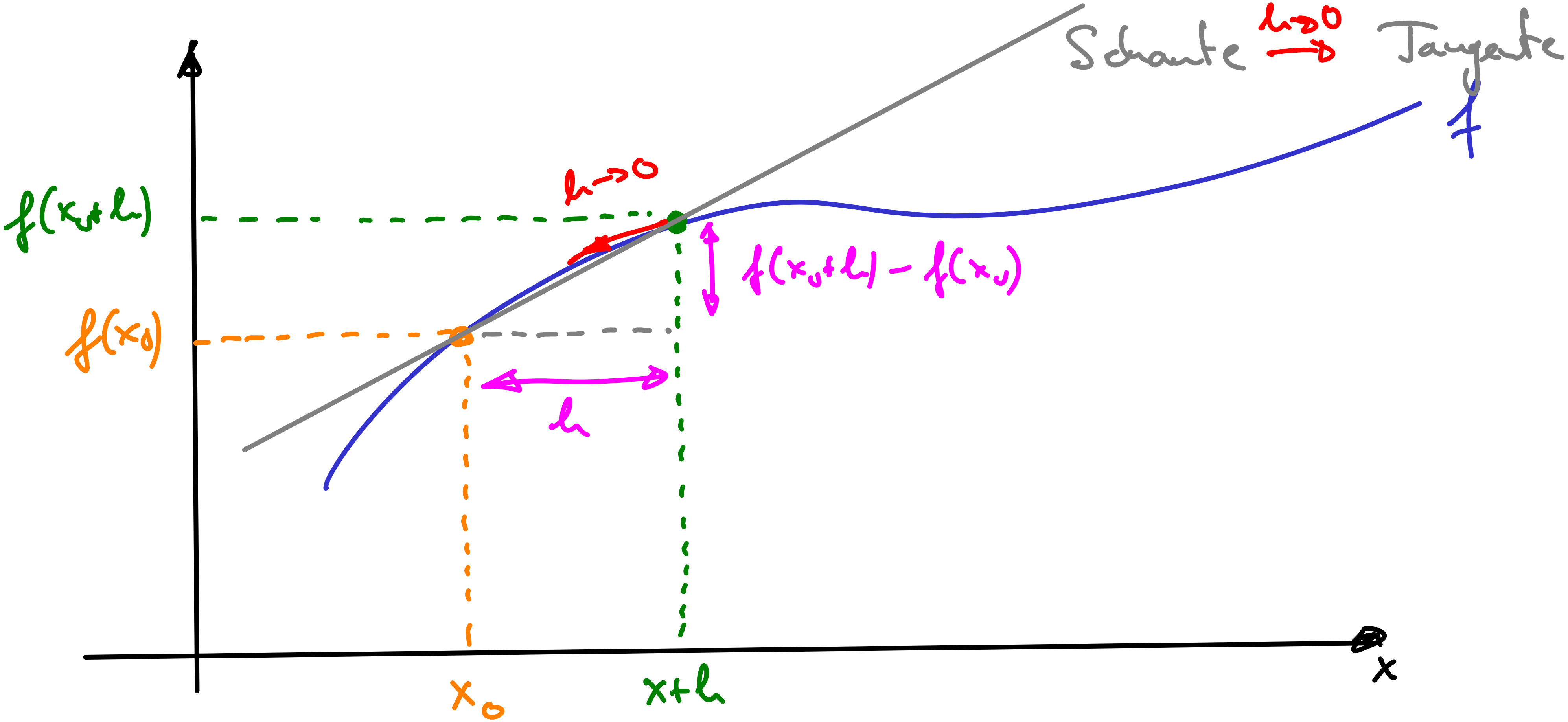


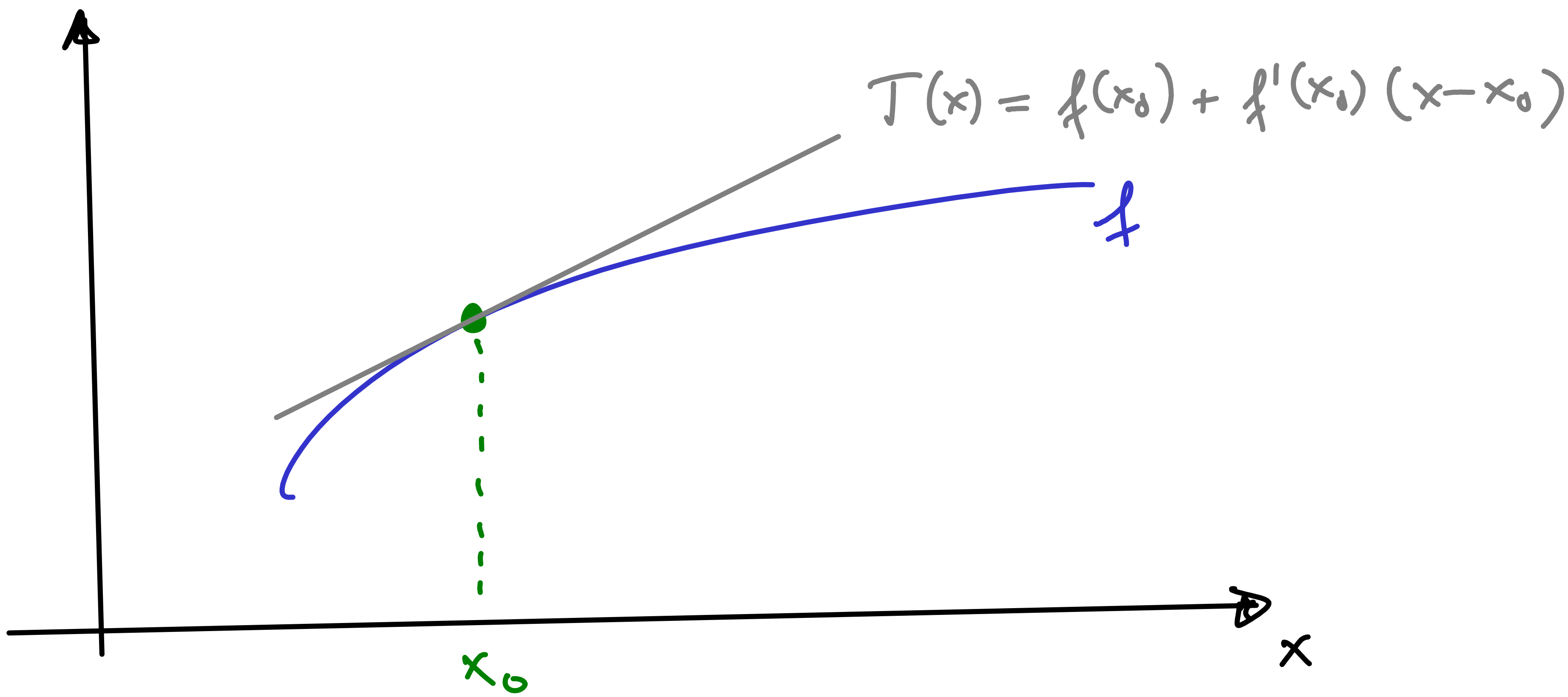
Sekante $\xrightarrow{h \to 0}$ Tangente
f
$h \to 0$
$f(x_0+h)$
$f(x_0+h) - f(x_0)$
$f(x_0)$
h
x_0
$x+h$
x

Notation

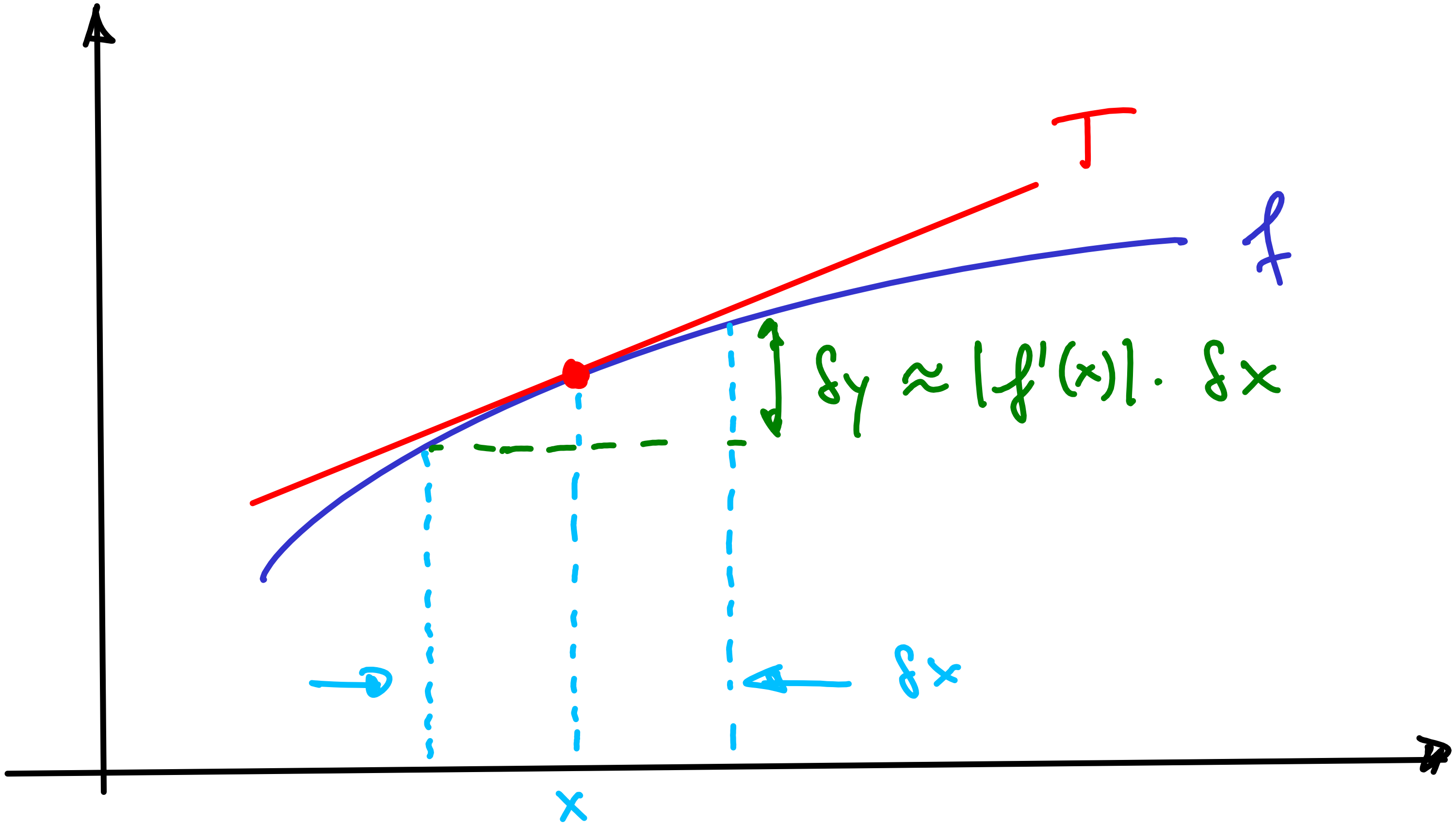
$$f' = \frac{df}{dx} = \left(\frac{d}{dx}\right) f$$

$$f'' = f^{(2)} = \frac{d^2 f}{dx^2} = \left(\frac{d}{dx}\right)^2 f$$

$$f'''''(x) = f^{(5)}(x)$$

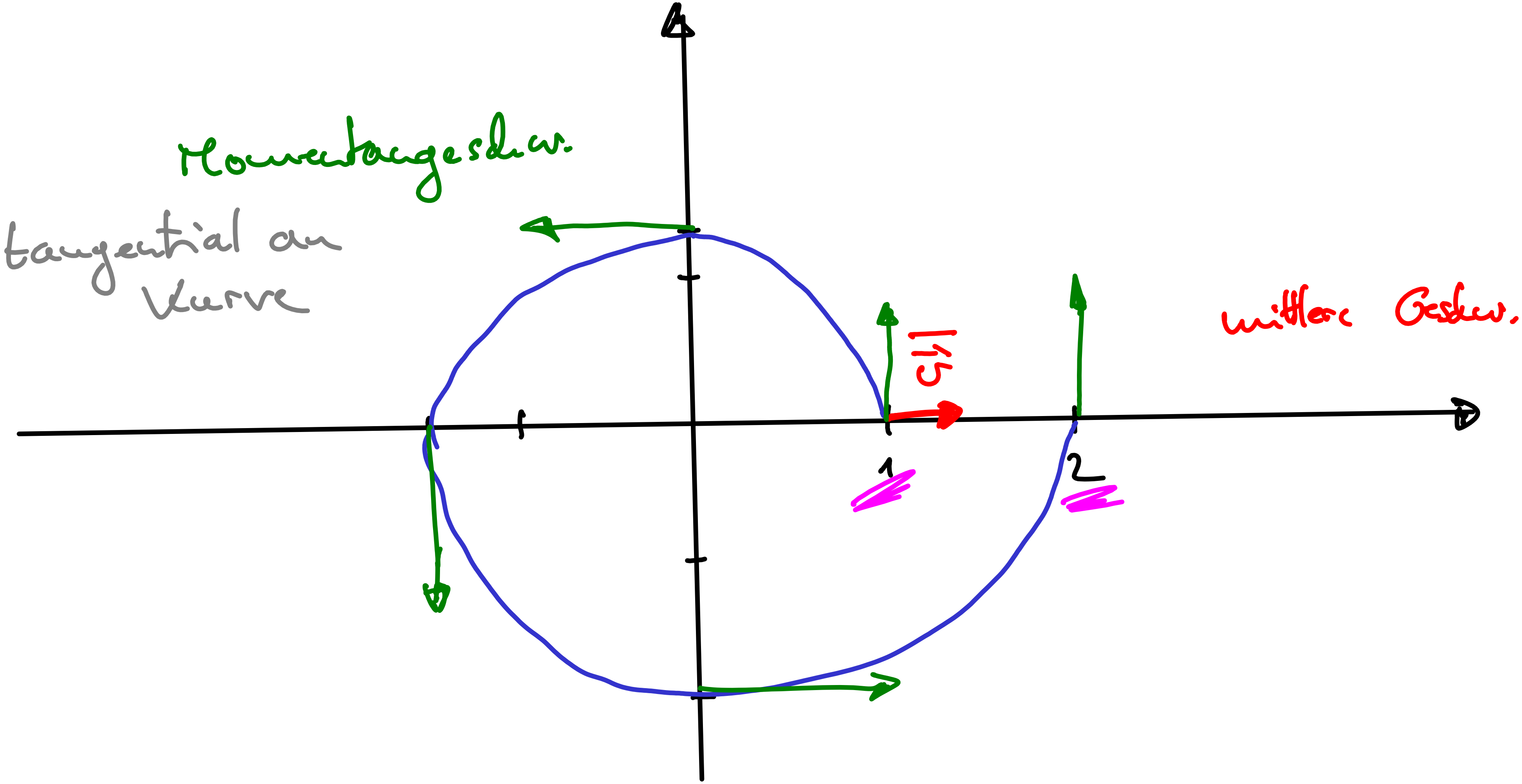


$T(x) = f(x_0) + f'(x_0)(x - x_0)$
f
x_0
x



$$x \in \left[x - \frac{\delta x}{2},\ x + \frac{\delta x}{2}\right]$$

Kurve $\vec{x} : [0, 2\pi] \to \mathbb{R}^2$



in Formeln: zunächst Kreis

$$\vec{x}(t) = \begin{pmatrix} r\cos t \\ r\sin t \end{pmatrix}, \quad 0 \le t \le 2\pi$$

Radius

Spirale

$$\vec{x}(t) = \begin{pmatrix} \left(1+\frac{t}{2\pi}\right)\cos t \\ \left(1+\frac{t}{2\pi}\right)\sin t \end{pmatrix}, \quad 0 \le t \le 2\pi$$

$$\dot{\vec{x}} = \begin{pmatrix} \frac{1}{2\pi}\cos t + \left(1+\frac{t}{2\pi}\right)(-\sin t) \\ \frac{1}{2\pi}\sin t + \left(1+\frac{t}{2\pi}\right)\cos t \end{pmatrix}$$

$$\overline{\vec{v}} = \frac{\vec{x}(2\pi) - \vec{x}(0)}{2\pi} = \frac{\begin{pmatrix} 2 \\ 0 \end{pmatrix} - \begin{pmatrix} 1 \\ 0 \end{pmatrix}}{2\pi} = \frac{1}{2\pi}\begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

Bsp. für Ableitungen:

$$f(x) = \frac{1}{x}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{1}{h} \left(\frac{1}{x+h} - \frac{1}{x} \right)$$

$$= \lim_{h \to 0} \frac{1}{h} \frac{x - (x+h)}{x(x+h)} = \lim_{h \to 0} \frac{-\cancel{h}}{\cancel{h}\, x\, (x+h)}$$

$$= -\frac{1}{x^2}$$

$$f(x) = a^x$$

$$f'(x) = \lim_{h \to 0} \frac{a^{x+h} - a^x}{h} = \lim_{h \to 0} \frac{a^x a^h - a^x}{h}$$

$$= a^x \underbrace{\lim_{h \to 0} \frac{a^h - 1}{h}}_{\text{hängt nicht von } x \text{ ab}}$$

Für $a = e = 2{,}718\ldots$ gilt $\lim_{h \to 0} \frac{e^h - 1}{h} = 1$

$$f(x) = \sin x$$

$$f'(x) = \lim_{h\to 0} \frac{\sin(x+h) - \sin x}{h} \qquad \text{(Add.-Th.)}$$

$$= \lim_{h\to 0} \frac{\sin x \cos h + \cos x \sin h - \sin x}{h}$$

$$= \lim_{h\to 0} \left(\sin x \, \frac{\cos h - 1}{h}\right) + \lim_{h\to 0} \left(\cos x \, \frac{\sin h}{h}\right)$$

$$= \sin x \lim_{h\to 0} \frac{\cos h - 1}{h} + \cos x \underbrace{\lim_{h\to 0} \frac{\sin h}{h}}_{= 1 \text{ (letzte Woche)}}$$

bleibt noch

$$\lim_{h\to 0} \frac{\cos h - 1}{h} = \lim_{h\to 0} \frac{\sqrt{1-\sin^2 h} - 1}{h} \cdot \frac{\sqrt{1-\sin^2 h} + 1}{\sqrt{1-\sin^2 h} + 1}$$

$$= \lim_{h\to 0} \frac{(1-\sin^2 h) - 1}{h\,\underbrace{\left(\sqrt{1-\sin^2 h}+1\right)}_{\xrightarrow[h\to 0]{} 2}} = \lim_{h\to 0}\left(- \underbrace{\frac{\sin h}{h}}_{\to 1} \; \frac{\overset{\to 0}{\sin h}}{\underbrace{\left(\sqrt{\ }+1\right)}_{\to 2}} \right)$$

$$= 0$$

Produktregel

$$(f\cdot g)'(x) = \lim_{h\to 0} \frac{f(x+h)\,g(x+h) - f(x)\,g(x)}{h}$$

$$= \lim_{h\to 0} \frac{[f(x+h) - f(x)]\,g(x+h) + f(x)\,g(x+h) - f(x)\,g(x)}{h}$$

$$= \lim_{h\to 0} \underbrace{\frac{f(x+h)-f(x)}{h}}_{\to f'(x)} \underbrace{g(x+h)}_{\to g(x)} + \lim_{h\to 0} f(x) \underbrace{\frac{g(x+h)-g(x)}{h}}_{\to g'(x)}$$

Idee Kettenregel

nähere g durch Tangente

$$[f(g(x))]' = \lim_{h \to 0} \frac{f(g(x+h)) - f(g(x))}{h}$$

$$= \lim_{\substack{h \to 0 \\ g'(x)h \to 0}} \frac{f(g(x) + \underbrace{g'(x)\cdot h}) - f(g(x))}{\underbrace{g'(x)\cdot h}} \cdot g'(x)$$

$$= f'(g(x)) \cdot g'(x)$$

Abl. der Umkehrfkt.

$$f^{-1}(f(x)) = x$$ ableiten $\Longrightarrow$

$$f^{-1\prime}(\underbrace{f(x)}_{=:y}) \cdot f'(\underset{=f^{-1}(y)}{x}) = 1$$

$\Longleftrightarrow$ $$f^{-1\prime}(y) = \frac{1}{f'(f^{-1}(y))}$$

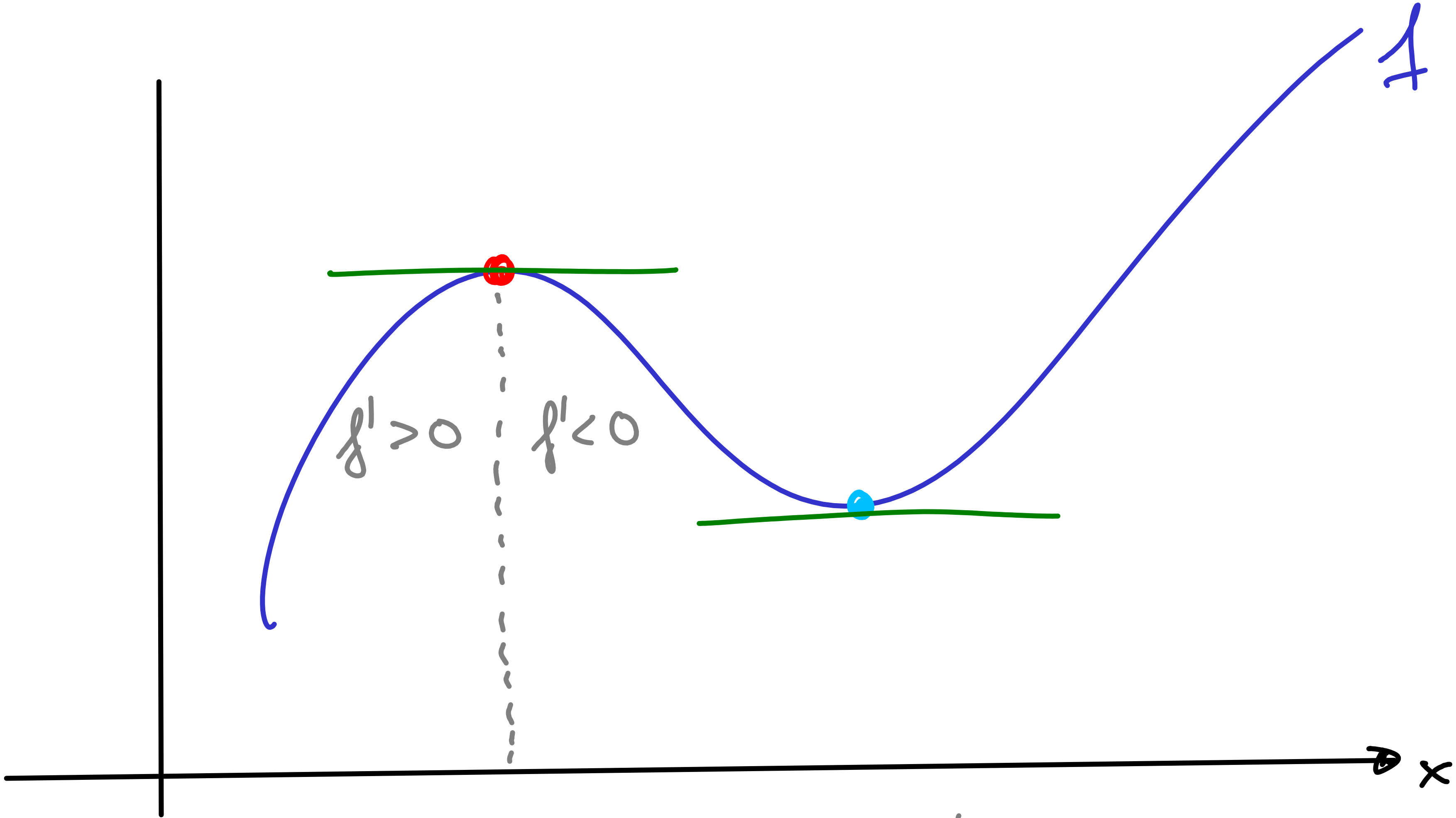
$f'(f^{-1}(y)) \neq 0$

nenne y nun wieder x

$$\boxed{f^{-1\prime}(x) = \frac{1}{f'(f^{-1}(x))}}$$

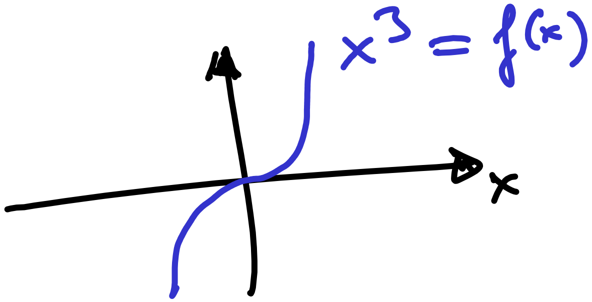
Bsp: $f^{-1}(x) = \log x$ $\qquad (f(x) = e^x = \exp(x))$

$$(\log x)' = \frac{1}{\exp'(\log x)} = \frac{1}{\exp(\log x)} = \frac{1}{x}$$



lokale Extrema $\Rightarrow f' = 0$

Umkehrung gilt nicht, z. B.



$f'(0) = 0$

aber kein Extremum