$$f: [0,\pi] \longrightarrow \mathbb{R}^2$$
$$x \longmapsto f(x) = \begin{pmatrix} \sin x \\ e^x + x \end{pmatrix}$$

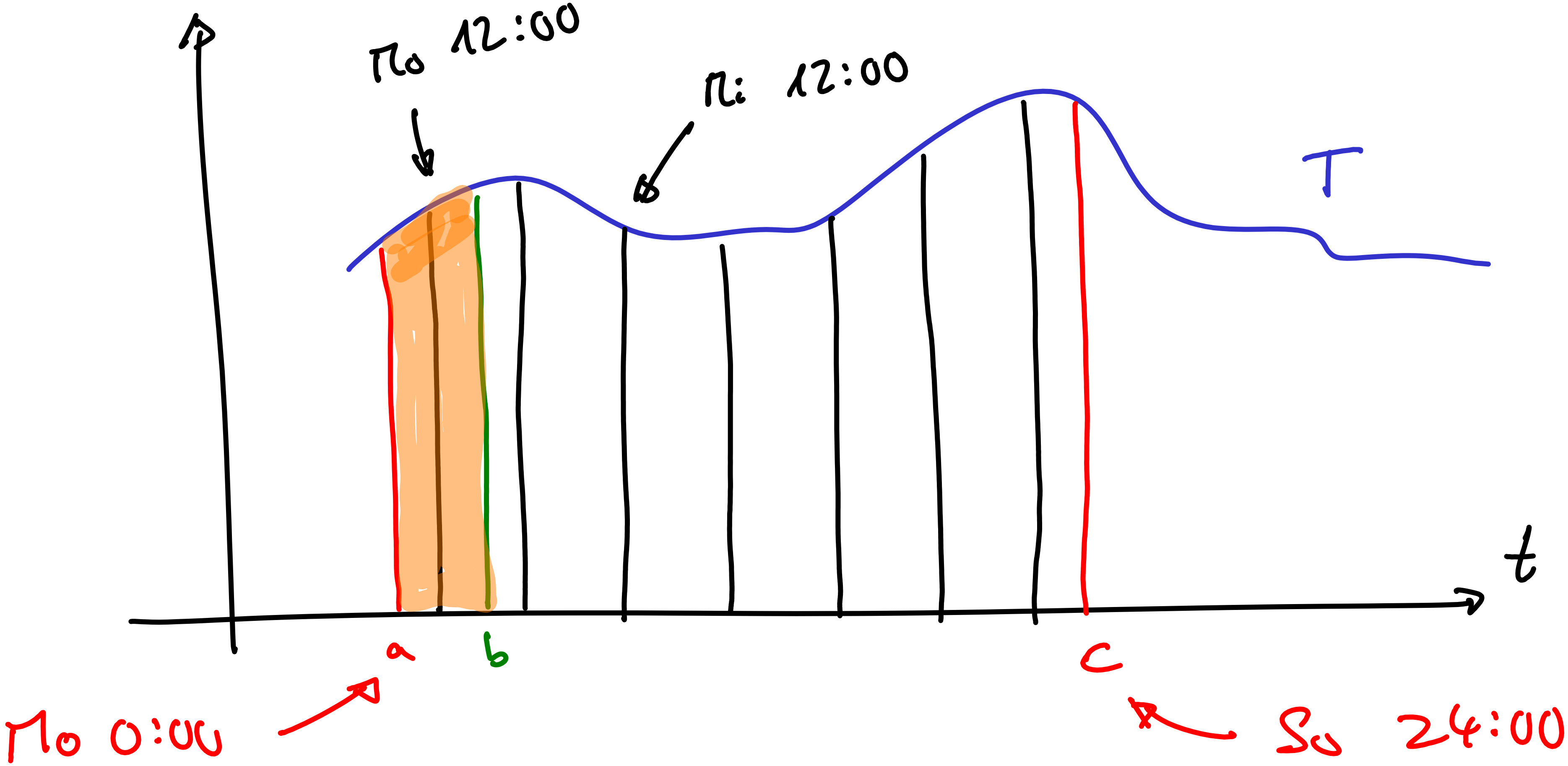
F mit $F' = f$ $\leftarrow$ gesucht

$$F: [0,\pi] \longrightarrow \mathbb{R}^2$$
$$x \longmapsto F(x) = \begin{pmatrix} -\cos x \\ e^x + \frac{1}{2}x^2 \end{pmatrix}$$

auch Stammfkt.:

$\tilde{F}(x) = \begin{pmatrix} -\cos x + 42 \\ e^x - 23 + \frac{1}{2}x^2 \end{pmatrix}$ hat auch f als Ableitung

hier $c = \begin{pmatrix} 42 \\ -23 \end{pmatrix}$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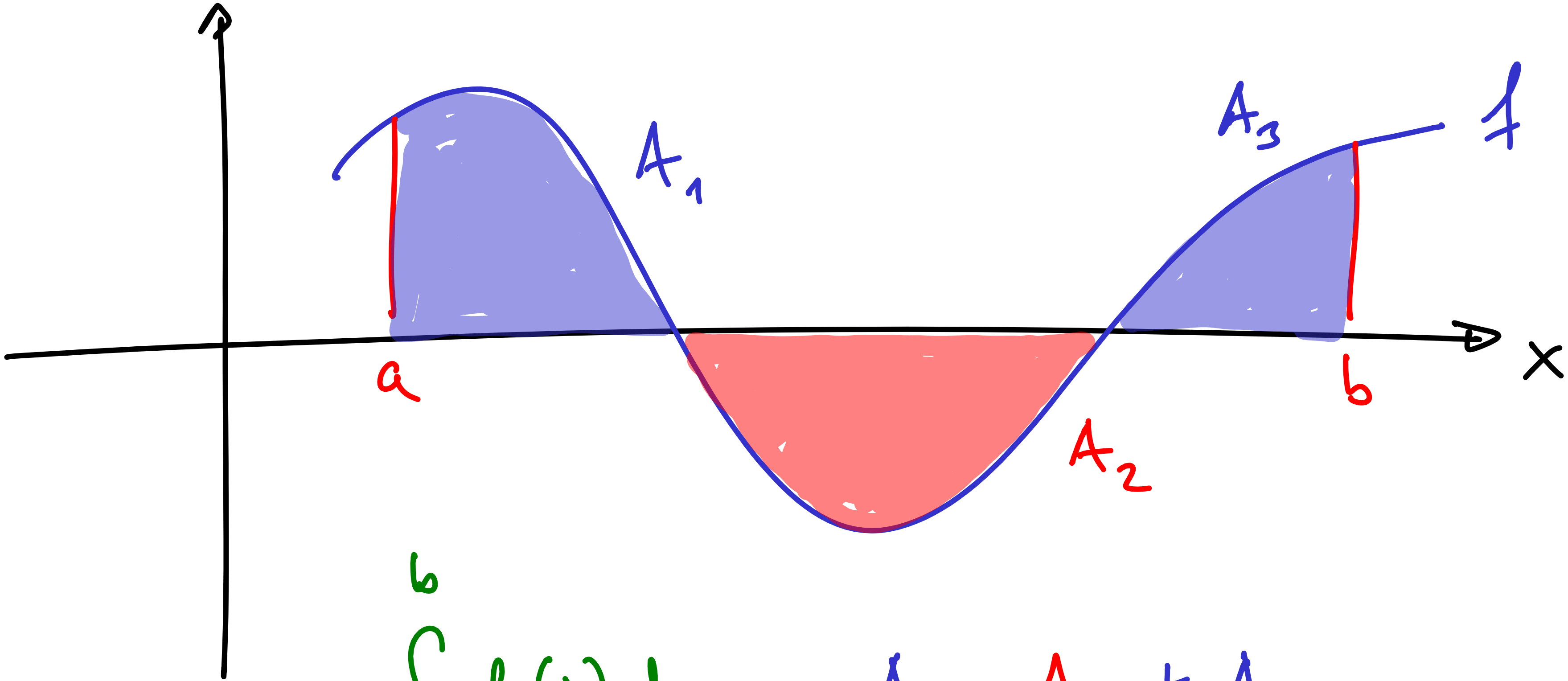
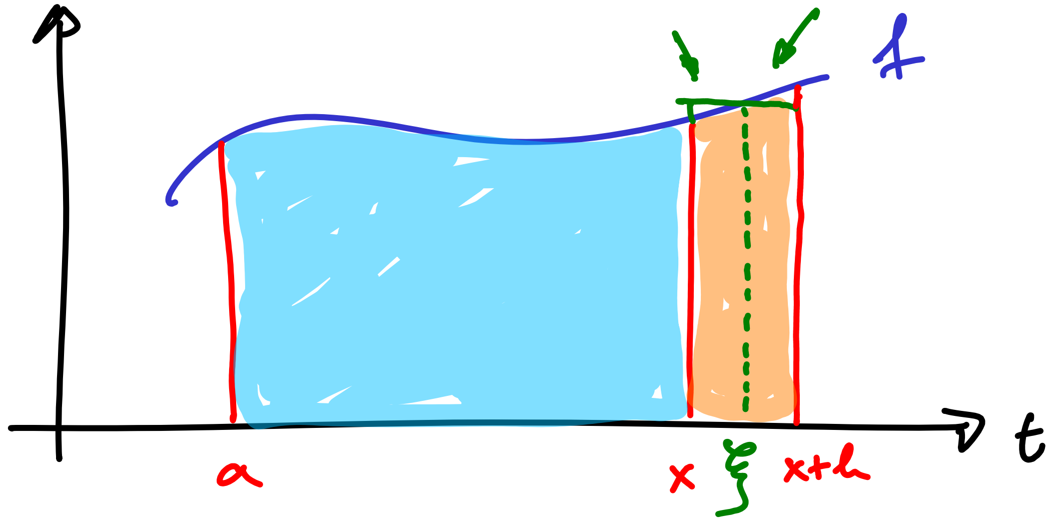


$$\int_a^b f(x)\,dx = A_1 - A_2 + A_3$$

Beweisidee zum Hauptsatz



$$F(x) = \int_a^x f(t)\,dt$$

z.z.: $F' = f$

es gibt ein $\xi \in [x, x+h]$ so dass $f(\xi)\cdot h =$

da f stetig

$$F'(x) = \lim_{h\to 0} \frac{F(x+h) - F(x)}{h} = \lim_{h\to 0} \frac{f(\xi)\cdot \cancel{h}}{\cancel{h}}$$

$$\xi \xrightarrow{h \to 0} x$$

$$= f(x)$$

Beispiele

$$\int_0^1 (7x^5 - 8x^2 + 3x)\,dx$$

$$= \left[\frac{7}{6}x^6 - \frac{8}{3}x^3 + \frac{3}{2}x^2\right]_0^1$$ ← Schreibweise für

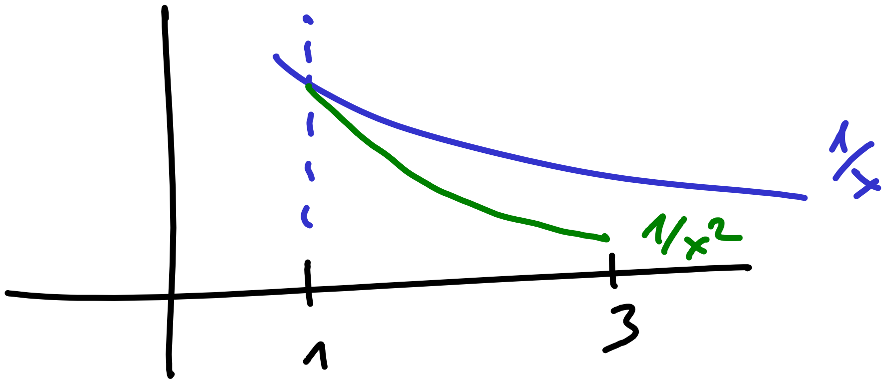
$$= \left(\frac{7}{6} - \frac{8}{3} + \frac{3}{2}\right) - (0)$$

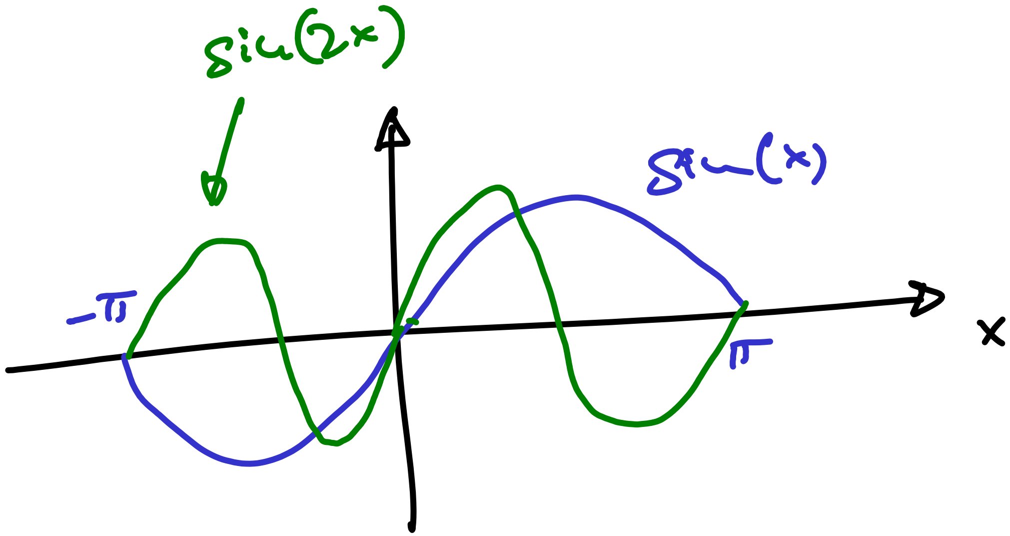
$$= \frac{7 - 16 + 9}{6} = 0$$

$$\int_1^3 \frac{dx}{x^2} = \int_1^3 \frac{1}{x^2}\,dx = \int_1^3 x^{-2}\,dx = \left[-x^{-1}\right]_1^3$$

$$= \left[-\frac{1}{x}\right]_1^3 = -\frac{1}{3} - (-1) = \frac{2}{3}$$

$$\int_1^3 \frac{dx}{x} = \left[\log x\right]_1^3 = \log 3 - \log 1 = \log \frac{3}{1} = \log 3$$





also Null

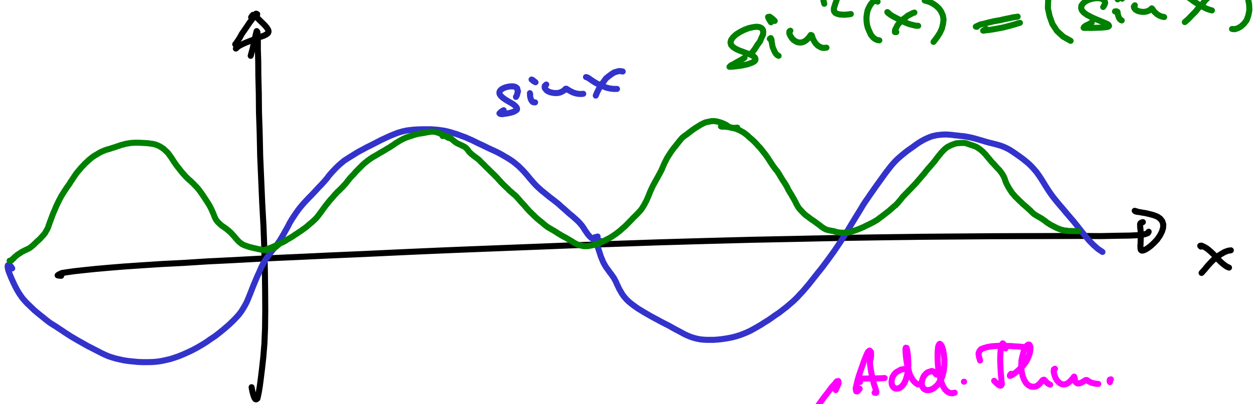
$$\int_{-\pi}^{\pi} \sin(nx)\,dx = \left[-\frac{1}{n}\cos(nx)\right]_{-\pi}^{\pi} = 0$$

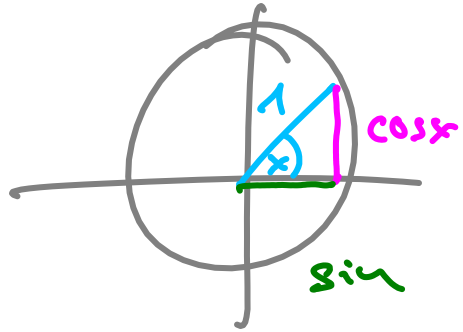
da $\cos(-x) = \cos x$

$$= -\frac{1}{n}\cos(n\pi) - \left(-\frac{1}{n}\cos(-n\pi)\right)$$

$$= -\frac{1}{n}(-1)^n + \frac{1}{n}(-1)^n = 0$$

$$\sin^2(x) = (\sin x)^2$$





Add. Thm.

$$\cos(2x) = \cos(x+x) = \cos^2 x - \sin^2 x$$

$$= 1 - 2\sin^2 x$$

Pythagoras $\sin^2 x + \cos^2 x = 1$

$$\Longrightarrow \quad \sin^2 x = \frac{1}{2} - \frac{1}{2}\cos(2x)$$

$$\int_{-\pi}^{\pi} \sin^2(nx)\,dx = \int_{-\pi}^{\pi} \left(\frac{1}{2} - \frac{1}{2}\cos(2nx) \right) dx$$

$$= \left[\frac{1}{2}x - \frac{1}{4n}\sin(2nx) \right]_{-\pi}^{\pi}$$

$$= \left(\frac{1}{2}\pi - 0 \right) - \left(-\frac{\pi}{2} - 0 \right) = \pi$$

$\sin(k\pi) = 0 \quad \forall k \in \mathbb{Z}$

Produktregel: $\left(f(x)\,g(x)\right)' = f'(x)\,g(x) + f(x)\,g'(x)$

$$\int_a^b \ldots \, dx$$

$$\Longrightarrow$$

$$\left[f(x)\,g(x)\right]_a^b = \underline{\int_a^b f'(x)\,g(x)\,dx} + \int_a^b f(x)\,g'(x)\,dx$$

$$\Leftrightarrow \int_a^b f'(x)\,g(x)\,dx = \left[f(x)\,g(x)\right]_a^b - \int_a^b f(x)\,g'(x)\,dx$$

Bspe zu partieller Integration:

$$\int_0^{\pi/2} \underset{g}{x} \, \underset{f'}{\cos x} \, dx = \left[x \sin x\right]_0^{\pi/2} - \int_0^{\pi/2} 1 \cdot \sin x \, dx$$

$$= \left[x \sin x\right]_0^{\pi/2} + \left[\cos x\right]_0^{\pi/2}$$

$$= \left[x \sin x + \cos x\right]_0^{\pi/2} = \frac{\pi}{2} \cdot 1 + 0 - (0 \cdot 0 + 1)$$

$$= \frac{\pi}{2} - 1$$

$$\int \log x \, dx = \int \underset{f'}{1} \cdot \underset{g}{\log x} \, dx = x \log x - \int x \cdot \frac{1}{x} \, dx$$

$$= x \log x - x$$

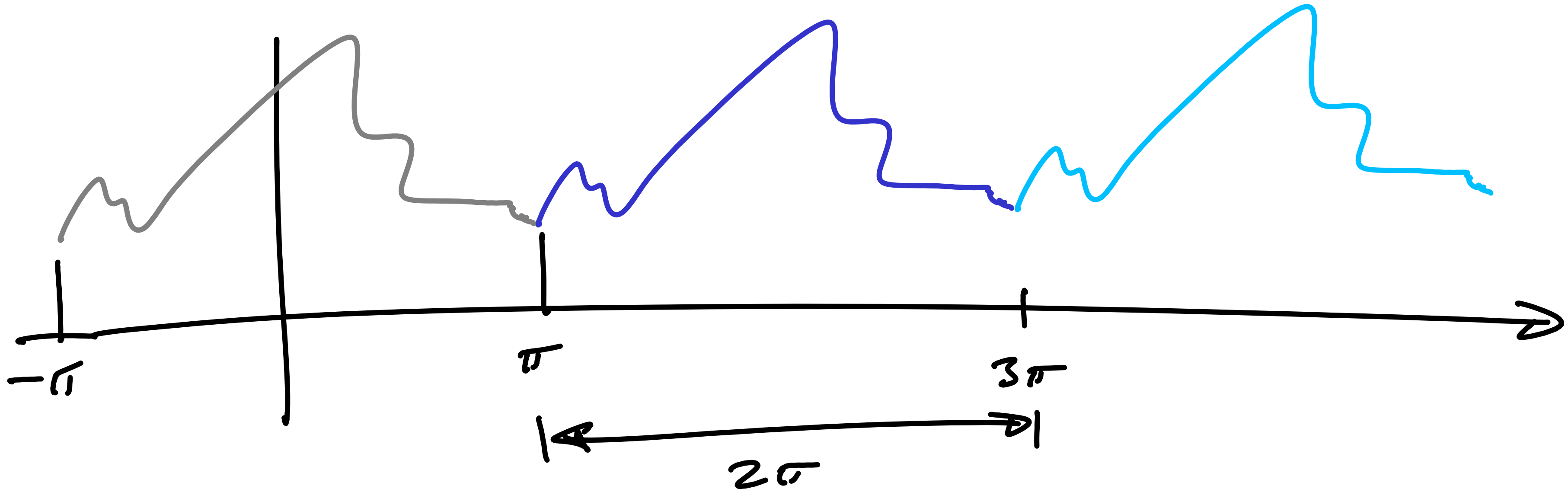
$$\int_{-\pi}^{\pi} \underset{f'}{\sin(nx)}\, \underset{g}{\sin(mx)}\, dx = \underbrace{\left[-\frac{\cos(nx)}{n}\sin(mx)\right]_{-\pi}^{\pi}}_{=0} + \int_{-\pi}^{\pi} \underset{f'}{\frac{\cos(nx)}{n}}\, \underset{g}{m\cos(mx)}\, dx$$

$$= \frac{m}{n}\left(\underbrace{\left[\frac{\sin(nx)}{n}\cos(mx)\right]_{-\pi}^{\pi}}_{=0} - \int_{-\pi}^{\pi} \frac{\sin(nx)}{n}\left(-\sin(mx)\cdot m\right) dx\right)$$

$$= \frac{m^2}{n^2}\int_{-\pi}^{\pi} \sin(nx)\sin(mx)\, dx$$

$$\Rightarrow \underbrace{\left(1-\frac{m^2}{n^2}\right)}_{\neq 0} \underbrace{\int_{-\pi}^{\pi} \sin(nx)\sin(mx)\, dx}_{=0} = 0$$

$$\Rightarrow$$



ist eine Summe von sin- & cos-Termen

$$\int_{-\pi}^{\pi} f(t)\,dt = \underbrace{\int_{-\pi}^{\pi} a_0\,dt}_{=2\pi a_0} + \left(\sum_n \underbrace{\int_{-\pi}^{\pi} a_n \cos(nt)\,dt}_{=0} + \underbrace{\int_{-\pi}^{\pi} b_n \sin(nt)\,dt}_{=0} \right)$$

$$\Rightarrow \quad a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(t)\,dt$$

$$\int_{-\pi}^{\pi} f(t)\sin(mt)\,dt = \underbrace{\int_{-\pi}^{\pi} a_0 \sin(mt)\,dt}_{=0}$$

$$+ \left(\sum_n \underbrace{\int_{-\pi}^{\pi} a_n \cos(nt)\sin(mt)\,dt}_{\substack{=0 \\ (\text{ÜA})}} + \underbrace{\int_{-\pi}^{\pi} b_n \sin(nt)\sin(mt)\,dt}_{= \begin{cases} \pi, & n = m \\ 0, & n \neq m \end{cases}} \right)$$

$$\Rightarrow \quad b_m = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t)\sin(mt)\,dt$$