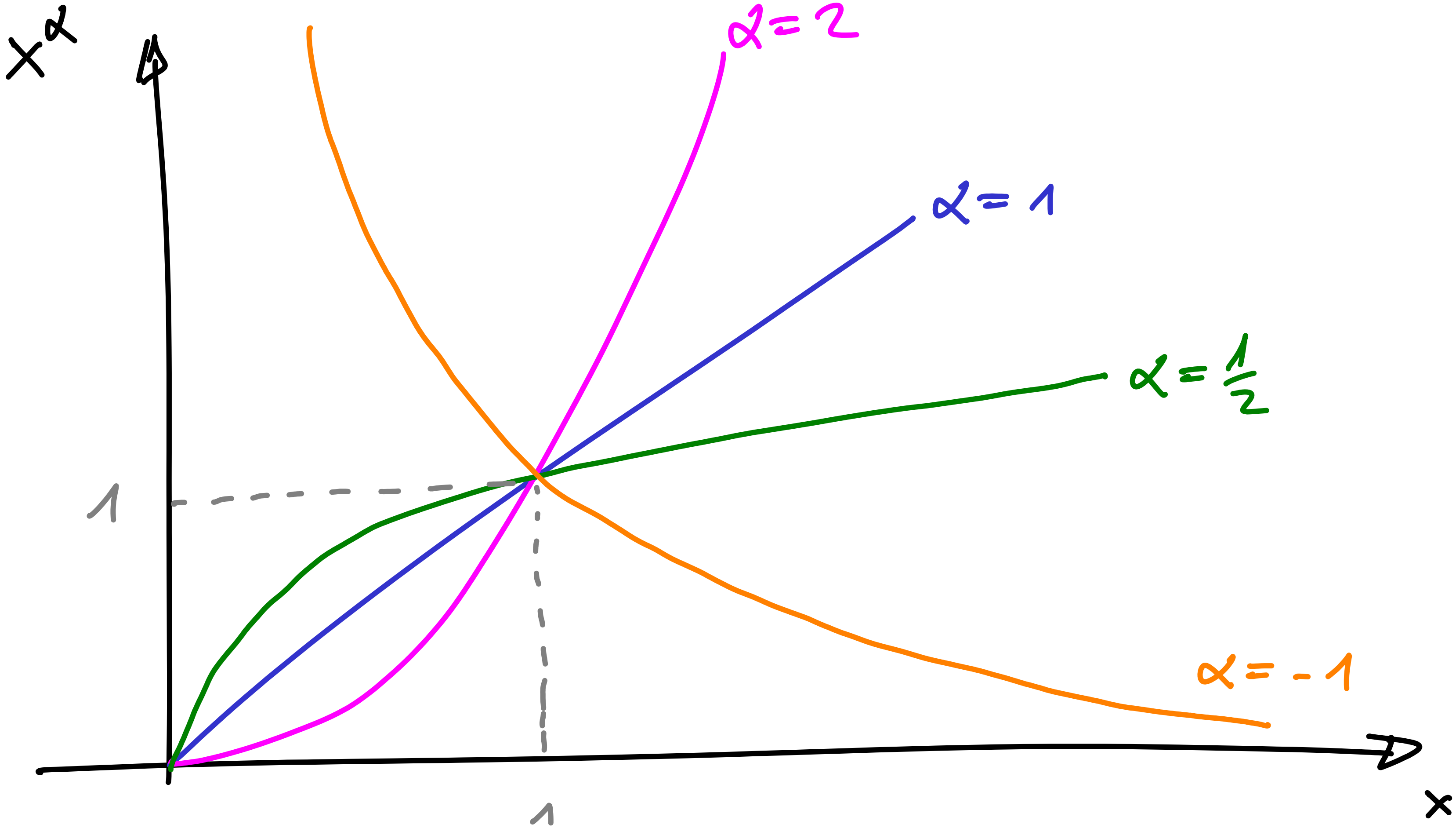
$$x^{\alpha\cdot\beta} = (x^{\alpha})^{\beta} \neq x^{(\alpha^{\beta})} = x^{\alpha^{\beta}}$$

$$\sqrt[3]{9^{-2}\cdot 3} = \left(\frac{1}{9^{2}}\cdot 3\right)^{1/3} = \left(\frac{1}{(3^{2})^{2}}\cdot 3\right)^{1/3}$$

$$= \left(\frac{1}{3^{4}}\cdot 3\right)^{1/3} = \left(\frac{1}{3^{3}}\right)^{1/3} = \frac{1}{3}$$



x^α
$\alpha = 2$
$\alpha = 1$
$\alpha = \frac{1}{2}$
$\alpha = -1$
1
1
x

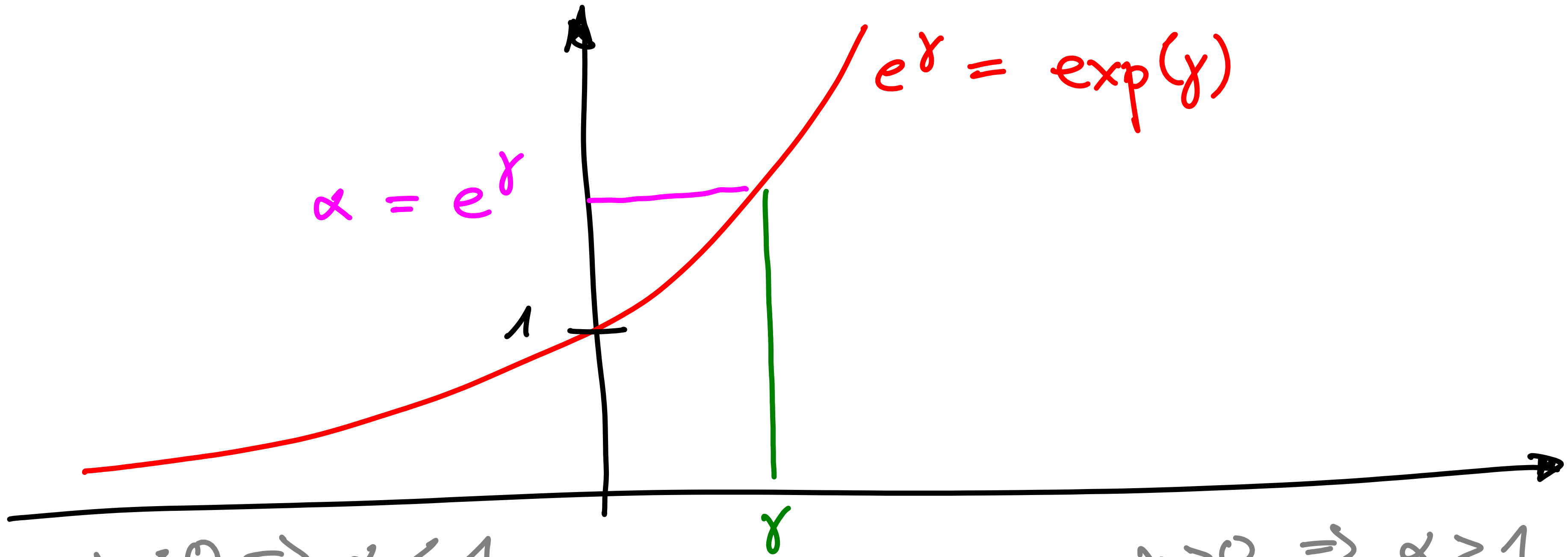
$\alpha = 1{,}06$ (6% Zinsen)

$G(0) = 100€$

Schuld nach halben Jahr

$G(\frac{1}{2}) = (1{,}06)^{1/2} \cdot 100€ \approx 102{,}96€ < 103€$

$\alpha^t = e^{\gamma t} = (e^\gamma)^t$ also $\alpha = e^\gamma$



$\gamma < 0 \Rightarrow \alpha < 1$ $\qquad$ $\gamma > 0 \Rightarrow \alpha > 1$

$$\alpha^{t/T} \underset{\alpha = e^{\gamma}}{=} \left(e^{\gamma}\right)^{t/T} = e^{\frac{\gamma t}{T}} \underset{\frac{\gamma}{T} = \lambda}{=} e^{\lambda t}$$

$$G(t) = e^{\lambda t}\, G(0)$$

$$G\left(\frac{1}{\lambda}\right) = e^{\lambda \cdot \frac{1}{\lambda}}\, G(0) = e \cdot G(0) \quad \lambda > 0$$

$$G\left(-\frac{1}{\lambda}\right) = e^{\lambda\left(-\frac{1}{\lambda}\right)}\, G(0) = e^{-1}\, G(0) = \frac{1}{e}\, G(0) \quad \lambda < 0$$

positiv

Radioaktiver Zerfall

$G(t)$ Menge zu Beginn des Intervalls $[t, t+T]$

$G(t+T)$ " am Ende ——— " ———

$[G]$ = Anzahl Atome

Zerfälle in $[t, t+T]$

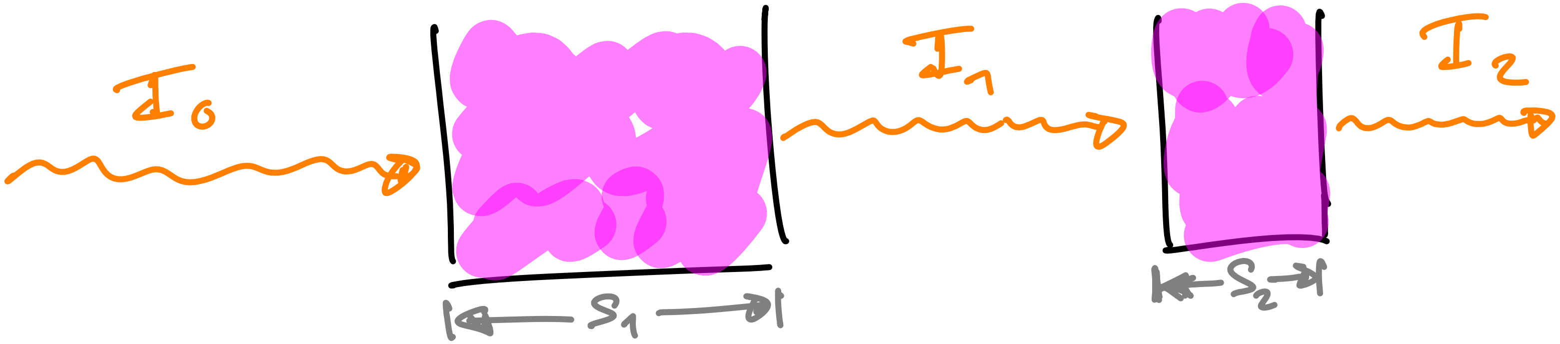
$$Z(t) = G(t) - G(t+T) = e^{-\lambda t} G(0) - e^{-\lambda(t+T)} G(0)$$

$$= e^{-\lambda t} \underbrace{\left[1 - e^{-\lambda T}\right] G(0)}_{\substack{\text{Anzahl der Zerfälle } [0,T] \\ = Z(0)}}$$

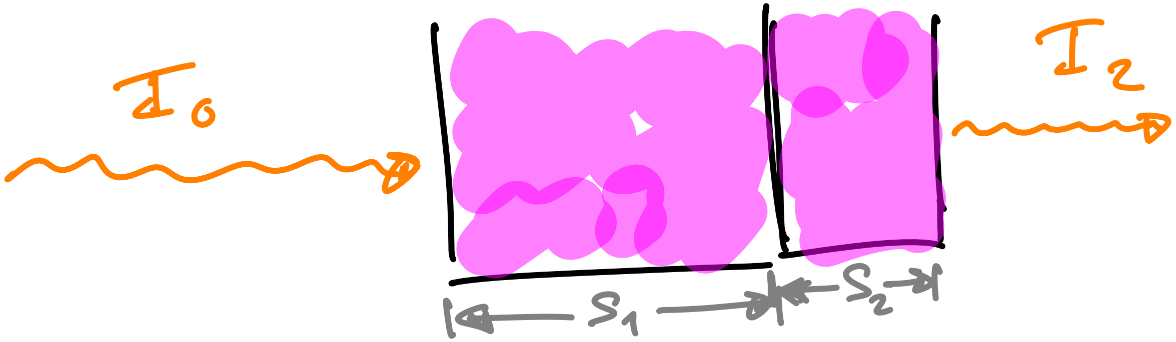
Zerfallene Menge bezogen auf Anfangsmenge

$$\frac{G(t) - G(t+T)}{G(t)} = \frac{e^{-\lambda t} - e^{-\lambda(t+T)}}{e^{-\lambda t}} = \underbrace{1 - e^{-\lambda T}}_{\text{hängt nicht von } t \text{ ab!}}$$

zu Lambert-Beer

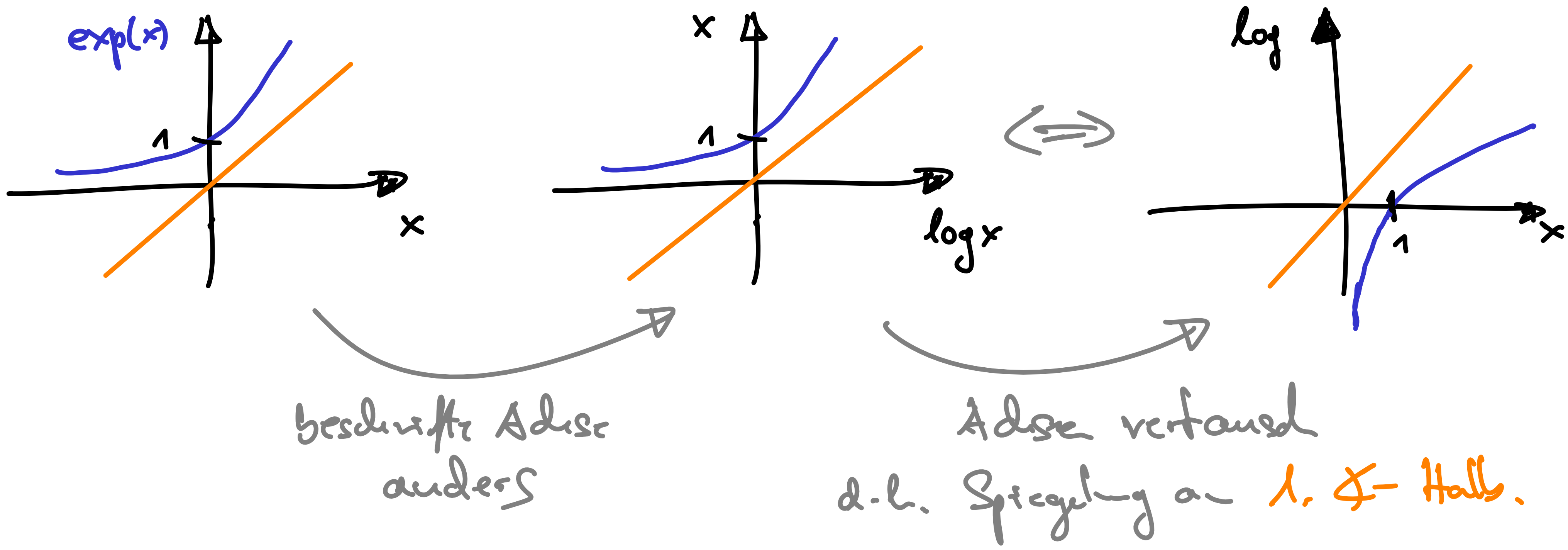


$$I_1 = \alpha_{s_1} \cdot I_0 \,, \quad I_2 = \alpha_{s_2} \cdot I_1 = \alpha_{s_1} \cdot \alpha_{s_2} \cdot I_0$$



$$I_2 = \alpha_{s_1+s_2} \cdot I_0$$

$$\alpha_{s_1+s_2} = \alpha_{s_1} \cdot \alpha_{s_2}$$ also exponentieller Zerfall



log-Rechenregeln

① $\log(xy) = \log x + \log y$

$$x = e^a,\ y = e^b \iff \log x = a,\ \log y = b$$

$$\log(xy) = \log(e^a \cdot e^b) \overset{\text{P.R.}}{=} \log(e^{a+b}) = a + b = \log x + \log y$$

② $\log(x^\alpha) = \alpha \cdot \log x$

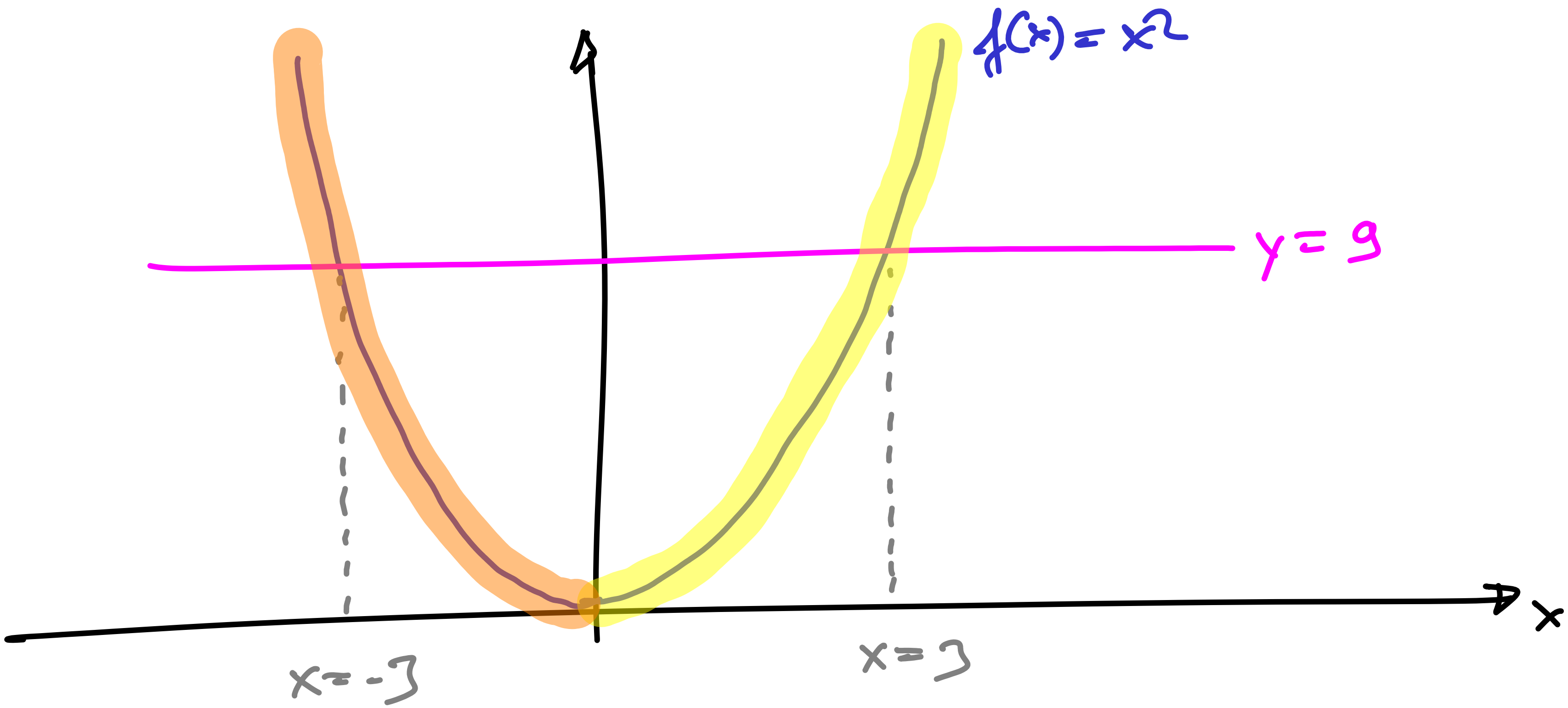
$x = e^\gamma \iff \gamma = \log x$

$\log(x^\alpha) = \log((e^\gamma)^\alpha) \overset{\text{P.R.}}{=} \log(e^{\gamma\cdot\alpha})$

$= \gamma \cdot \alpha = \alpha \cdot \log x$

③ $\log\left(\frac{1}{x}\right) = -\log x$ (② mit $\alpha = -1$)

④ $\log(1) = \log(e^0) = 0$



$f: \mathbb{R} \to \mathbb{R}_0^+ = [0, \infty)$ ist nicht injektiv

$x \mapsto x^2$ also nicht umkehrbar

$\tilde{f}: \mathbb{R}_0^+ \longmapsto \mathbb{R}_0^+$ ist injektiv & umkehrbar

$x \longmapsto x^2$ mit $\tilde{f}^{-1}: y \longmapsto \sqrt{y}$

$\tilde{\tilde{f}}: \mathbb{R}_0^- \longmapsto \mathbb{R}_0^+$ inj. & umkehrbar $\tilde{\tilde{f}}^{-1}: y \longmapsto -\sqrt{y}$

$x \longmapsto x^2$