ÜA 26 b)

$$(\overset{x}{10^{-5}}, \overset{y}{10^{-7}}) \qquad (10^{5}, 10^{1})$$

$$\log_{10} y = m \log_{10} x + b \overset{}{\underset{10^{\cdots}}{\iff}} y = 10^{m \log_{10} x + b}$$

$$= x^{m} \cdot 10^{b}$$

Punkte einsetzen:

$$\text{(i)} \quad -7 = m(-5) + b$$

$$\text{(ii)} \quad 1 = m(5) + b$$

$$\text{(i)+(ii)} \quad -6 = 2b \implies b = -3$$

$$\text{(ii)}-\text{(i)} \quad 8 = 10m \implies m = \frac{8}{10} = \frac{4}{5}$$

also

$$y = x^{4/5} \cdot 10^{-3}$$

ÜA 45 b)

z. Zt. $t=0$ (x in Metern)

$$x_S(t) = 10 + v \cdot t$$

$$x_A(t) = 0 + 100v \cdot t$$

$$x_S(t) = x_A(t) \iff 10 + vt = 100\,vt$$

löse nach t auf: $t = \frac{10}{99 \cdot v}$

einsetzen entweder in x_S oder x_A (egal)

$$x_A\left(\frac{10}{99v}\right) = 100 \cdot \frac{10}{99} = \frac{1000}{99}$$

a)

$$10 + \frac{10}{100} + \frac{10}{100^2} + \frac{10}{100^3} + \dots$$

zunächst legt A 10 m zurück

in der Zeit schafft S $\frac{10}{100}$ m, die muss A auf und laufe etc.

$$= 10\left(1 + \frac{1}{100} + \left(\frac{1}{100}\right)^2 + \left(\frac{1}{100}\right)^3 + \dots\right)$$

$$= 10 \cdot \sum_{j=0}^{\infty} \left(\frac{1}{100}\right)^j = 10 \cdot \frac{1}{1 - \frac{1}{100}} = 10 \cdot \frac{100}{99}$$ ☺

$$= \sum_{j=0}^{\infty} \left(\frac{1}{100}\right)^j \cdot 10$$

$$\boxed{\sum_{j=0}^{\infty} x^j = \frac{1}{1-x} \quad \text{falls } |x| < 1}$$

geometrische Reihe

$|x| < 1$

$$\sum_{j=1}^{\infty} x^j = -1 + \sum_{j=0}^{\infty} x^j = -1 + \frac{1}{1-x} = \frac{x}{1-x}$$

$$\sum_{j=1}^{\infty} x^j = \sum_{j=0}^{\infty} x^{j+1} = x\sum_{j=0}^{\infty} x^j = x \cdot \frac{1}{1-x}$$

Nachklausur 09/10

7 b)

$$p' = -\frac{\alpha}{T_0 - \gamma z}\, p \qquad (*)$$

$$p = C\,(T_0 - \gamma z)^{\beta}$$

einsetzen, also zuerst ableiten

$$p' = C\beta\,(T_0 - \gamma z)^{\beta-1}\,(-\gamma)$$

jetzt einsetzen in $(*)$

$$\underbrace{-\cancel{C}\beta\gamma\,(T_0 - \gamma z)^{\beta-1}}_{=p'} = -\frac{\alpha}{T_0-\gamma z}\,\underbrace{\cancel{C}\,(T-\gamma z)^{\beta}}_{=p}$$

$$\Leftrightarrow \quad -\beta\gamma\, \cancel{(T_0-\gamma z)^{\beta-1}} = -\alpha\, \cancel{(T-\gamma z)^{\beta-1}}$$

$$\Leftrightarrow \quad \beta = \frac{\alpha}{\gamma}$$

Peter fährt mit 180 km/h nach München

auf der Rückweg schafft er nur noch $90 \frac{km}{h}$

gesucht: Durchschnittsgeschw.

Sei s die Strecke bis München

$\Rightarrow$ Peter legt $2s$ zurück

Für die erste Hälfte benötigt er $\frac{s}{180\, km/h}$

... zweite ---- $\frac{s}{90\, km/h}$

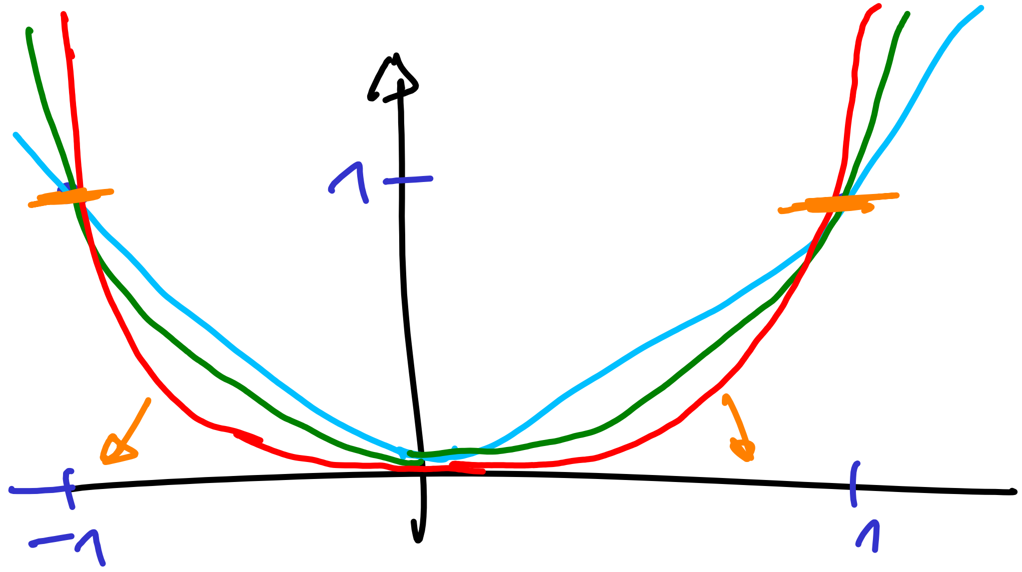
$$\frac{\text{Gesamtstrecke}}{\text{Gesamtzeit}} = \frac{2s}{\frac{s}{180\,\frac{km}{h}} + \frac{s}{90\,km/h}} = \frac{2}{\frac{1}{180} + \frac{1}{90}}\ km/h$$

$$= \frac{2 \cdot 180}{1 + 2} = 120\ km/h$$

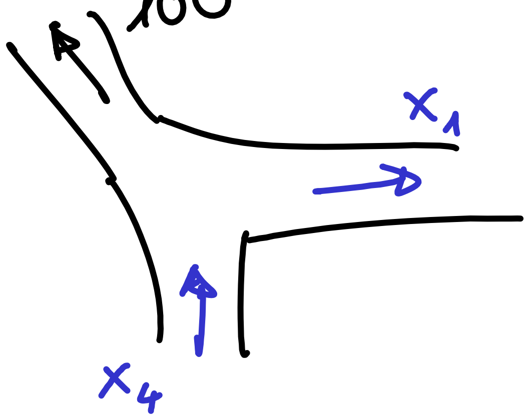
Ü A 44 b)

$$\lim_{n\to\infty} x^{2n} = \lim_{n\to\infty} (x^2)^n = \begin{cases} 0 & , \ -1 < x^2 < 1 \\ 1 & , \ x^2 = 1 \end{cases}$$

$$= \begin{cases} 0 & , \ -1 < x < 1 \\ 1 & , \ x = 1 \text{ oder } x = -1 \end{cases}$$



ÜA le6



$$x_4 - x_1 = 100$$

etc.

$$x_4 - x_1 = 100$$

$$x_2 + x_1 = 200$$

$$x_3 + x_2 = 250$$

$$x_4 - x_3 = 50$$

diktiert

korrigiert

$$\begin{aligned}
-x_1 \quad\quad\quad\quad\quad\quad + x_4 &= 100 \\
+x_1 + x_2 \quad\quad\quad\quad\quad\quad &= 200 \\
x_2 + x_3 \quad\quad\quad &= 250 \\
-x_3 + x_4 &= 50
\end{aligned}$$

$$\left(\begin{array}{cccc|c}
-1 & 0 & 0 & 1 & 100 \\
+1 & 1 & 0 & 0 & 200 \\
0 & 1 & 1 & 0 & 250 \\
0 & 0 & -1 & 1 & 50
\end{array}\right) \quad \cancel{\cdot 1}$$

$$
\left(\begin{array}{cccc|c}
-1 & 0 & 0 & 1 & 100 \\
0 & 1 & 0 & +1 & \cancel{100}\ 300 \\
0 & 1 & 1 & 0 & 250 \\
0 & 0 & -1 & 1 & 50
\end{array}\right)
\quad -1
$$

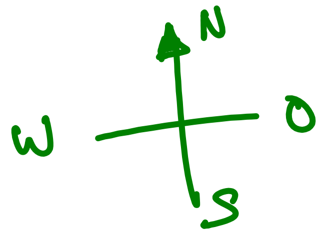
$$
\left(\begin{array}{cccc|c}
-1 & 0 & 0 & 1 & 100 \\
0 & 1 & 0 & +1 & 300\ \cancel{100} \\
0 & 0 & 1 & -1 & -50\ \cancel{150} \\
0 & 0 & -1 & 1 & 50
\end{array}\right)
$$

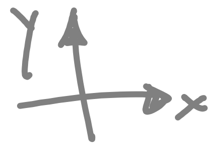
$$
\left(\begin{array}{cccc|c}
-1 & 0 & 0 & 1 & 100 \\
0 & 1 & 0 & +1 & 300\ \cancel{100} \\
0 & 0 & 1 & 1 & 150 \\
0 & 0 & 0 & \cancel{2}\ 0 & \cancel{200}\ 0
\end{array}\right)
$$

$$f = x^2 + y^3$$

$$f' = \begin{pmatrix} 2x \\ 3y^2 \end{pmatrix}$$ Gradient; an der Stelle $(3,1)$: $\begin{pmatrix} 6 \\ 3 \end{pmatrix}$

Nachklausur 09/10, 1





a) $\vec{v}_1 = \frac{9}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix}$

b) $\vec{w} = \frac{4}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix} = \sqrt{8}\begin{pmatrix}1\\-1\end{pmatrix}$

c) $\frac{\vec{v}_2}{|\vec{v}_2|} = \begin{pmatrix}0\\1\end{pmatrix}$

d) $\vec{u}_1 + \vec{w} = \vec{v}_1 \Rightarrow |\vec{u}_1| = |\vec{v}_1 - \vec{w}| = \left|\frac{5}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix}\right| = 5$

e) $\vec{u}_2 + \vec{w} = \vec{v}_2 \Rightarrow |\vec{u}_2|^2 = |\vec{v}_2 - \vec{w}|^2$

und wegen $|\vec{u}_1| = |\vec{u}_2|$ gilt

$$25 = \left|\, |\vec{v}_2|\begin{pmatrix}0\\1\end{pmatrix} - \frac{4}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix}\right|^2 = \left(-\frac{4}{\sqrt{2}}\right)^2 + \left(|\vec{v}_2| + \frac{4}{\sqrt{2}}\right)^2$$

$$= 8 + \left(|\vec{v}_2| + \frac{4}{\sqrt{2}}\right)^2 \Leftrightarrow \vec{v}_2 + \frac{4}{\sqrt{2}} = \pm\sqrt{17}$$

$$\Leftrightarrow |\vec{v}_2| = -\frac{4}{\sqrt{2}} \pm \sqrt{17}$$

Welches VZ?

$\frac{4}{\sqrt{2}} < 4$, $\sqrt{17} > 4$ und da $|\vec{v}_2| > 0 \Rightarrow$ Plus

also $|\vec{v}_2| = \sqrt{17} - \frac{4}{\sqrt{2}} = \sqrt{17} - \sqrt{8}$

Probe:

$$\vec{u}_2 = \vec{v}_2 - \vec{w} = (\sqrt{17} - \sqrt{8}) \begin{pmatrix} 0 \\ 1 \end{pmatrix} - \sqrt{8} \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} -\sqrt{8} \\ \sqrt{17} \end{pmatrix}$$

$$\Rightarrow |\vec{u}_2| = \sqrt{8 + 17} = 5$$