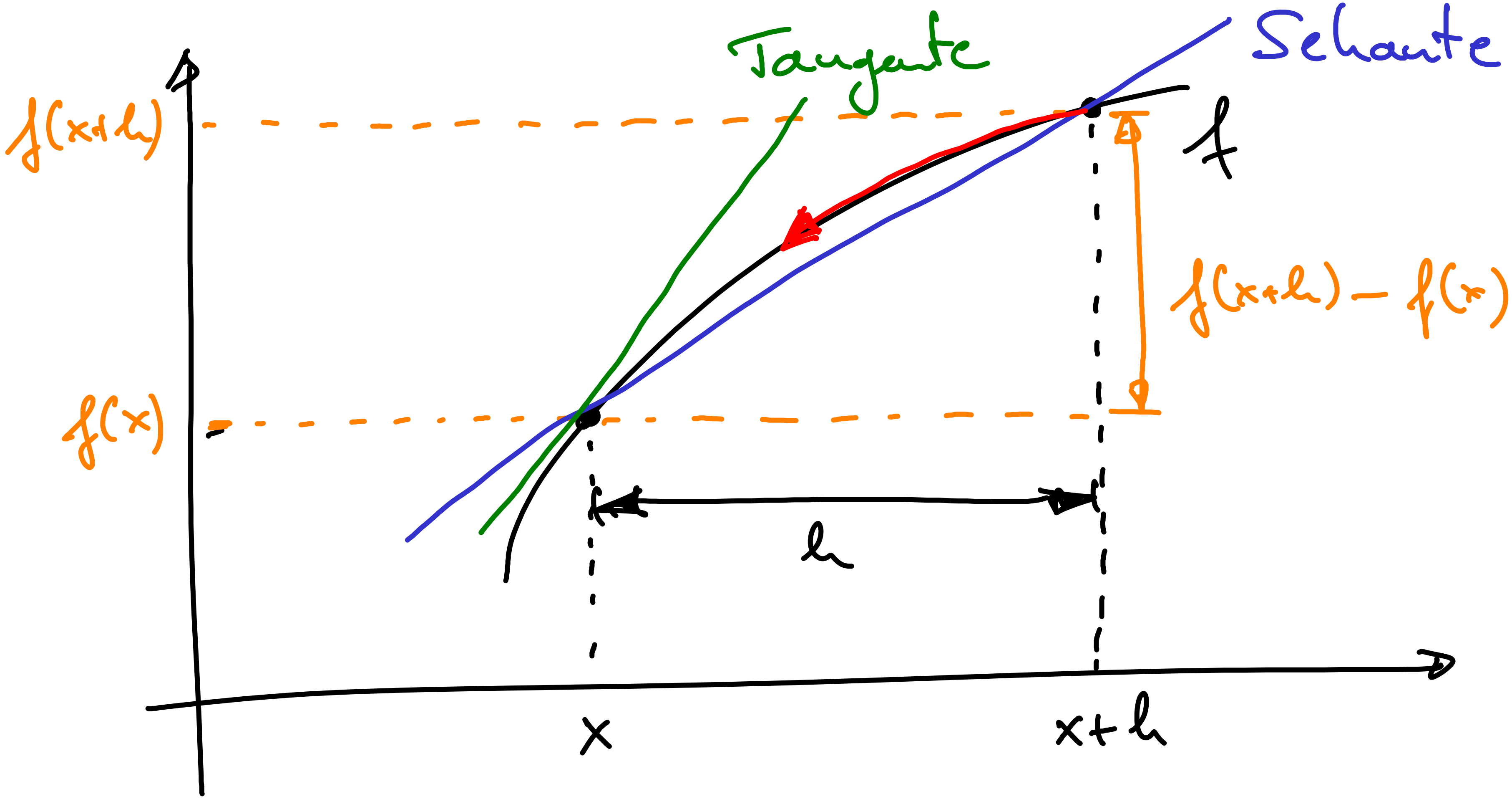


Sekante $\xrightarrow[h \to 0]{}$ Tangente

Ableitung $f'(x)$: Steigung der Tangente an der Stelle x

Notation

$$f^{(2)} = f''$$

$$f^{(5)} = f'''''$$

$$f^{(0)} = f$$

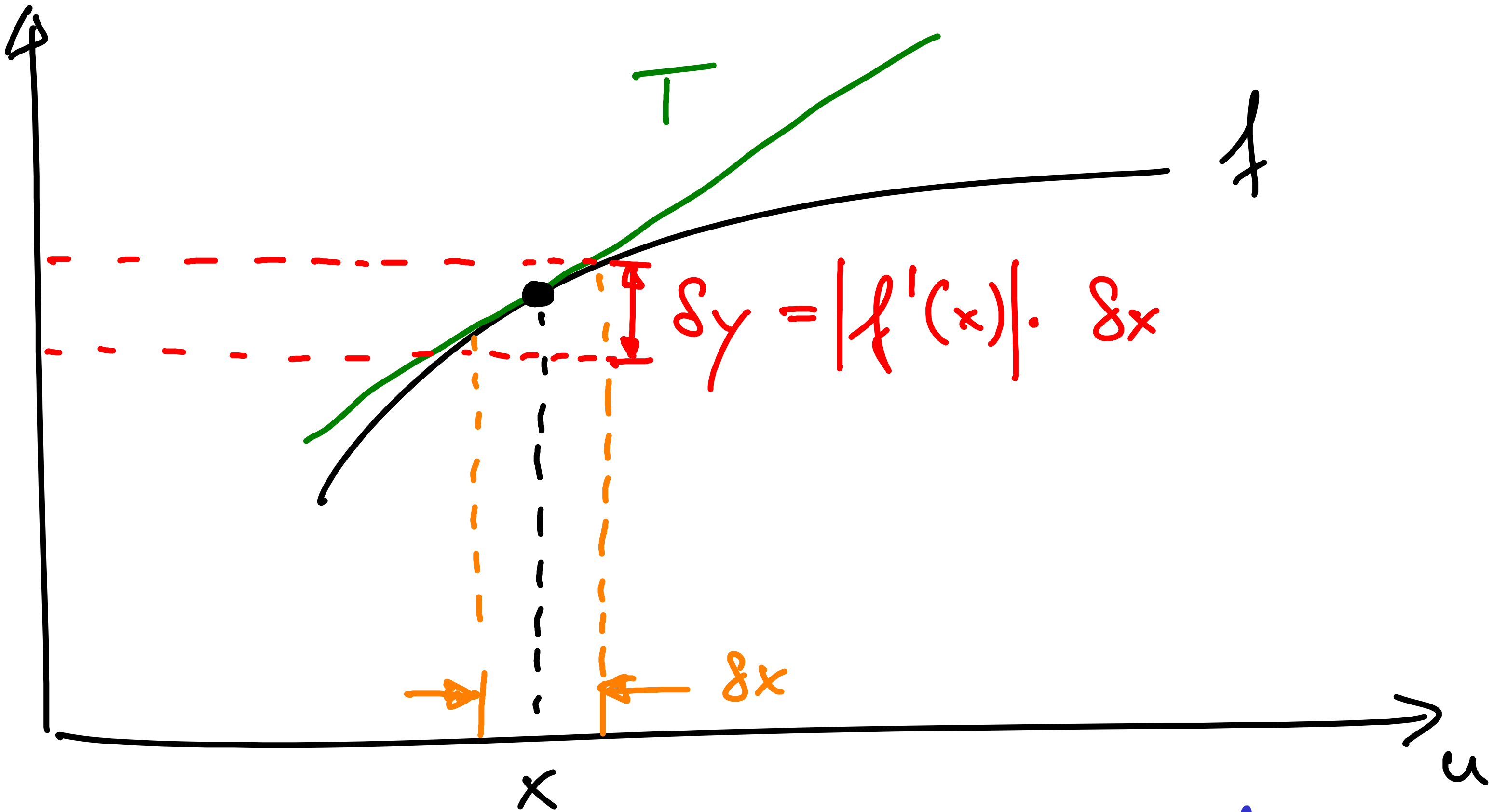
$$f' = \frac{df}{dx}$$

$$f'(x) = \frac{df}{dx}(x)$$

$$f'(x) = \frac{d}{dx} f(x)$$

$$f''(x) = \frac{d^2}{dx^2} f(x)$$

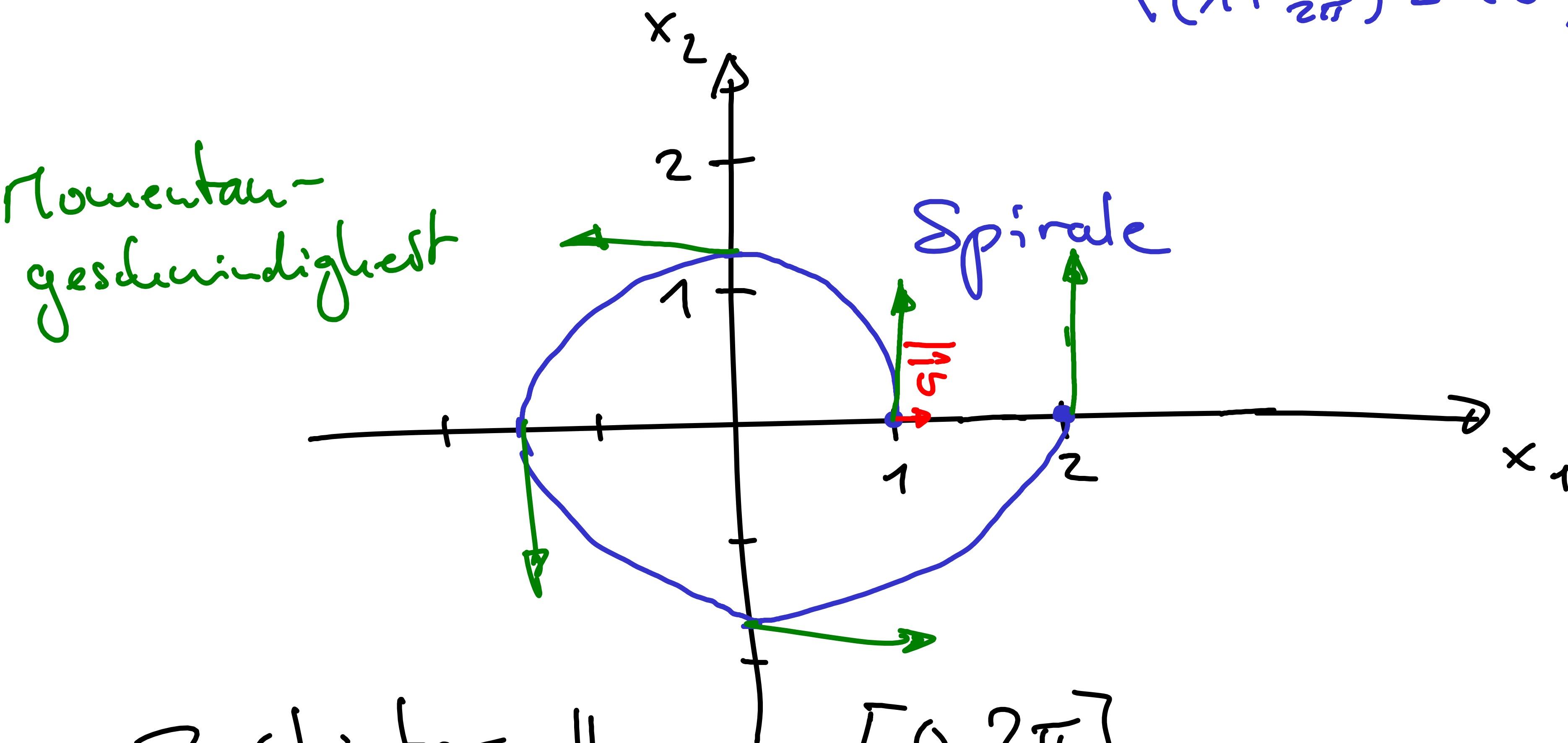
$$= \frac{d^2 f}{dx^2}(x)$$



eigentlich: suche max/min von f auf Intervall $\left[x - \frac{\delta x}{2}, x + \frac{\delta x}{2}\right]$

Kurve: $\vec{x}: \mathbb{R} \supset [0,2\pi] \longrightarrow \mathbb{R}^2$

$$t \longmapsto \begin{pmatrix} (1+\frac{t}{2\pi})\cos t \\ (1+\frac{t}{2\pi})\sin t \end{pmatrix}$$



Zeitintervall $[0, 2\pi]$

mittlere Geschw.: $\overline{\vec{v}} = \dfrac{\vec{x}(2\pi) - \vec{x}(0)}{2\pi}$

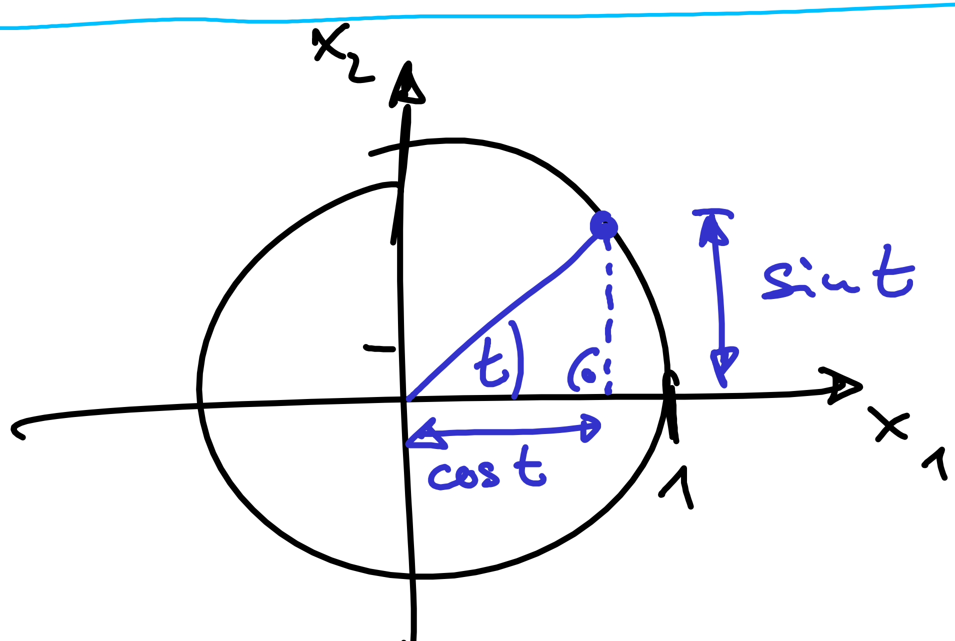
Zeitintervall $[0, 2\pi]$

mittlere Geschw.: $$\overline{\vec{v}} = \frac{\vec{x}(2\pi) - \vec{x}(0)}{2\pi}$$

$$= \frac{\begin{pmatrix} 2 \\ 0 \end{pmatrix} - \begin{pmatrix} 1 \\ 0 \end{pmatrix}}{2\pi} = \frac{1}{2\pi}\begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

gleich mit Ableitungsregeln

$$\frac{d\vec{x}}{dt} = \begin{pmatrix} \frac{1}{2\pi}\cos t - \left(1 + \frac{t}{2\pi}\right)\sin t \\ \frac{1}{2\pi}\sin t + \left(1 + \frac{t}{2\pi}\right)\cos t \end{pmatrix}$$



Wdh.: Kreis

$$\vec{x}(t) = \begin{pmatrix} \cos t \\ \sin t \end{pmatrix}$$

(Radius 1)

$$f(x) = x^n \quad , \quad n \in \mathbb{N}$$

$$f'(x) = \lim_{h \to 0} \frac{(x+h)^n - x^n}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)(x+h)\cdots(x+h) - x^n}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{x^n} + n h x^{n-1} + h^2(\dots) + \dots + h^n \cancel{- x^n}}{h}$$

$$= \lim_{h \to 0} \left(n x^{n-1} + h(\dots) + \dots + h^{n-1} \right)$$

$$= n x^{n-1}$$

$$f(x) = e^x = \exp(x)$$

$$f'(x) = \lim_{h\to 0} \frac{e^{x+h} - e^x}{h} = \lim_{h\to 0} \frac{e^x \cdot e^h - e^x}{h}$$

$$= e^x \underbrace{\lim_{h\to 0} \frac{e^h - 1}{h}}_{\text{hängt nicht von } x \text{ ab}}$$

Wert $e = 2{,}718\ldots$ so gewählt, dass

$$\lim_{h\to 0} \frac{e^h - 1}{h} = 1$$

damit $(e^x)' = e^x$

$$(f\cdot g)'(x) = \lim_{h\to 0} \frac{f(x+h)\,g(x+h) - f(x)\,g(x)}{h}$$

$$= \lim_{h\to 0} \frac{[f(x+h) - f(x)]\,g(x+h) + f(x)\,g(x+h) - f(x)\,g(x)}{h}$$

$$= \lim_{h\to 0} \left[\frac{f(x+h)-f(x)}{h}\,g(x+h) + f(x)\,\frac{g(x+h)-g(x)}{h} \right]$$

$$= f'(x)\,g(x) + f(x)\,g'(x)$$

$$f(x) = x \cdot \sin x + 5x^2$$

$$f'(x) = \sin x + x \cos x + 5 \cdot 2x$$

Umkehrfkt.:

$$f^{-1}(f(x)) = x$$ Ableiten (der Glg.) nach x

$$\underbrace{f^{-1\prime}(f(x))}_{\text{äußere Abl.}} \cdot \underbrace{f'(x)}_{\text{innere Abl.}} = 1$$

$\Longrightarrow$ falls $f'(x) \neq 0$ $$f^{-1\prime}(f(x)) = \frac{1}{f'(x)}$$

nenne $f(x) =: y$, $f^{-1}(y) = x$, und damit

$$f^{-1\prime}(y) = \frac{1}{f'(f^{-1}(y))}$$

$$f(x) = \log x$$

$$f'(x) = \frac{1}{\exp(\log x)} = \frac{1}{x}$$

Abl. d. Umkehrfkt. des log

$$f(x) = \arcsin x$$

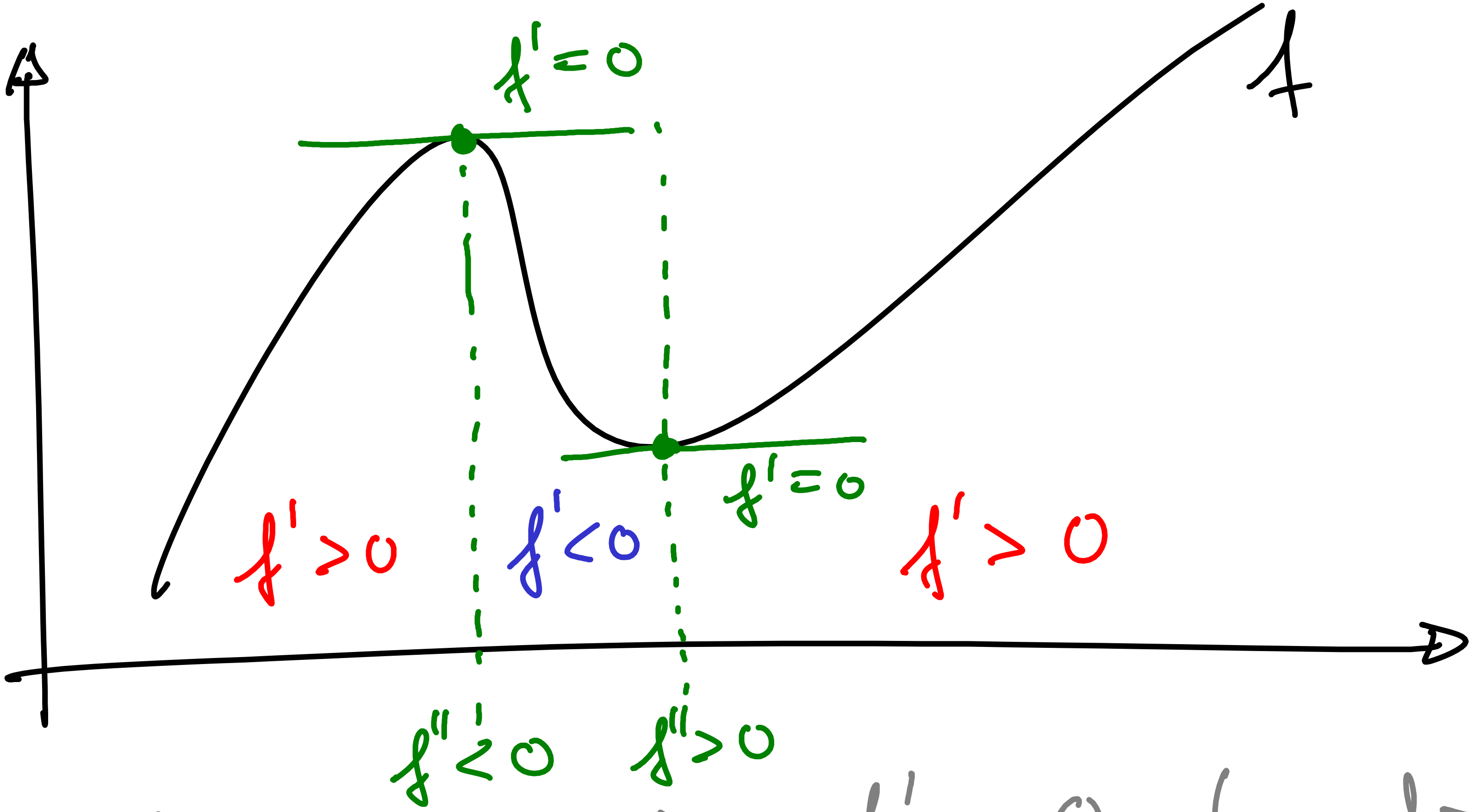
$$f'(x) = \frac{1}{\underline{\cos}\left(\arcsin x\right)} \quad , \quad \sin^2 x + \cos^2 x = 1$$

Abl. d. sin

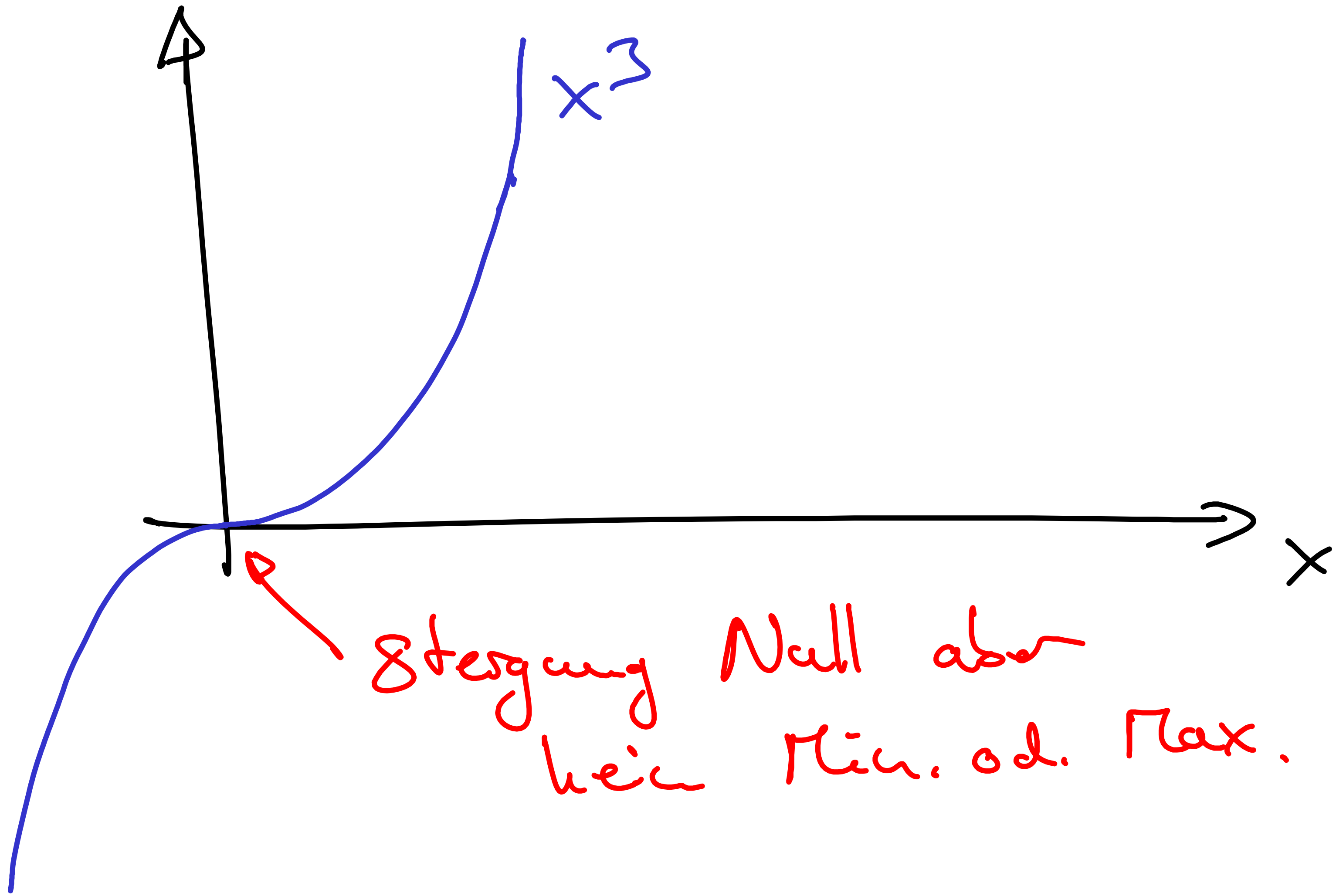
$$= \frac{1}{\sqrt{1 - \sin^2(\arcsin x)}} = \frac{1}{\sqrt{1 - x^2}}$$

Vorsicht mit dem Zweig,
d. h. dem VZ der Wurzel

$$\left(\sin^2 x = (\sin x)^2 \right)$$



lokales Extremum $\Longrightarrow$ $f' = 0$ (an der Stelle)


x^3
x
Steigung Null aber
kein Min. od. Max.