$$(x^\alpha)^\beta \neq x^{(\alpha\beta)}$$

z.B.

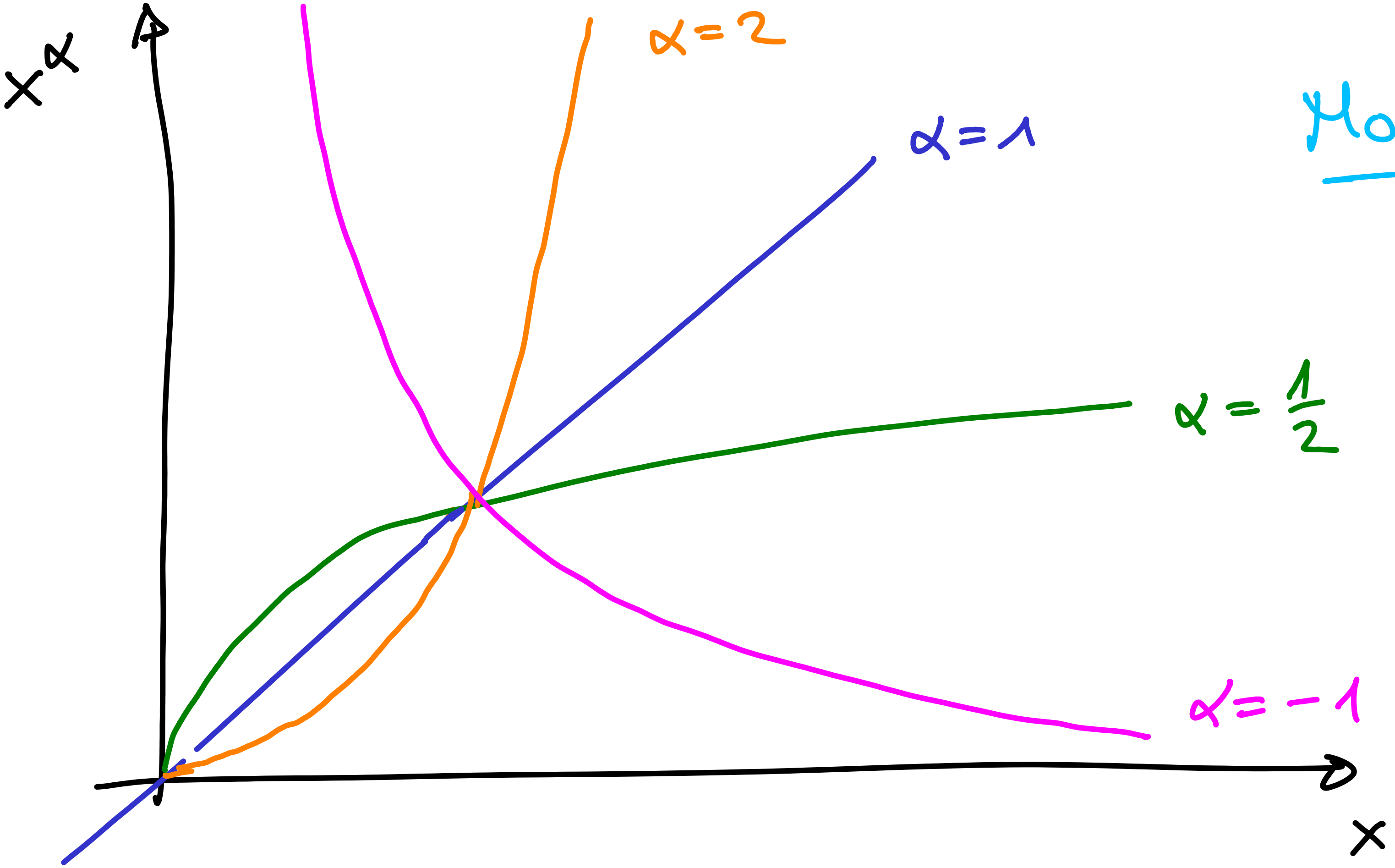
$$(2^2)^3 = 4^3 = 64$$

$$2^{(2^3)} = 2^8 = 256$$

$$\sqrt[3]{9^{-2} \cdot 3} = \sqrt[3]{(3^2)^{-2} \cdot 3} = \left(3^{-4} \cdot 3^{1}\right)^{1/3}$$

$$= \left(3^{-3}\right)^{\frac{1}{3}} = 3^{-1} = \frac{1}{3}$$

Monotonie



$\alpha = 1{,}06$ (6% Zinsen)

$G(0) = 100\,€$

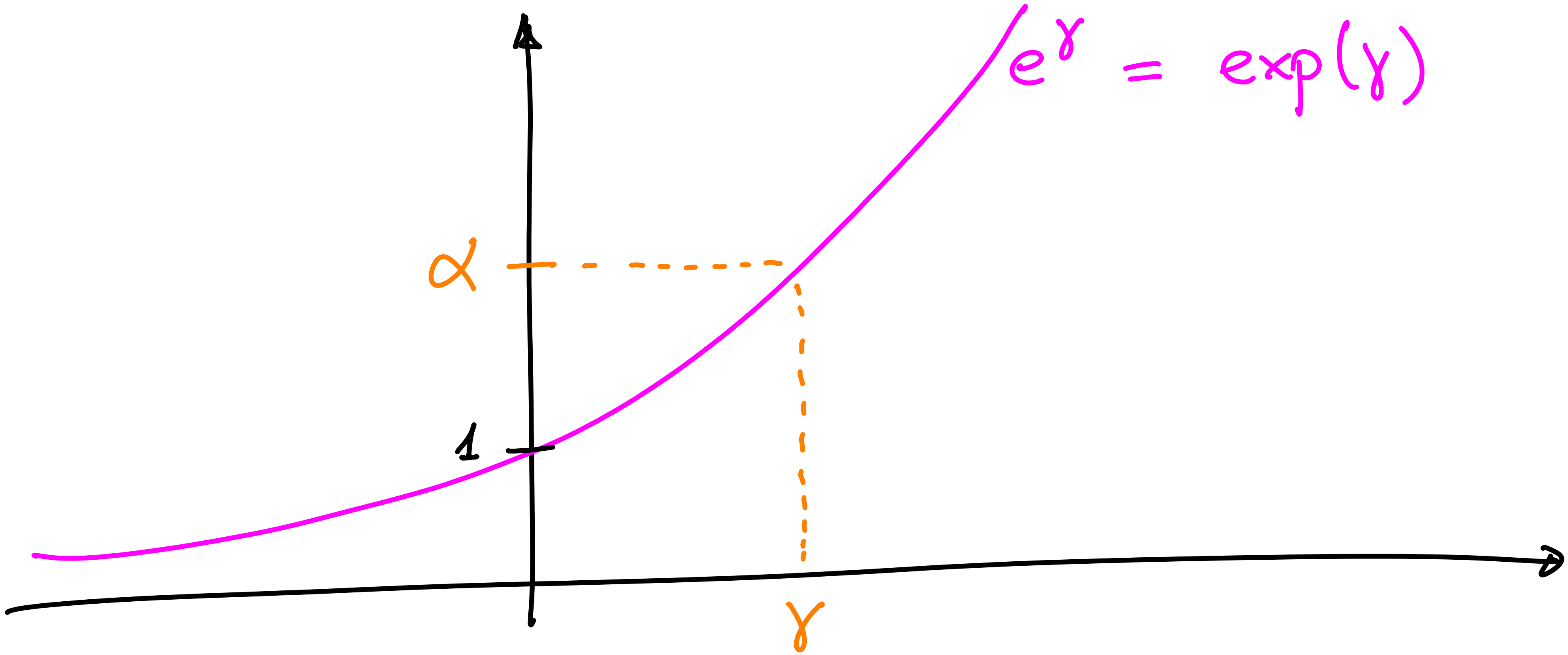
$t = \frac{1}{2}$

Rückzahlung nach halben Jahr

$G(\frac{1}{2}) = (1{,}06)^{1/2} \cdot 100\,€ \approx 102{,}96\,€ < 103\,€$

("Zinseszins rückwärts")

$$\alpha^t = e^{\gamma t} = (e^\gamma)^t \quad \text{deshalb} \quad \alpha = e^\gamma$$



$\alpha > 1 \implies \gamma > 0$

$\alpha < 1 \implies \gamma < 0$

$$\alpha^{t/T} \underset{\alpha = e^{\gamma}}{=} \left(e^{\gamma}\right)^{t/T} = e^{\frac{\gamma t}{T}} \underset{\lambda = \frac{\gamma}{T}}{=} e^{\lambda t}$$

$$G(t) = e^{\lambda t}\, G(0)$$

$$G\left(\tfrac{1}{\lambda}\right) = e^{\lambda \cdot \frac{1}{\lambda}}\, G(0) = e \cdot G(0), \quad \lambda > 0$$

$$G\left(-\tfrac{1}{\lambda}\right) = e^{-\lambda \frac{1}{\lambda}}\, G(0) = \tfrac{1}{e}\, G(0), \quad \lambda < 0$$

radioaktiver Zerfall

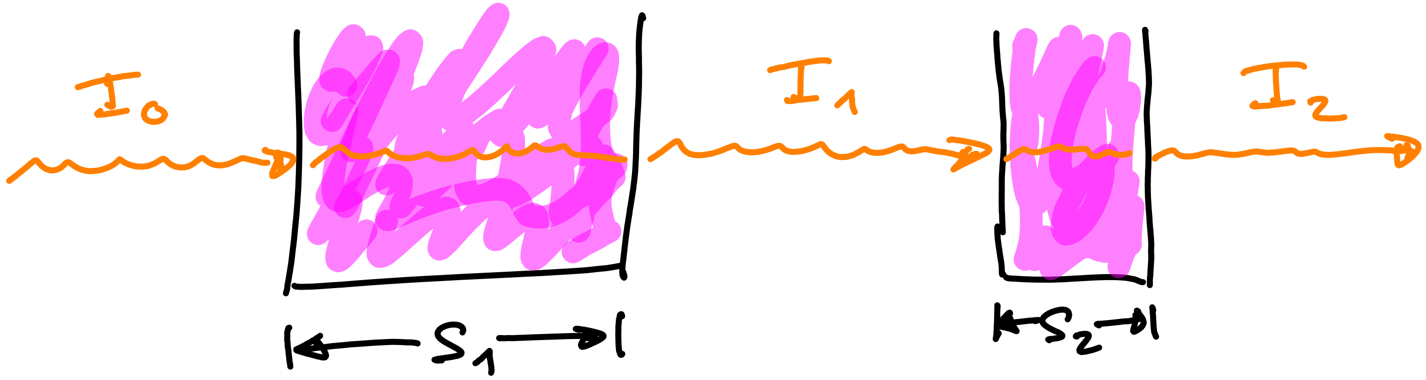
$G(t)$ Menge zu Beginn des Intervalls $[t, t+T]$

$G(t+T)$ " am Ende ——— " ———

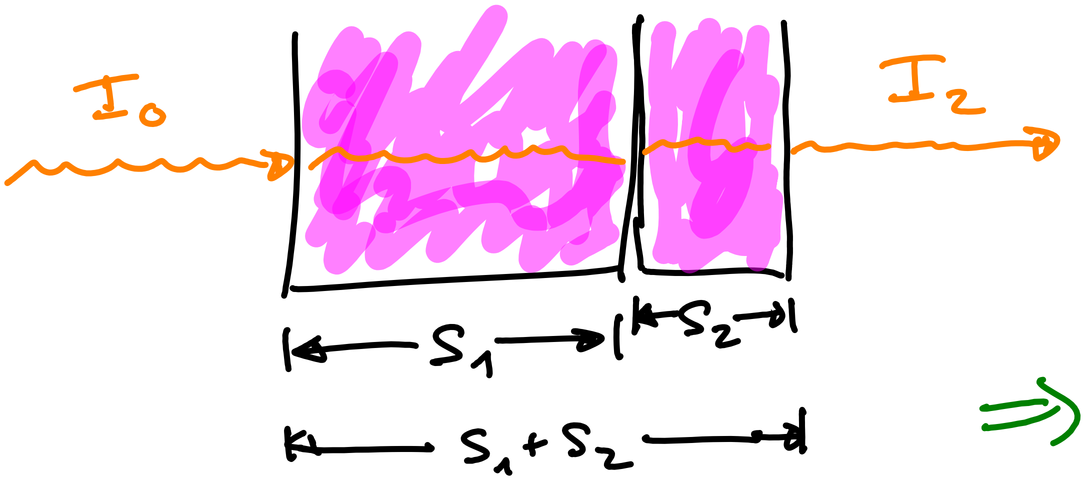
$$\frac{G(t+T)}{G(t)} = \frac{e^{-\lambda(t+T)}\,G(0)}{e^{-\lambda t}\,G(0)} = e^{-\lambda T}$$

hängt <u>nicht</u> von t ab!

zu Lambert-Beer

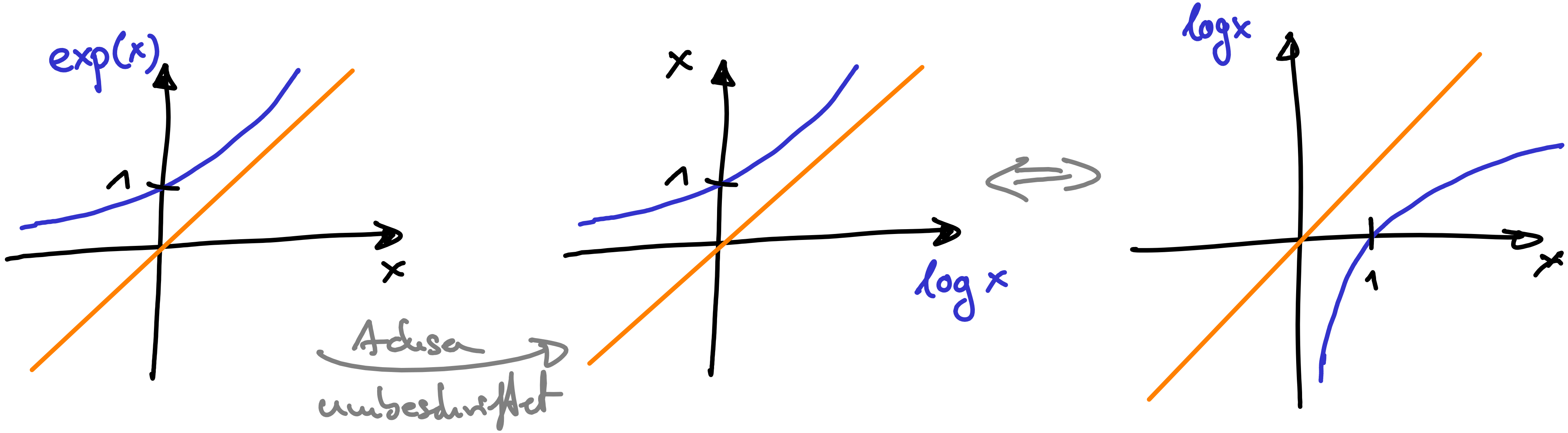


$$I_1 = \alpha_{s_1} \cdot I_0 \quad , \quad I_2 = \alpha_{s_2} I_1 = \alpha_{s_2} \cdot \alpha_{s_1} \cdot I_0$$



$$I_2 = \alpha_{s_1+s_2} I_0$$

$$\Rightarrow \alpha_{s_1+s_2} = \alpha_{s_1} \cdot \alpha_{s_2}$$



also gespiegelt an erste Winkelhalbierende

log-Rechenregeln

① $\log(xy) = \log x + \log y$

$x = e^a,\ y = e^b \iff \log x = a,\ \log y = b$

$\log(xy) = \log(e^a \cdot e^b) \overset{\text{P.R.}}{=} \log(e^{a+b})$

$\underset{\text{Umkehrfkt.}}{=} a+b = \log x + \log y$

② $\log(x^\alpha) = \alpha \log x$

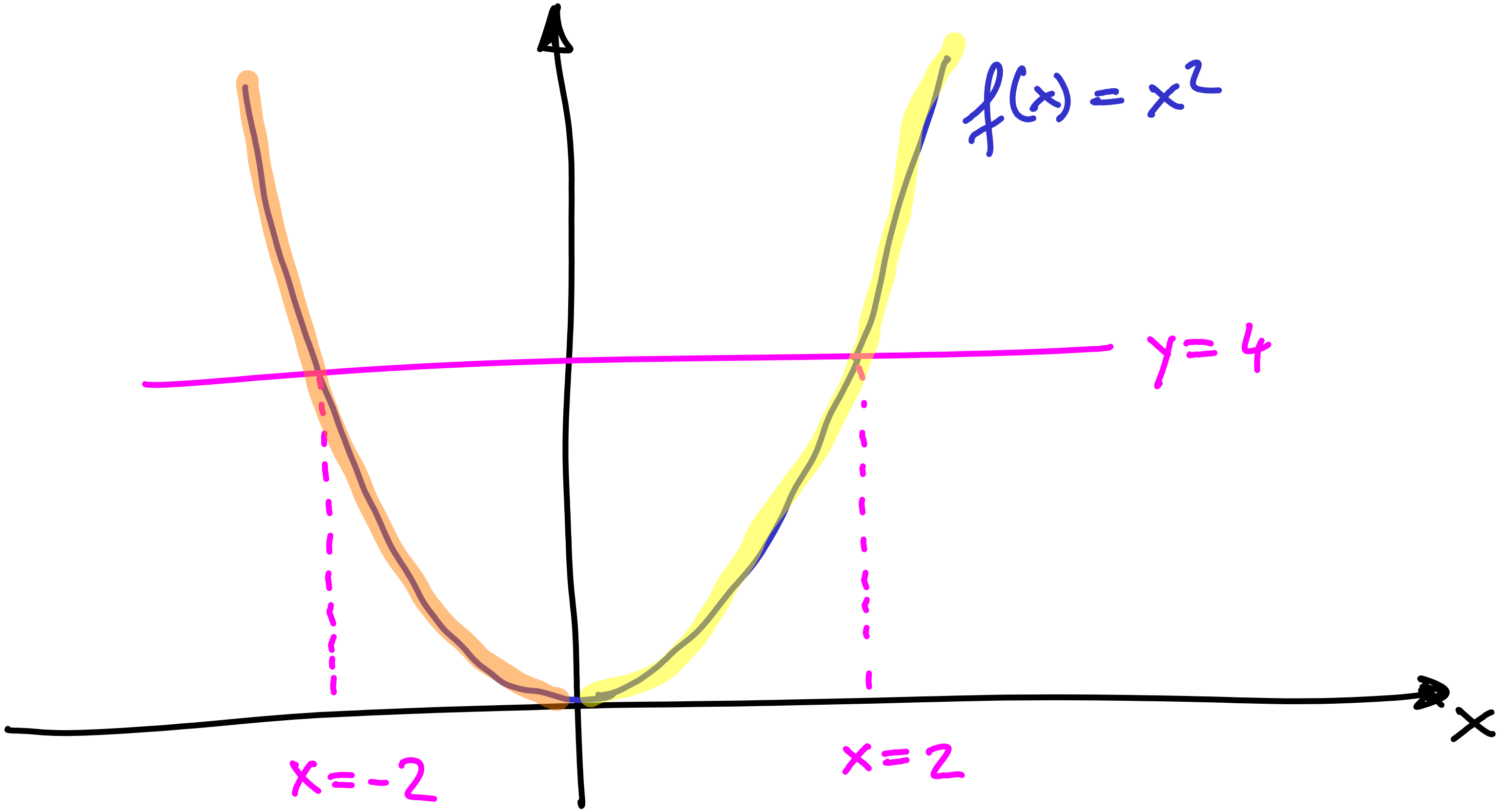
$x = e^\gamma \iff \gamma = \log x$

$\log(x^\alpha) = \log((e^\gamma)^\alpha) \overset{\text{P.R.}}{=} \log(e^{\gamma\cdot\alpha}) \overset{\text{Umkehrfkt.}}{=} \gamma\cdot\alpha$

$= \alpha \cdot \log x$

③ $\log\left(\frac{1}{x}\right) = -\log x$ (② mit $\alpha = -1$)

④ $\log(1) = \log(e^0) = 0$



$f: \mathbb{R} \to \mathbb{R}_0^+ = [0, \infty)$ ist <u>nicht</u> injektiv, d.h.
$x \mapsto x^2$ auch <u>nicht</u> umkehrbar

$\tilde{f}: [0, \infty) \to [0, \infty)$ ist umkehrbar mit
$x \mapsto x^2$ $\tilde{f}^{-1}: \underset{\substack{\cap \\ [0,\infty)}}{y} \mapsto \sqrt{y}$

$$\tilde{f} : (-\infty, 0] \longrightarrow [0, \infty)$$
$$x \longmapsto x^2$$

ist umkehrbar und $\tilde{f}^{-1}(y) = -\sqrt{y}$

$$\tilde{f}^{-1} : [0, \infty) \longrightarrow (-\infty, 0]$$