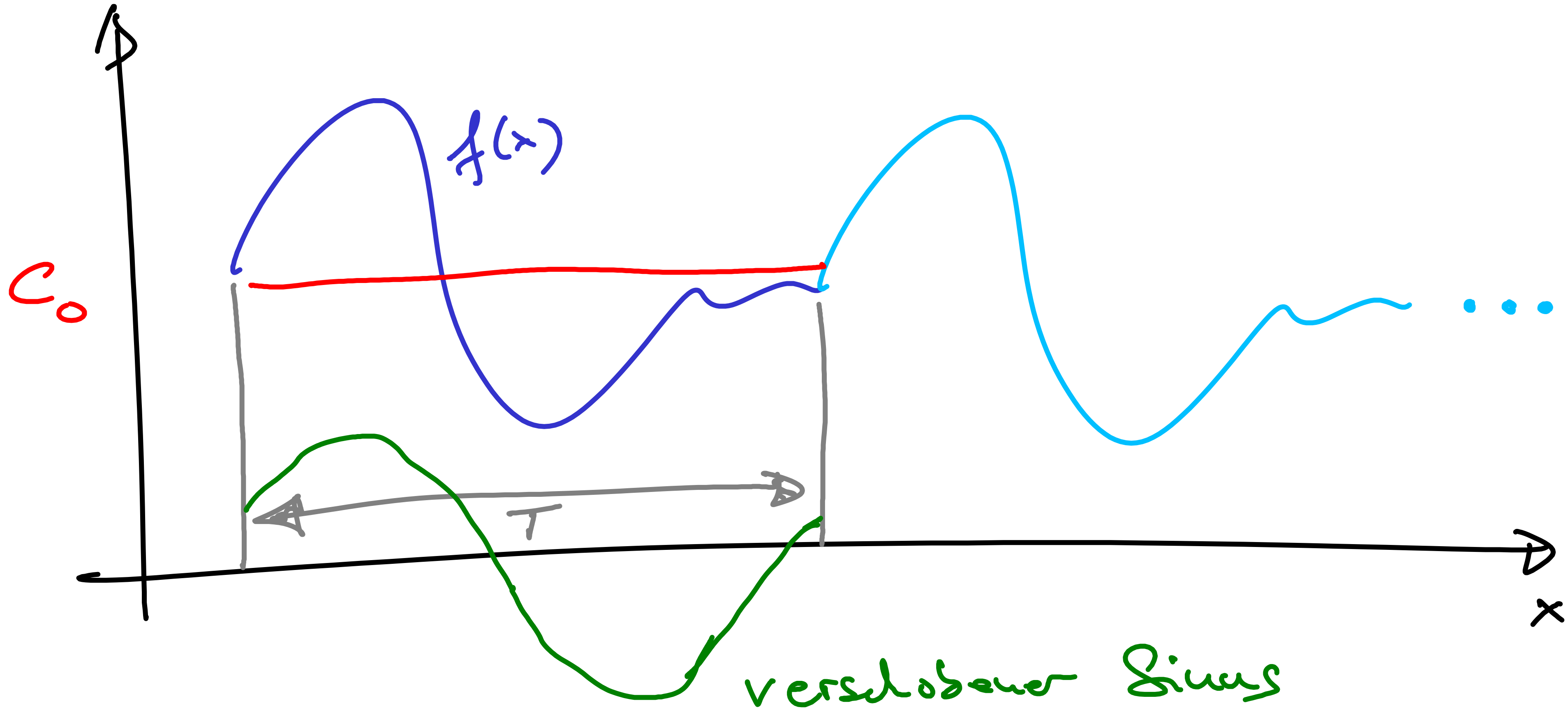
$$\lim_{n\to\infty} f(a_n) = f\left(\lim_{n\to\infty} a_n\right) = f(a)$$

Stetigkeit



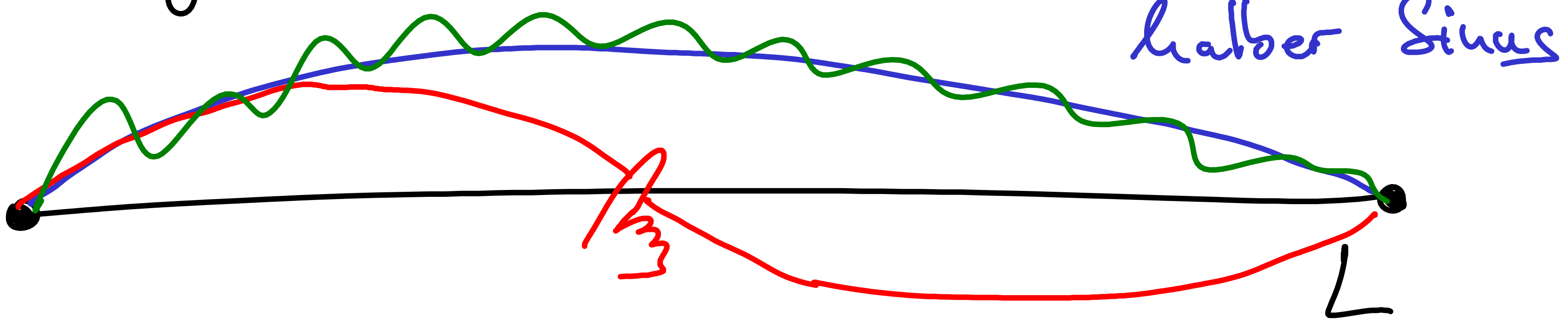
(z. B. $-\frac{1}{n}$)

$$\lim_{n\to\infty} f(a_n) = 0 \neq f\left(\lim_{n\to\infty} a_n\right) = f(0) = 1$$



$$f(x) = c_0 + c_1 \sin\left(\frac{2\pi}{T}x + \varphi_1\right) + \dots$$

Saite, Länge L



Wellenlänge: $\lambda_0 = 2L$

Flageolett-Ton

Frequenz: $\nu_0 = \frac{c}{\lambda_0}$ (c: Ausbreitungsgeschw.)

$\nu_1 = 2\nu_0$

Kreisfrequenz: $\omega_0 = 2\pi\nu_0$

Grundton

Klangfarbe mit Beiträgen von

$\nu_0,\ \nu_1 = 2\nu_0,\ 3\nu_0,\ \ldots$

geometrische Folge

① $q > 1$ Widerspruchsbeweis

Annahme: q^n beschränkt d.h. $\exists r > 0$:

$$|q^n| < r \quad \forall n$$

$$\Leftrightarrow \quad q^n < r \quad \forall n \qquad \text{(log monoton wachsend)}$$

$$\Leftrightarrow \quad n \log q < \log r \quad \Big| \cdot \frac{1}{\log q} \leftarrow \text{positiv}$$

$$\Leftrightarrow \quad n < \frac{\log r}{\log q}$$

↯ Widerspruch (wähle n groß genug)

also unbeschränkt ⇒ nicht konvergent

② $|q| < 1$: Behauptung $q^n \longrightarrow 0$

$$|q^n - 0| = |q^n| = |q|^n < \varepsilon$$

$$\underset{q \neq 0}{\Longleftrightarrow} \quad n \log|q| < \log \varepsilon \quad \Big| \; \frac{1}{\log|q|} \leftarrow \text{negativ}$$

$$\Longleftrightarrow \quad n > \frac{\log \varepsilon}{\log|q|}$$

Wähle $n_0 \in \mathbb{N}$ mit $n_0 > \dfrac{\log \varepsilon}{\log|q|}$:

$$|q^n - 0| < \varepsilon \quad \forall \; n > n_0 \qquad \square$$

geometrische Reihe

$$S_n := \sum_{k=0}^{n} q^k = 1 + q + q^2 + \dots + q^n \quad | \, q$$

$$q S_n = q + q^2 + \dots + q^n + q^{n+1}$$

Differenz

$$S_n - q S_n = 1 - q^{n+1}$$

$$\Leftrightarrow \quad S_n = \frac{1 - q^{n+1}}{1-q}$$

$|q| < 1$

$$\sum_{k=0}^{\infty} q^k = \lim_{n\to\infty} S_n = \frac{1}{1-q} \quad \square$$

$$\lim_{n\to\infty} q^{n+1} = 0$$