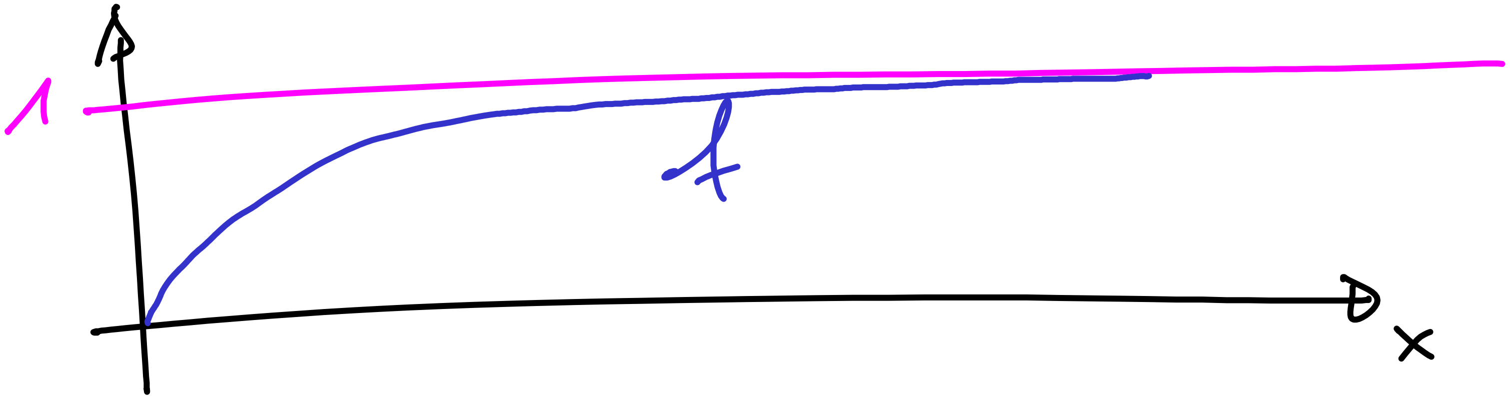
$$f(x) = 1 - e^{-\lambda x}$$

$$\lim_{x \to \infty} f(x) = 1 \quad , \quad f(0) = 1 - 1 = 0$$



Funktionen dieses Typs lösen Relaxationsgln.

$$f' + 3f = 6$$

Differenzialgleichung, da f und auch Ableitungen von f auftauchen

Rate $f(x) = a + b e^{-\lambda x}$

Einsetzen in DGL + Zeigen, dass f mit geeigneten Konstanten a, b, λ die DGL erfüllt

$$f'(x) = -\lambda b e^{-\lambda x} \quad \text{in DGL:}$$

$$\underbrace{-\lambda b e^{-\lambda x}}_{=f'} + \underbrace{3a + 3b e^{-\lambda x}}_{=3f} = 6$$

mit $a = 2$ und $\lambda = 3$ klappt's sogar für beliebige $b \in \mathbb{R}$

$$f(x) = 2 + b e^{-3x}, \quad b \in \mathbb{R}$$

löst die DGL

z.B. begrenztes Wachstum

$$f' = \underset{\sim\sim\sim\sim\sim}{6 - 3f}$$ "Änderung d. Pop.-Dichte mit der Zeit x"

"Geburtenrate − Sterberate"

f : Populationsdichte

x : Zeit

für kleine f wächst die Pop.-Dichte

für $f > 2$ schrumpft die Pop.-Dichte

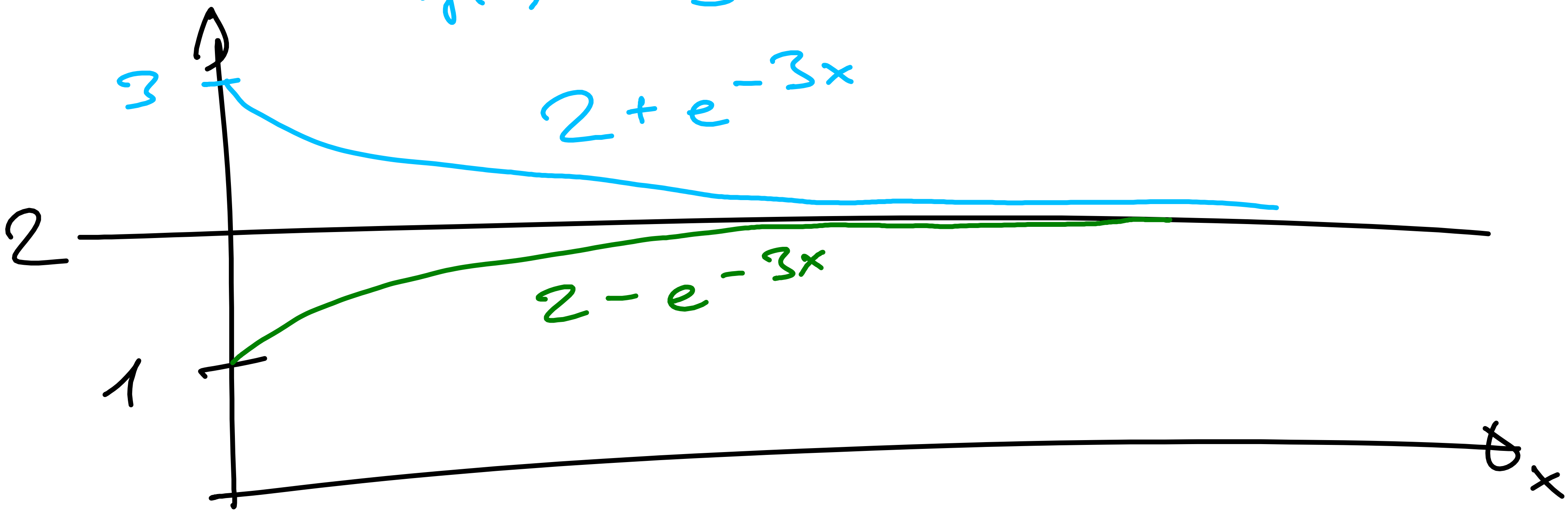
Wir wissen

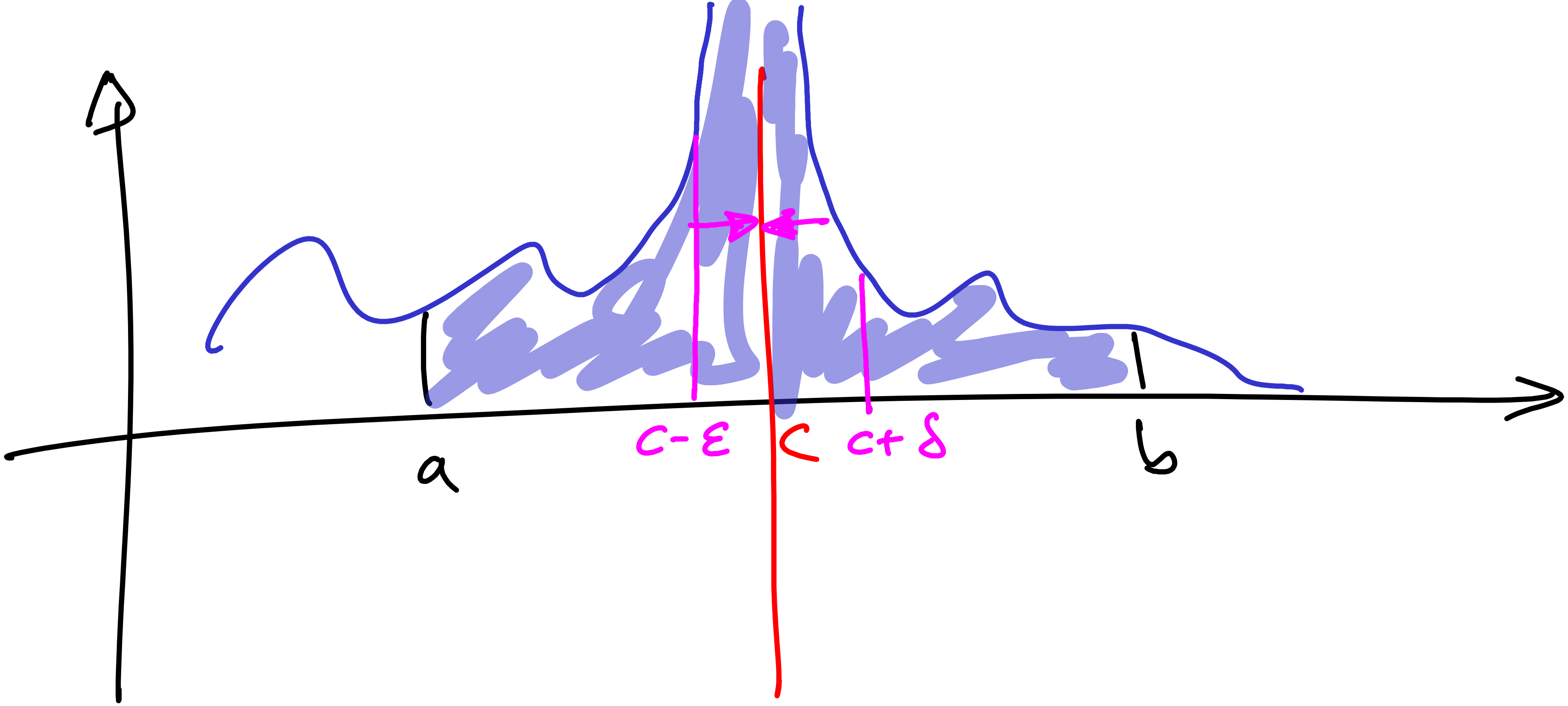
$$f(x) = 2 + b\, e^{-3x}, \quad f(0) = 2 + b$$

b wird durch $f(0)$ festgelegt

z. B. $f(0) = 1 \Rightarrow b = -1$

$f(0) = 3 \Rightarrow b = 1$



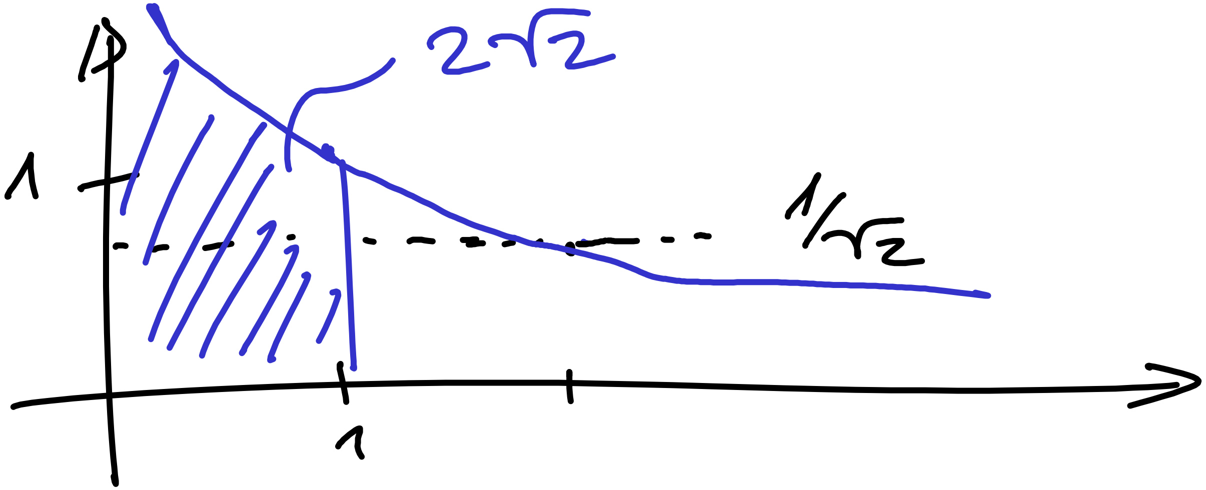


Beispiel

$$\int_0^2 \frac{dx}{\sqrt{x}} = \lim_{\varepsilon \to 0+} \int_\varepsilon^2 x^{-1/2}\, dx$$

$$= \lim_{\varepsilon \to 0+} \left[2x^{1/2} \right]_\varepsilon^2 = \lim_{\varepsilon \to 0+} \left(2\sqrt{2} - 2\sqrt{\varepsilon} \right)$$

$$= 2\sqrt{2}$$

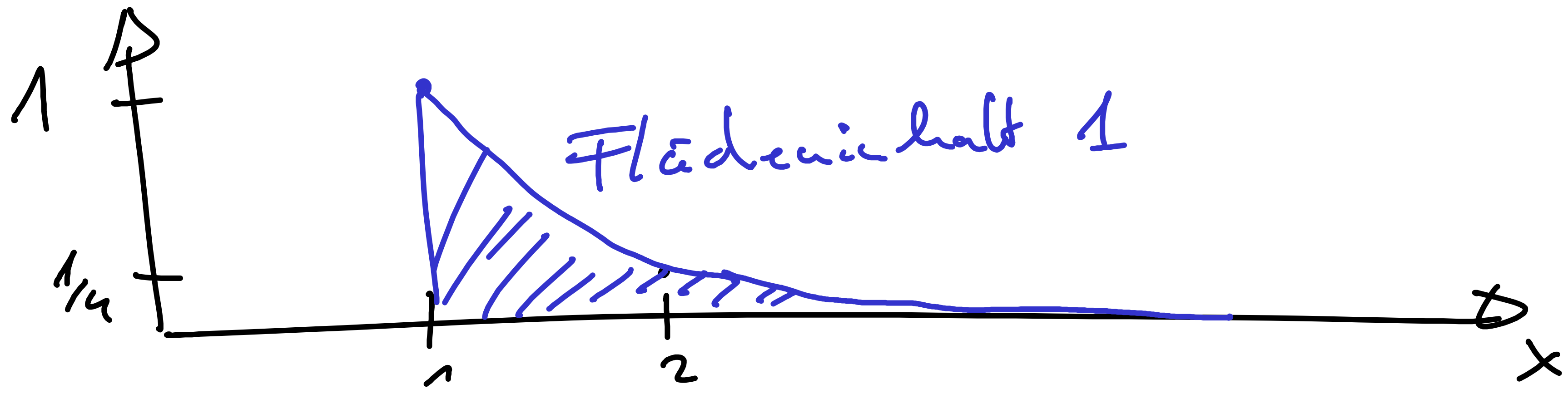


Beispiel:

$$\int_1^\infty \frac{dx}{x^2} = \lim_{b\to\infty} \int_1^b x^{-2}\,dx$$

$$= \lim_{b\to\infty} \left[-x^{-1}\right]_1^b = \lim_{b\to\infty} \left(-\frac{1}{b} - (-1)\right)$$

$$= \lim_{b\to\infty} \left(1 - \frac{1}{b}\right) = 1$$

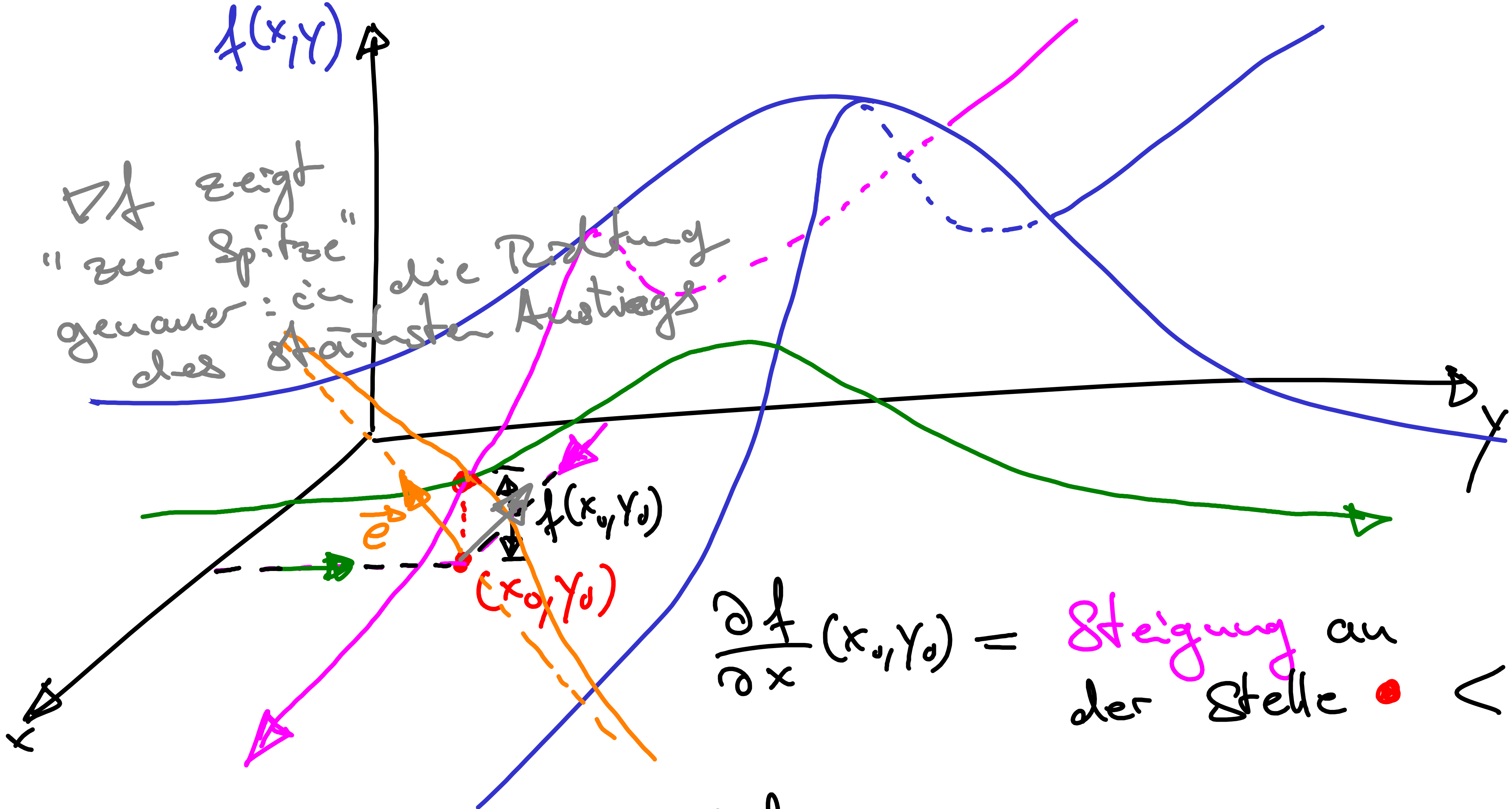

1
1/4
1
2
x
Flächeninhalt 1

partielle Ableitungen, Bsp

$$f(s,t) = s e^t + \sin(st)$$

$$\frac{\partial f}{\partial s}(s,t) = e^t + \cos(st) \cdot t$$

$$\frac{\partial f}{\partial t}(s,t) = s e^t + \cos(st) \cdot s$$



$\frac{\partial f}{\partial x}(x_0,y_0) =$ Steigung an der Stelle ● < 0

$\frac{\partial f}{\partial y}(x_0,y_0) =$ Steigung an der Stelle ● > 0

$\frac{\partial f}{\partial \vec{e}}(x_0,y_0) =$ Steigung an der Stelle ●

$$f(s,t) = s e^{-t} + \sin(st)$$

$$\frac{\partial f}{\partial s}(s,t) = e^{t} + \cos(st) \cdot t$$

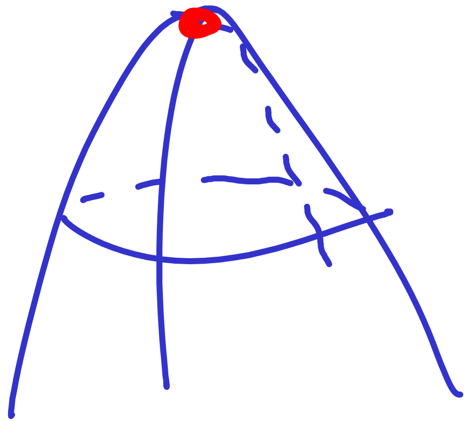
$$\frac{\partial f}{\partial t}(s,t) = s e^{t} + \cos(st) \cdot s$$

2. part. Abl.

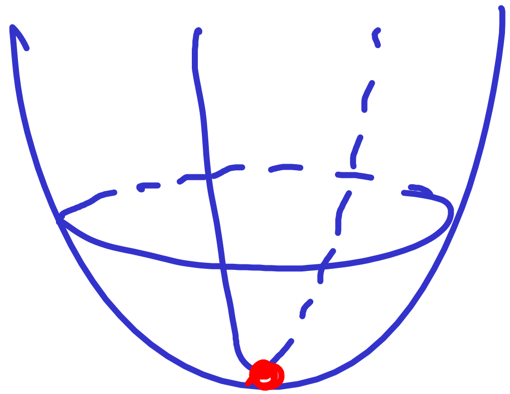
$$\frac{\partial^2 f}{\partial s^2}(s,t) = -\sin(st) \cdot t^2$$

$$\frac{\partial^2 f}{\partial t \partial s} = e^{t} - \sin(st) \cdot st$$

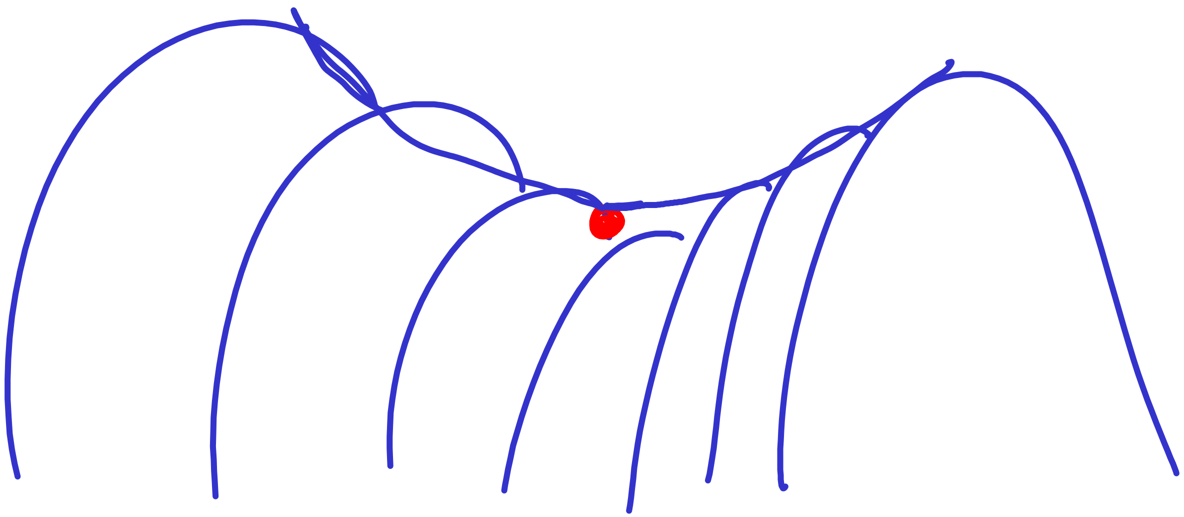
$$\frac{\partial^2 f}{\partial s \partial t} = e^t - \sin(st) \cdot st$$



Maxima



Minimum



Sattel