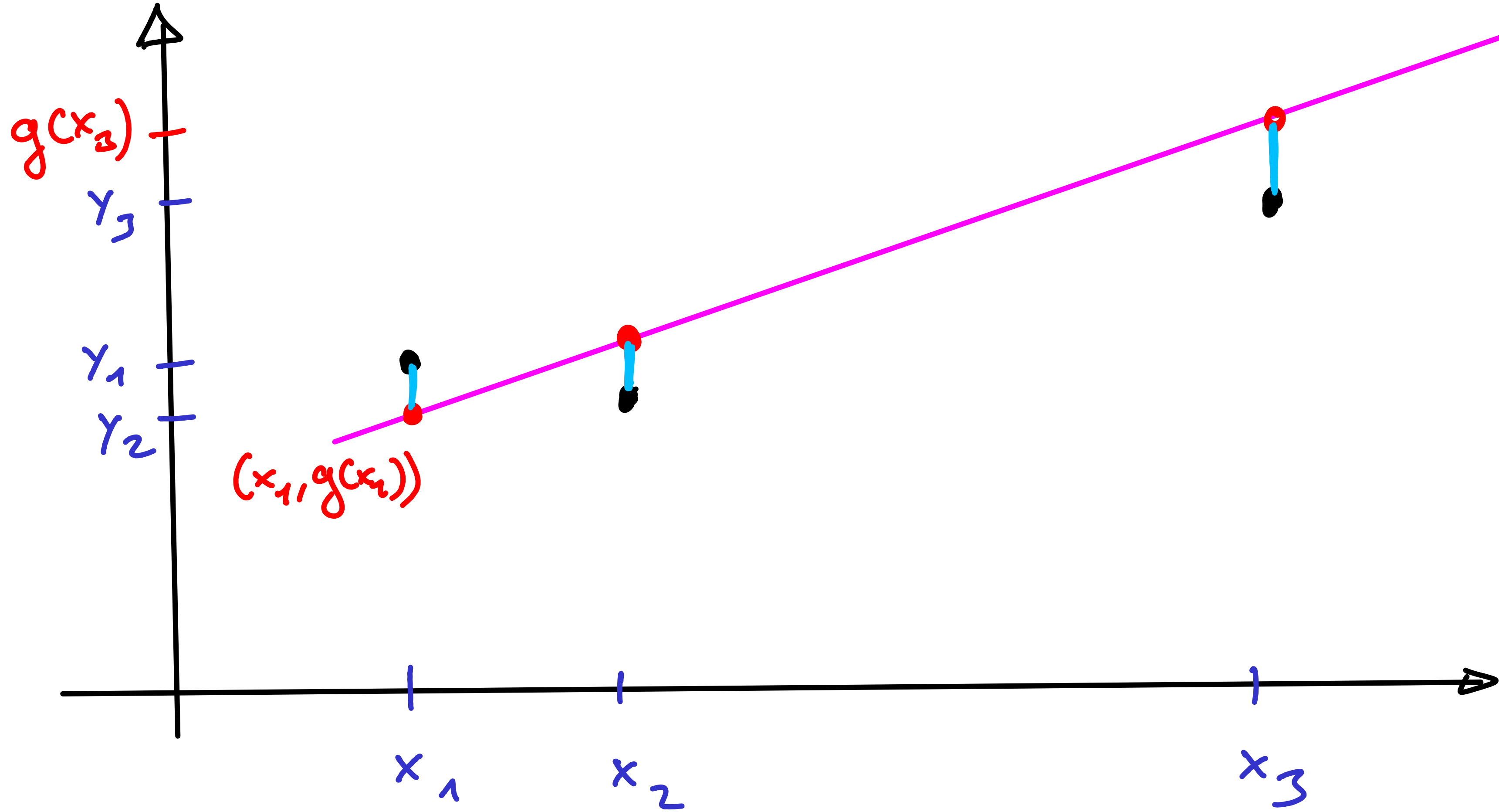


$$\underset{D(m,b)}{D} = \sqrt{\sum_{i=1}^{3}\left(\underset{mx_i+b}{g(x_i)} - y_i\right)^2}$$

$$f(b,m) = \sum_{i=1}^{n} (m x_i + b - y_i)^2$$

$$\frac{\partial f}{\partial b} = 0 \iff \sum_{i=1}^{n} 2(m x_i + b - y_i) = 0 \quad \Big| \cdot \frac{1}{2}$$

$$\iff \underbrace{\sum_{i=1}^{n} m x_i}_{= m \underbrace{\sum_{i=1}^{n} x_i}_{= n \cdot \bar{x}}} + \underbrace{\sum_{i=1}^{n} b}_{= nb} - \underbrace{\sum_{i=1}^{n} y_i}_{\text{analog } = n\bar{y}} = 0$$

$$\bar{y} = \frac{1}{n} \sum_{i=1}^{n} y_i$$

mit $\bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i$

$$\iff m \cdot \cancel{n} \cdot \bar{x} + \cancel{n} \cdot b = \cancel{n}\, \bar{y} \quad \Big| \cdot \frac{1}{n}$$

$$\iff b = \bar{y} - m\bar{x}$$

$$\sum_{i=1}^{n}(x_i-\bar{x})(y_i-\bar{y})$$

$$= \sum_{i=1}^{n}\left(x_i y_i - x_i\bar{y} - \bar{x}y_i + \bar{x}\bar{y}\right)$$

$$= \sum_{i=1}^{n} x_i y_i - \bar{y}\underbrace{\sum_{i=1}^{n} x_i}_{=n\bar{x}} - \bar{x}\underbrace{\sum_{i=1}^{n} y_i}_{=n\bar{y}} + n\bar{x}\bar{y}$$

$$= \sum_{i=1}^{n} x_i y_i - n\bar{x}\bar{y}$$ ← so stand es ursprünglich da

analog $$\sum_{i=1}^{n}(x_i-\bar{x})^2 = \sum_{i=1}^{n} x_i^2 - n\bar{x}^2$$ steht auch in LGS

Dürfte mir durch

$$\underbrace{\sum_{i=1}^{n} (x_i - \bar{x})^2}_{\geq 0}$$

teilen ?

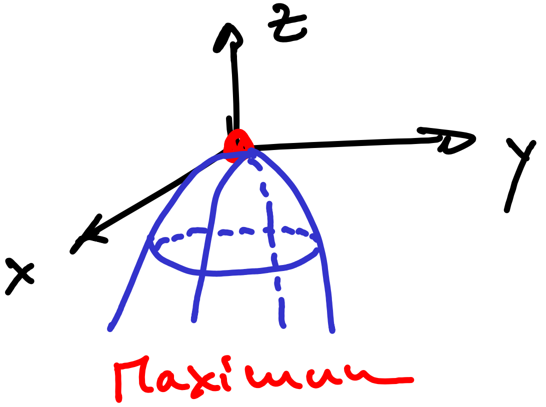
und ganze Summe $= 0$ nur dann, wenn

jedes $x_i = \bar{x}$ d.h. wenn alle x_i gleich – war

aber ausgeschlossen!

$\rightarrow$ "Ja!"

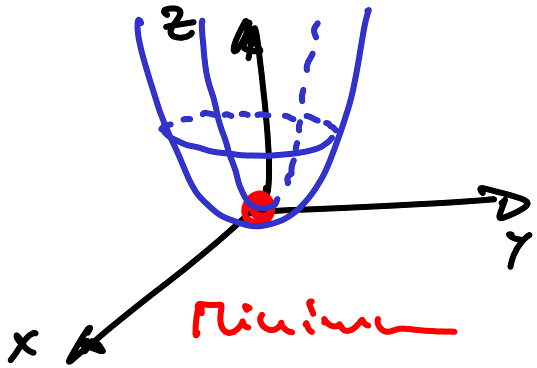
Definitheit



z.B. $z = -x^2 - y^2$

$$z'' = \begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix}$$

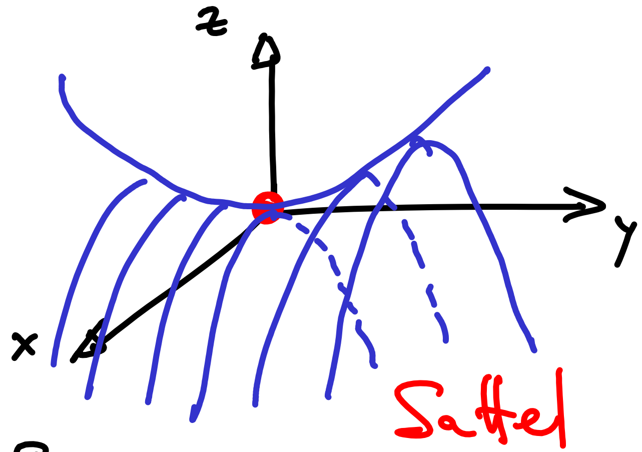
negativ definit



z.B. $z = x^2 + y^2$

$$z'' = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$$

positiv definit



z.B. $z = y^2 - x^2$

$$z'' = \begin{pmatrix} -2 & 0 \\ 0 & 2 \end{pmatrix}$$

indefinit

Definitheit von 2x2-Matrizen

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}, \quad \vec{x} = \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\vec{x}^T A \vec{x} = (x \quad y) \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = (x \quad y) \begin{pmatrix} ax + by \\ cx + dy \end{pmatrix}$$

$$= ax^2 + \underbrace{bxy + cxy}_{2bxy} + dy^2$$

symmetrische Matrix: $b = c$

quadratische Ergänzung

$$= a\left(x^2 + 2\frac{b}{a}xy\right) + dy^2$$

$$= a\left(\left(x + \frac{b}{a}y\right)^2 - \frac{b^2}{a^2}y^2\right) + dy^2$$

$$= a\underbrace{\left(x + \frac{b}{a}y\right)^2}_{\geq 0} + \left(d - \frac{b^2}{a}\right)\underbrace{y^2}_{\geq 0}$$

Falls $a > 0$ und $ad - b^2 > 0$

dann ist A pos. definit

$$A = \begin{pmatrix} a & b \\ b & d \end{pmatrix}$$

2. Ableitung aus Reg.-Analyse

$$\begin{pmatrix} 2n & 2n\bar{x} \\ 2n\bar{x} & 2\sum_{i=1}^{n} x_i^2 \end{pmatrix}$$

① $2n > 0$

② $4n \sum_{i=1}^{n} x_i^2 - 4n^2 \bar{x}^2$

$$= 4n \underbrace{\left(\sum_{i=1}^{n} x_i^2 - n \bar{x}^2 \right)}_{> 0 \text{ wenn nicht alle } x_i \text{ gleich}}$$

kenne wir von oben (Nenner von m)

$\Rightarrow f''$ pos. def. d.h. Minimum ☺