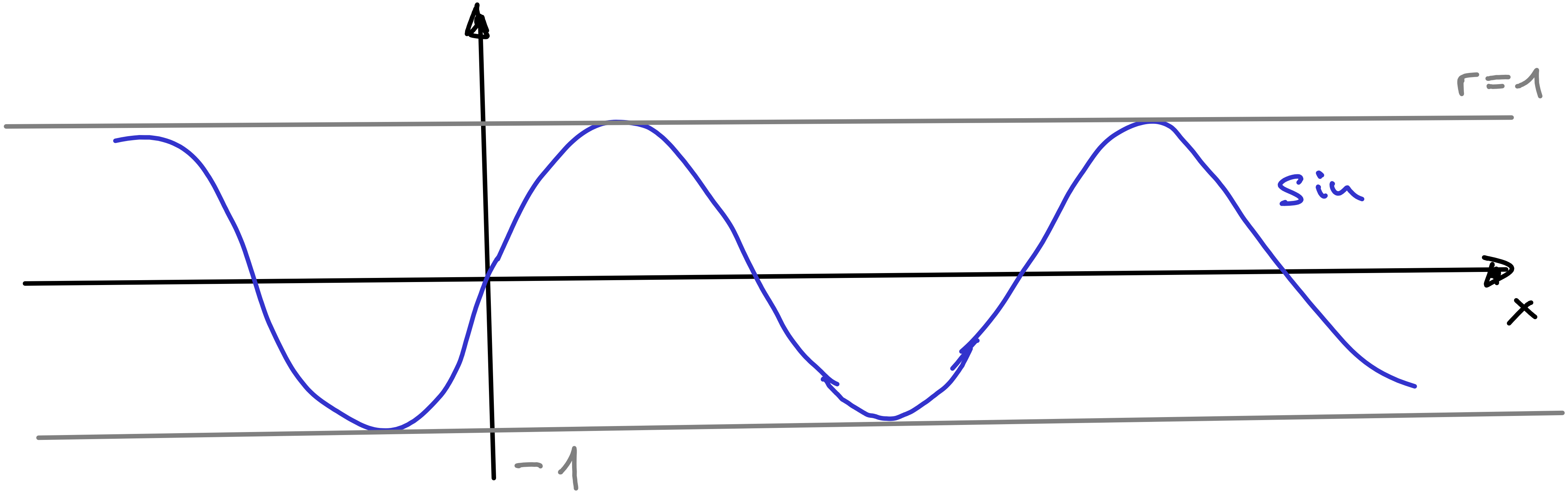
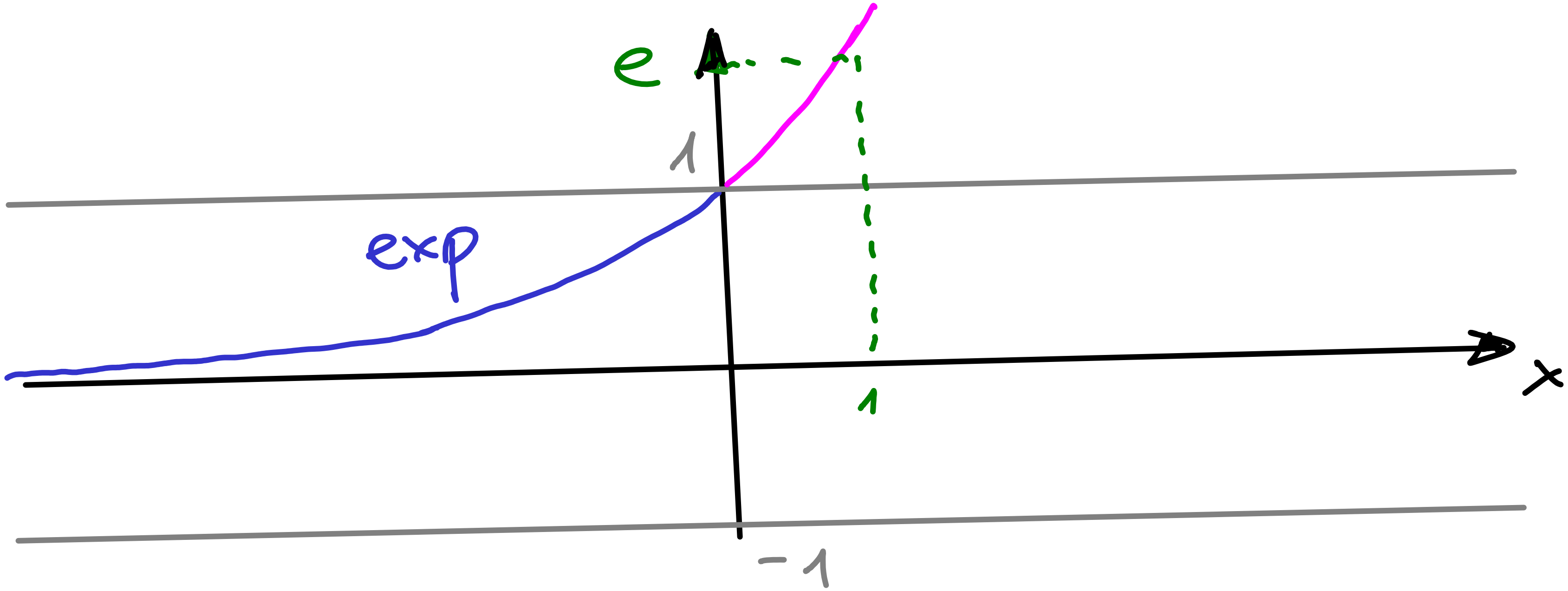


$$f(x) = 1 + \sin x$$

$$0 \leq f(x) \leq 2 \quad \forall\, x \in \mathbb{R}$$

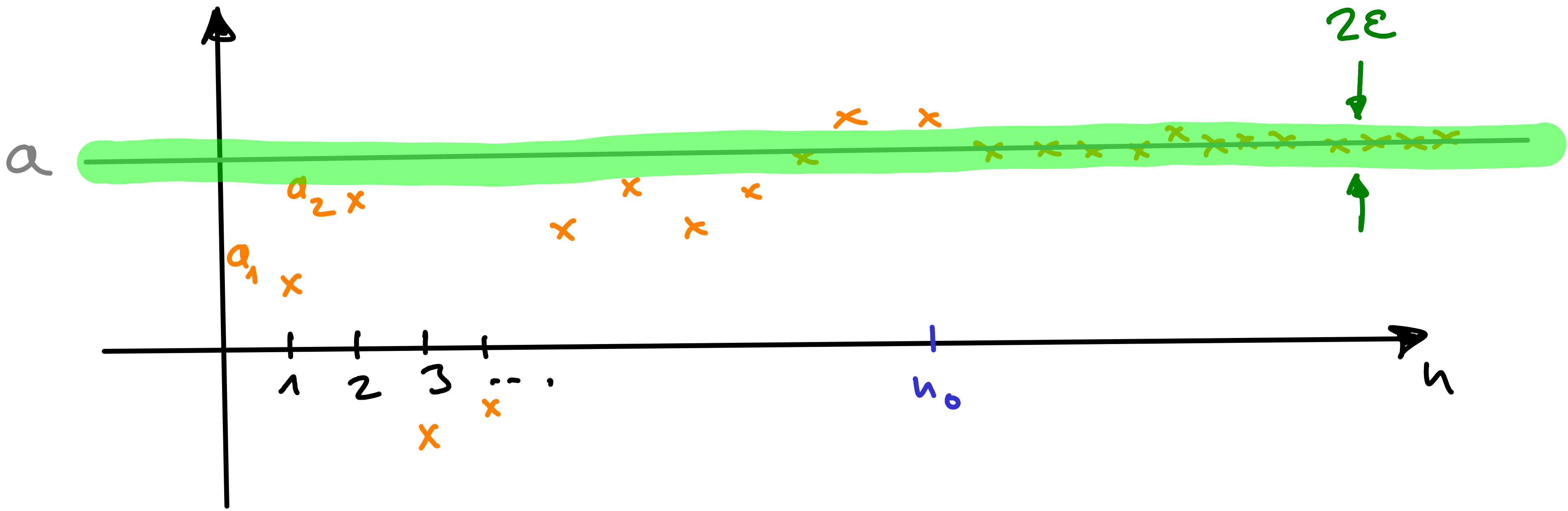
$$\Rightarrow r = 2 \quad (\text{z.B.})$$



$$\exp: (-\infty, 1] \longrightarrow (0, e]$$

ist beschränkt, z.B.

$$|\exp(x)| \leq e \quad \forall \; x \in (-\infty, 1]$$



a
a_1
a_2
2ε
1
2
3
...
n_0
n

$a_n = \frac{1}{n}$, $n \in \mathbb{N}$

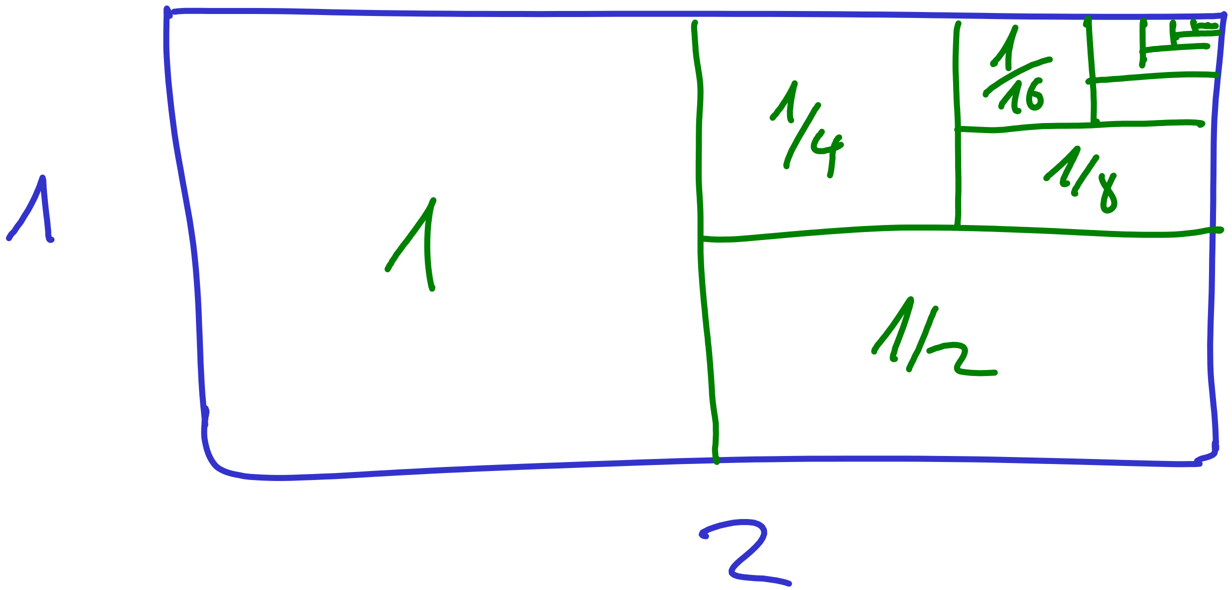
Behauptung: $\lim_{n \to \infty} a_n = 0$

hätten wir gerne

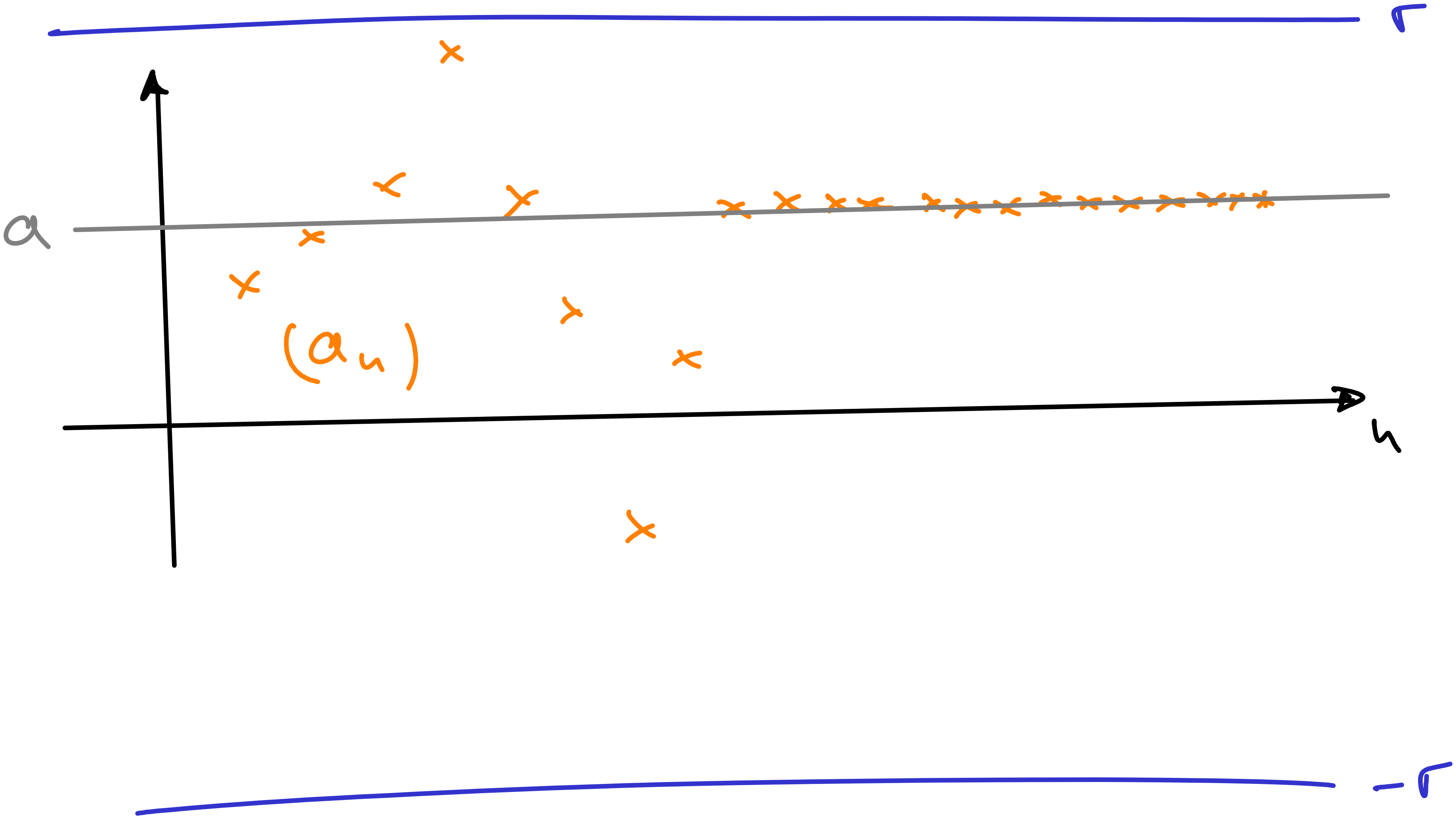
$$|a_n - 0| = \left|\frac{1}{n} - 0\right| = \left|\frac{1}{n}\right| = \underbrace{\frac{1}{n} < \varepsilon}$$

$$\Longleftrightarrow \quad n > \frac{1}{\varepsilon}$$

Wähle $n_0 \geq \frac{1}{\varepsilon}$ $\square$



Fläche: $2 = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots$

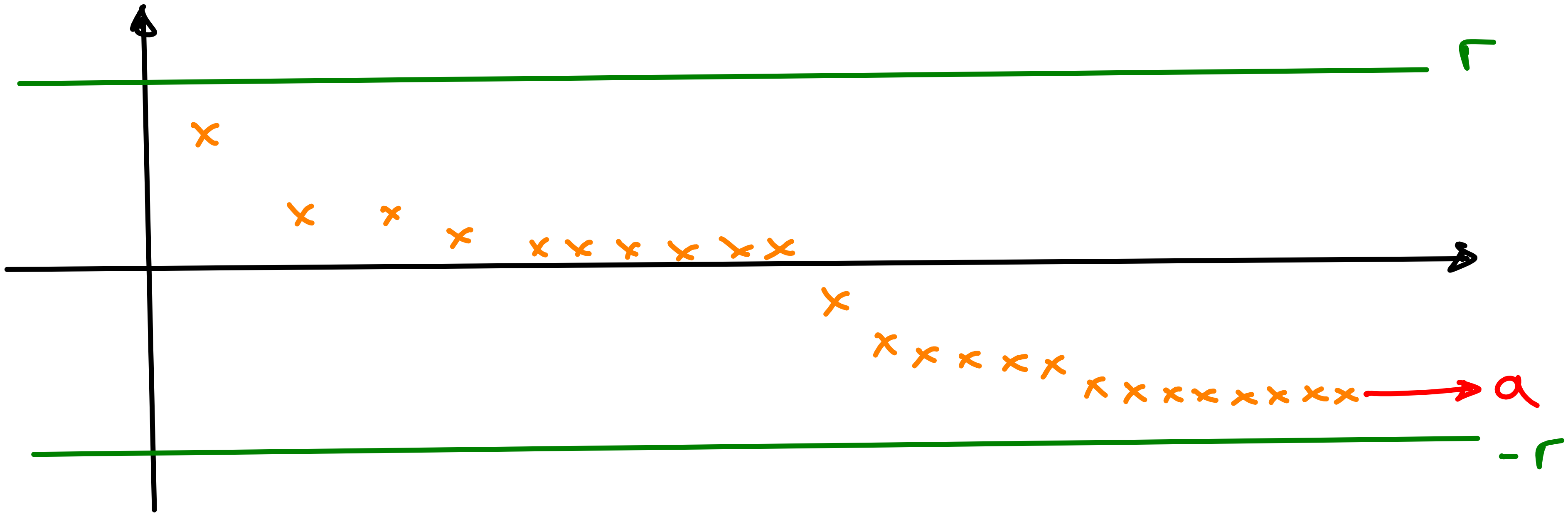


r
a
(an)
n
-r

Folge a_n sei monoton fallend, d.h.

$$a_{n+1} \leq a_n \quad \forall n$$

und beschränkt, d.h. $|a_n| \leq r \quad \forall n$



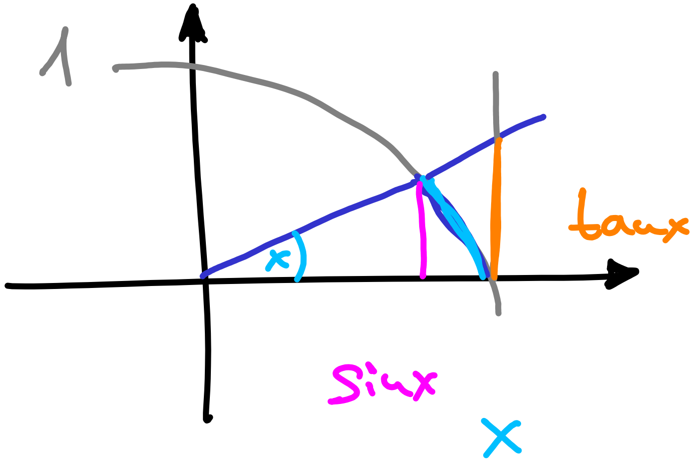
Bsp: $\frac{1}{n}$

$\left|\frac{1}{n}\right| \leq 1 \quad \forall\, n \in \mathbb{N}$ beschränkt

$\frac{1}{n+1} < \frac{1}{n}$ monoton fallend

$\Rightarrow \frac{1}{n}$ konvergiert

$$\frac{\sin\left(\frac{1}{n}\right)}{\frac{1}{n}}$$



für $0 < x < \frac{\pi}{2}$:

$$\sin x < x < \tan x$$

$$\Leftrightarrow \quad \frac{1}{\sin x} > \frac{1}{x} > \frac{1}{\tan x} = \frac{\cos x}{\sin x} \quad \Big| \cdot \sin x$$

$$\Leftrightarrow \quad 1 > \frac{\sin x}{x} > \cos x$$

d.h. $1 \xleftarrow{n\to\infty} \cos\left(\frac{1}{n}\right) < \frac{\sin\left(\frac{1}{n}\right)}{\frac{1}{n}} < 1$

$$\boxed{\Rightarrow \lim_{n\to\infty} \frac{\sin\left(\frac{1}{n}\right)}{\frac{1}{n}} = 1}$$

$$\lim_{n\to\infty} \frac{n^{42} - 32 n^{27}}{12n + n^{42}} = \lim_{n\to\infty} \frac{1 - 32 \frac{1}{n^{15}}}{\frac{12}{n^{41}} + 1}$$

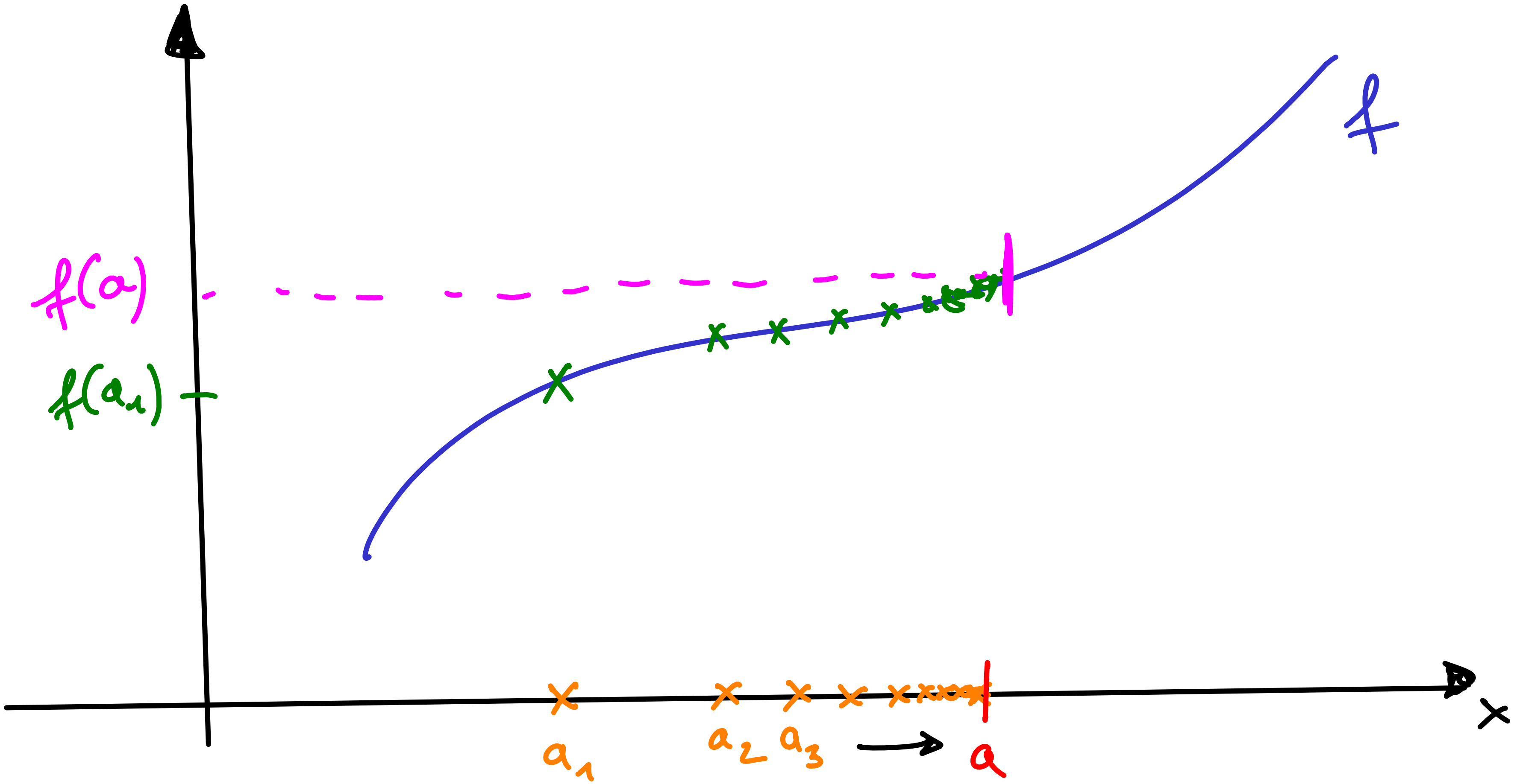
Produkt

$$= \frac{\lim_{n\to\infty}\left(1 - \frac{32}{n^{15}}\right)}{\lim_{n\to\infty}\left(\frac{12}{n^{41}} + 1\right)}$$

Summe

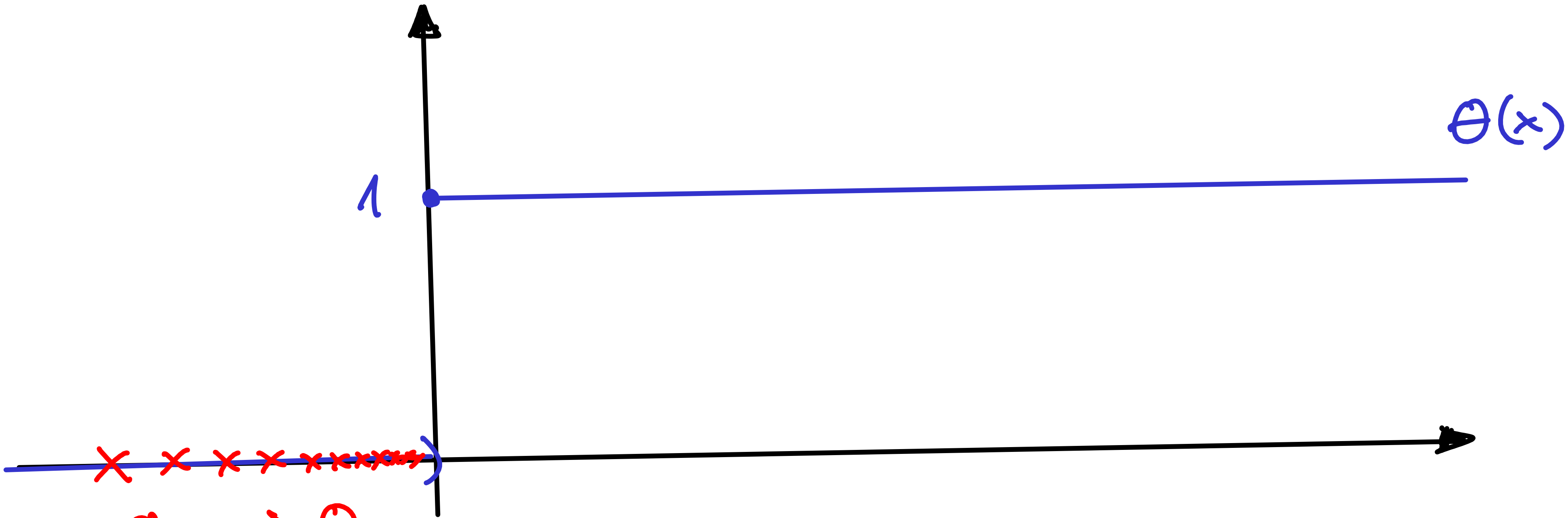
$$= \frac{\lim_{n\to\infty} 1 - \lim_{n\to\infty} \frac{32}{n^{15}}}{\lim_{n\to\infty} \frac{12}{n^{41}} + \lim_{n\to\infty} 1}$$

$$= \frac{1 - 0}{0 + 1} = 1$$



$$\lim_{n\to\infty} f(a_n) = f\Big(\underbrace{\lim_{n\to\infty} a_n}_{=a}\Big) = f(a)$$

Stetigkeit von f



$a_n \to 0$

$a_n < 0 \ \forall n$

$$\lim_{n\to\infty} \theta(a_n) = \lim_{n\to\infty} 0 = 0$$

$$\neq \theta\left(\lim_{n\to\infty} a_n\right) = \theta(0) = 1$$

geom. Summe

$$S_n = \sum_{k=0}^{n} q^k = 1 + q + q^2 + \dots + q^n \quad | \cdot q$$

$$q \cdot S_n = q + q^2 + \dots + q^n + q^{n+1}$$

Differenz

$$S_n - q S_n = 1 - q^{n+1} \quad \Big| \ \frac{1}{1-q} \quad (q \neq 1)$$

$$\Rightarrow \quad S_n = \frac{1 - q^{n+1}}{1-q} \quad \forall\, q \neq 1$$

Also

$$\lim_{n\to\infty} S_n = \sum_{k=0}^{\infty} q^k = \lim_{n\to\infty} \frac{1 - q^{n+1}}{1-q} = \frac{1}{1-q}$$

$\xrightarrow[n\to\infty]{} 0$