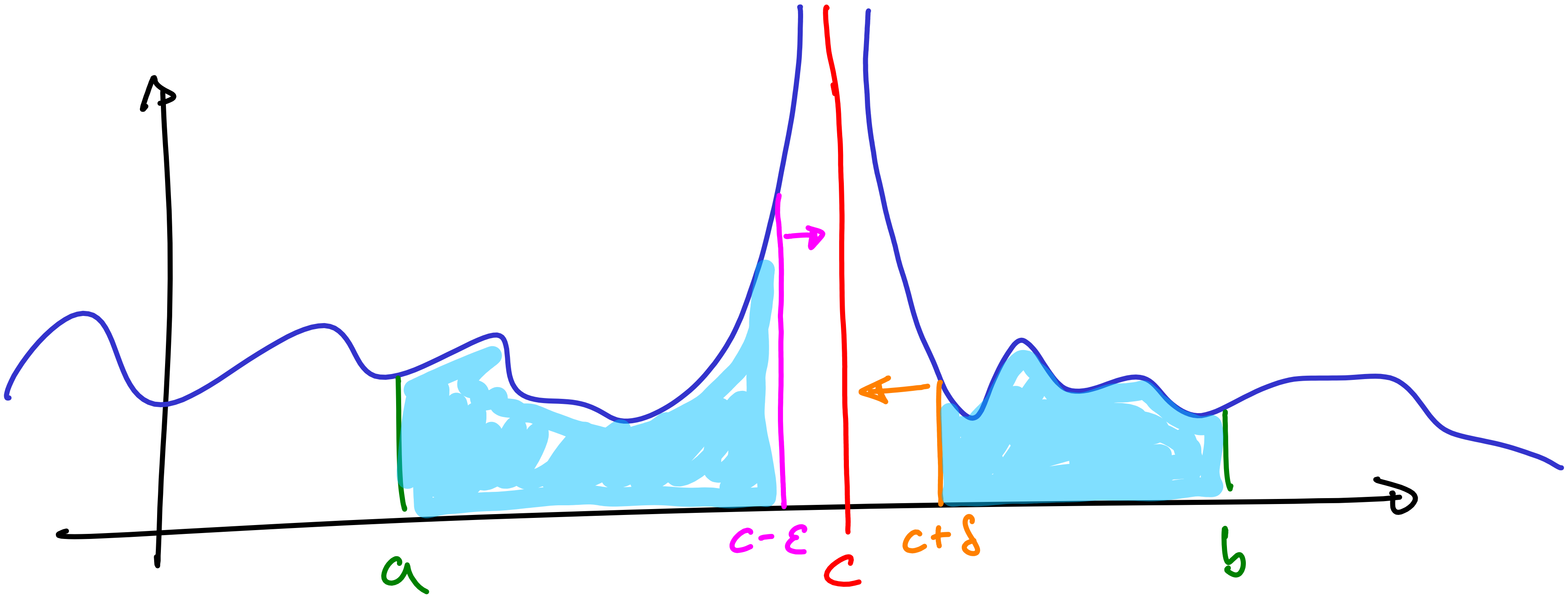


Beispiel:

$$\int_{-1}^{2} \frac{dx}{\sqrt{|x|}} = \int_{-1}^{0} \frac{dx}{\sqrt{|x|}} + \int_{0}^{2} \frac{dx}{\sqrt{|x|}}$$

$$= \lim_{\varepsilon \to 0+} \int_{-1}^{-\varepsilon} \frac{dx}{\sqrt{-x}} + \lim_{\delta \to 0+} \int_{\delta}^{2} \frac{dx}{\sqrt{x}}$$

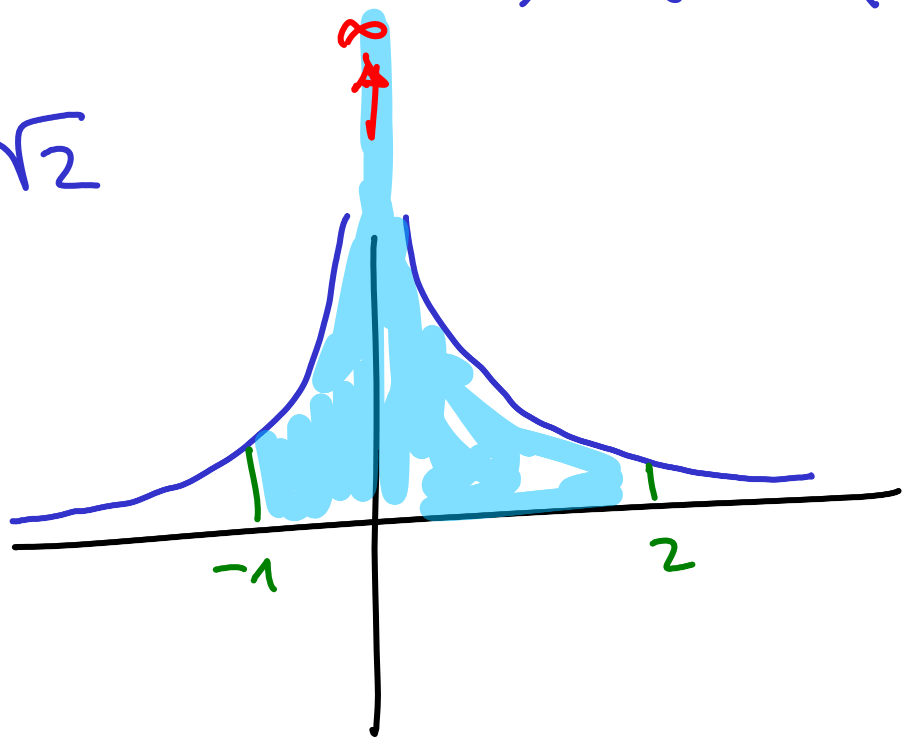
$$\frac{1}{\sqrt{-x}} = (-x)^{-1/2}$$

$$\frac{1}{\sqrt{x}} = x^{-1/2}$$

$$= \lim_{\varepsilon \to 0+} \left[-2(-x)^{1/2}\right]_{-1}^{-\varepsilon} + \lim_{\delta \to 0+} \left[2x^{1/2}\right]_{\delta}^{2}$$

$$= \lim_{\varepsilon \to 0+} \left(-2\sqrt{\varepsilon} - \left(-2\,(1)^{1/2}\right)\right) + \lim_{\delta \to 0+} \left(2\sqrt{2} - 2\sqrt{\delta}\right)$$

$$= 2 + 2\sqrt{2}$$

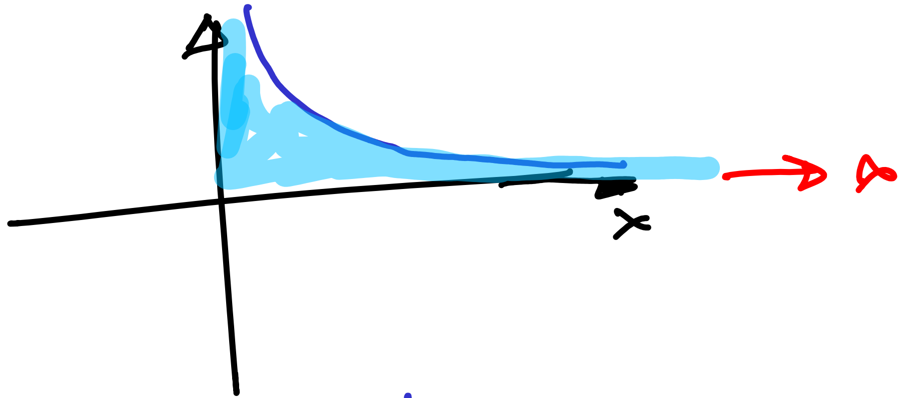


später <u>kurz</u>:

$$\int_0^2 \frac{dx}{\sqrt{x}} = \left[2\sqrt{x}\right]_0^2 = 2\sqrt{2} - 0 = 2\sqrt{2}$$

↑ hier versteckt sich ein Grenzwert

$$\int_1^\infty \frac{dx}{x^2}$$



$$= \lim_{b\to\infty} \int_1^b \frac{dx}{x^2} = \lim_{b\to\infty} \left[-\frac{1}{x}\right]_1^b = \lim_{b\to\infty}\left(-\frac{1}{b}-\left(-\frac{1}{1}\right)\right)$$

$$= 1$$

kurz: $\int_1^\infty \frac{dx}{x^2} = \left[-\frac{1}{x}\right]_1^\infty = 0-(-1)) = 1$

anderes Beispiel! (Skizze qualitativ wie oben)

$$\int_1^\infty \frac{dx}{x} = \left[\log x\right]_1^\infty = \infty$$

da $\lim_{b\to\infty} \log b = \infty$

Calc-zilla: $f(x,y) = y^x = e^{\log(y^x)} = e^{x \cdot \log y}$

$$\frac{\partial f}{\partial x}(x,y) = \log y \cdot e^{x \cdot \log y} = y^x \cdot \log y$$

...und nach y?

$$\frac{\partial f}{\partial y}(x,y) = x \cdot y^{x-1}$$

$$f: \mathbb{R}^2 \to \mathbb{R}$$

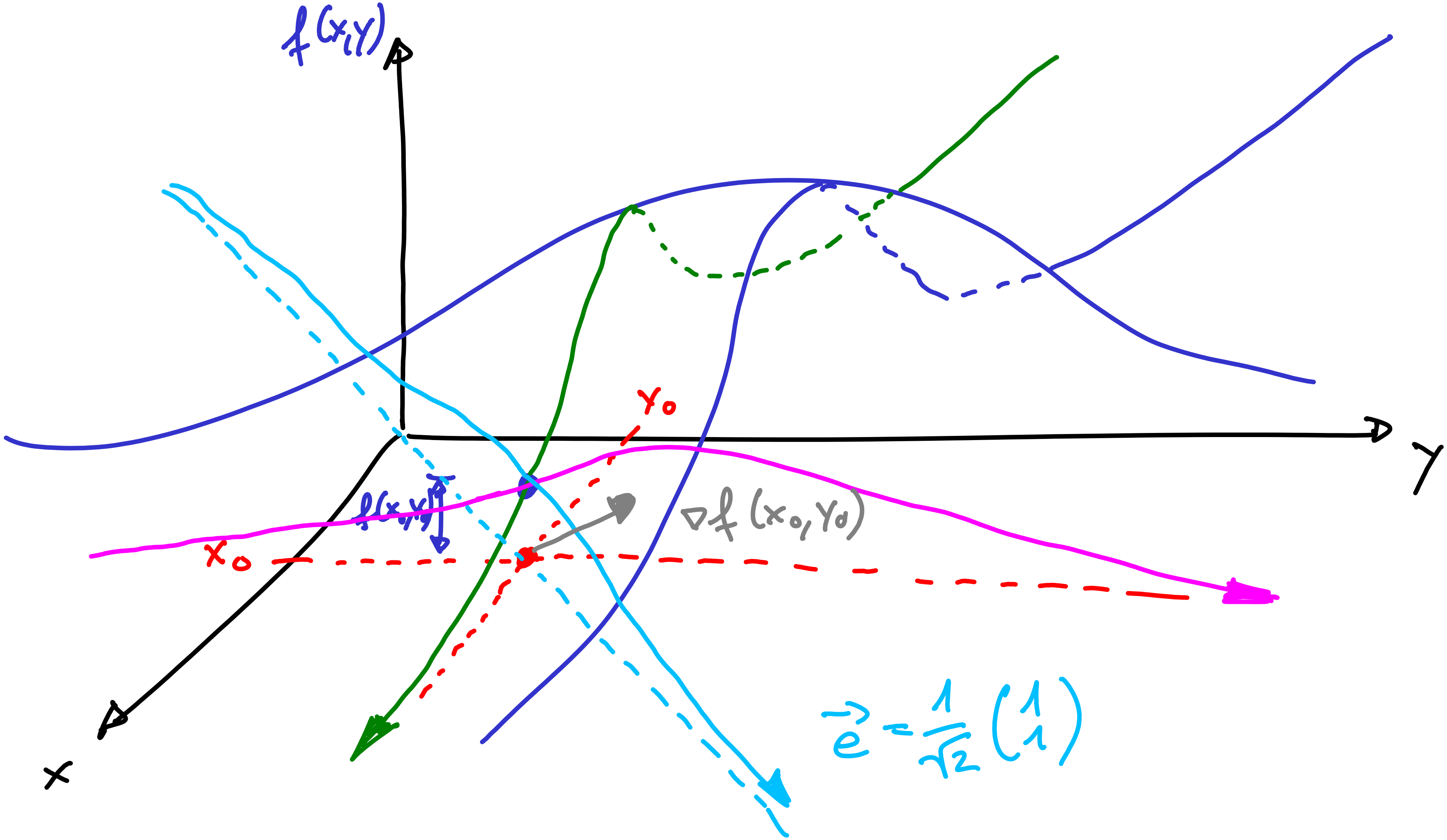
$$(x_1, x_2) \mapsto f(x_1, x_2)$$

$$\vec{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, \quad \vec{e} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\frac{\partial f}{\partial x_1}(x_1, x_2) = \lim_{h \to 0} \frac{f(\vec{x} + h\vec{e}) - f(\vec{x})}{h}$$

$$\vec{x} + h\vec{e} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + h \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} x_1 + h \\ x_2 \end{pmatrix}$$

$$= \lim_{h \to 0} \frac{f(x_1 + h, x_2) - f(x_1, x_2)}{h}$$



$\frac{\partial f}{\partial x}(x_0,y_0)$ = Steigung des grünen Weges an der Stelle (x_0,y_0) < 0

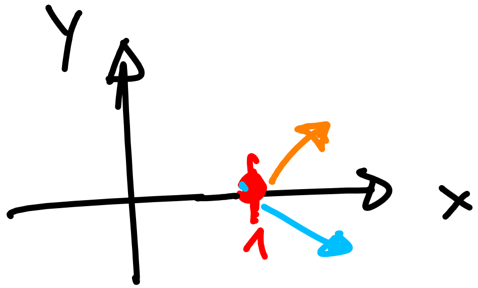
$\frac{\partial f}{\partial y}(x_0,y_0)$ = Steigung des rosa Weges an der Stelle (x_0,y_1) > 0

$\frac{\partial f}{\partial \vec{e}}(x_0, y_0) =$ Steigung des blauen Weges an der Stelle (x_0, y_0)

Bsp: $f(x,y) = x\,e^y$ Stelle $(1,0)$

Richtung $\vec{e} = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix}$

$\vec{e} = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}$



$$\frac{\partial f}{\partial x} = e^y, \quad \frac{\partial f}{\partial y} = x \cdot e^y$$

$$\frac{\partial f}{\partial \vec{e}}(1,0) = \left(\frac{\partial f}{\partial x}(1,0), \frac{\partial f}{\partial y}(1,0)\right) \cdot \frac{1}{\sqrt{2}}\begin{pmatrix}1\\\pm 1\end{pmatrix}$$

$$= (1, 1) \cdot \frac{1}{\sqrt{2}}\begin{pmatrix}1\\\pm 1\end{pmatrix} = 0$$

Matrix-Prod.

$\sqrt{2}$

Skalar-produkt

$$= \begin{pmatrix}1\\1\end{pmatrix} \cdot \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix}$$

$$\frac{\partial f}{\partial \vec{e}}(1,0) = \nabla f(1,0) \cdot \vec{e}$$

Erinnerung: Tangente

Funktion f, Stelle x_0

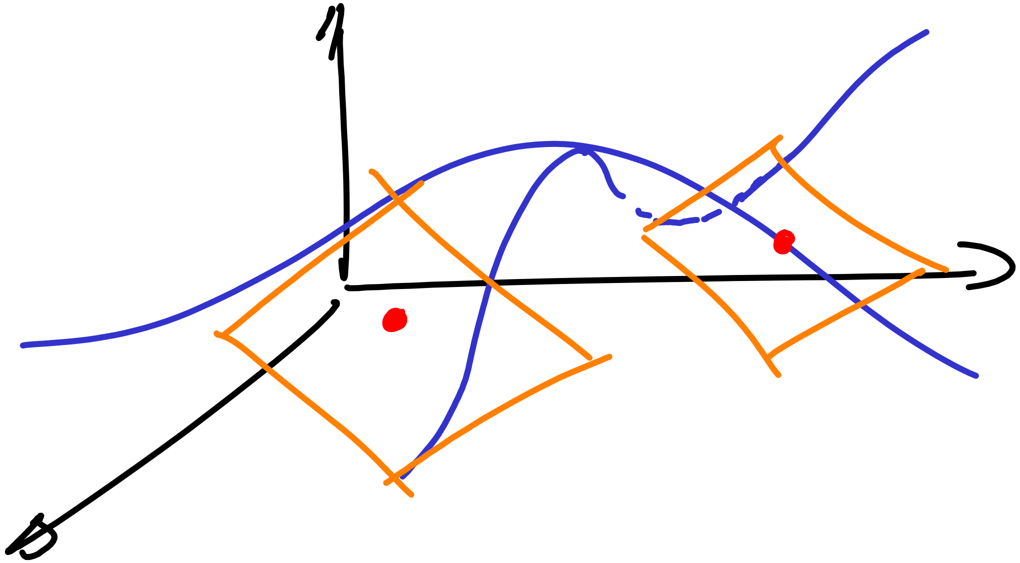
Tangente: $t(x) = f(x_0) + f'(x_0) \cdot (x - x_0)$

analog mehrdimensional

Funktion f, Stelle $\vec{x}_0$

Tangentialebene

$$T(\vec{x}) = f(\vec{x}_0) + \nabla f(\vec{x}_0) \cdot (\vec{x} - \vec{x}_0)$$



zweite Ableitung $\quad f(x,y) = x \cdot \sin y$

$$\frac{\partial f}{\partial x}(x,y) = \sin y \quad , \quad \frac{\partial f}{\partial y}(x,y) = x \cdot \cos y$$

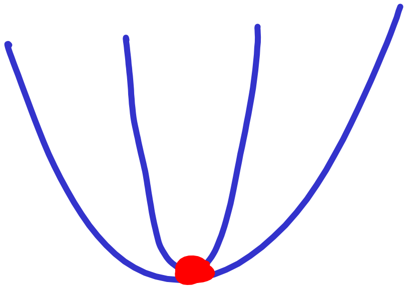
$$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right) = \frac{\partial}{\partial x}(x \cdot \cos y) = \cos y$$

$$\frac{\partial^2 f}{\partial y \partial x} = \frac{\partial}{\partial y}(\sin y) = \cos y$$

immer gleich ?

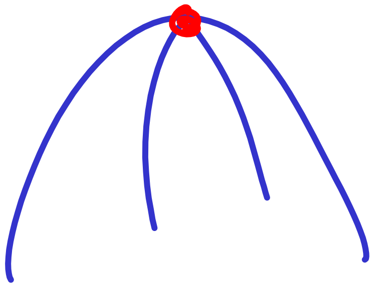
$$\frac{\partial^2 f}{\partial x^2} = 0 \,, \quad \frac{\partial^2 f}{\partial y^2} = -x \cdot \sin y$$

$$H = \begin{pmatrix} 0 & \cos y \\ \cos y & -x \cdot \sin y \end{pmatrix}$$



Min.

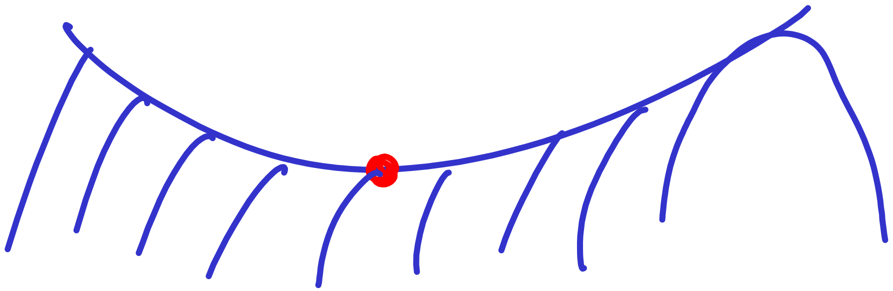
H pos. def.



Max

H neg. def.

mit Matlab ausprobieren ?



Sattel

H indefinit