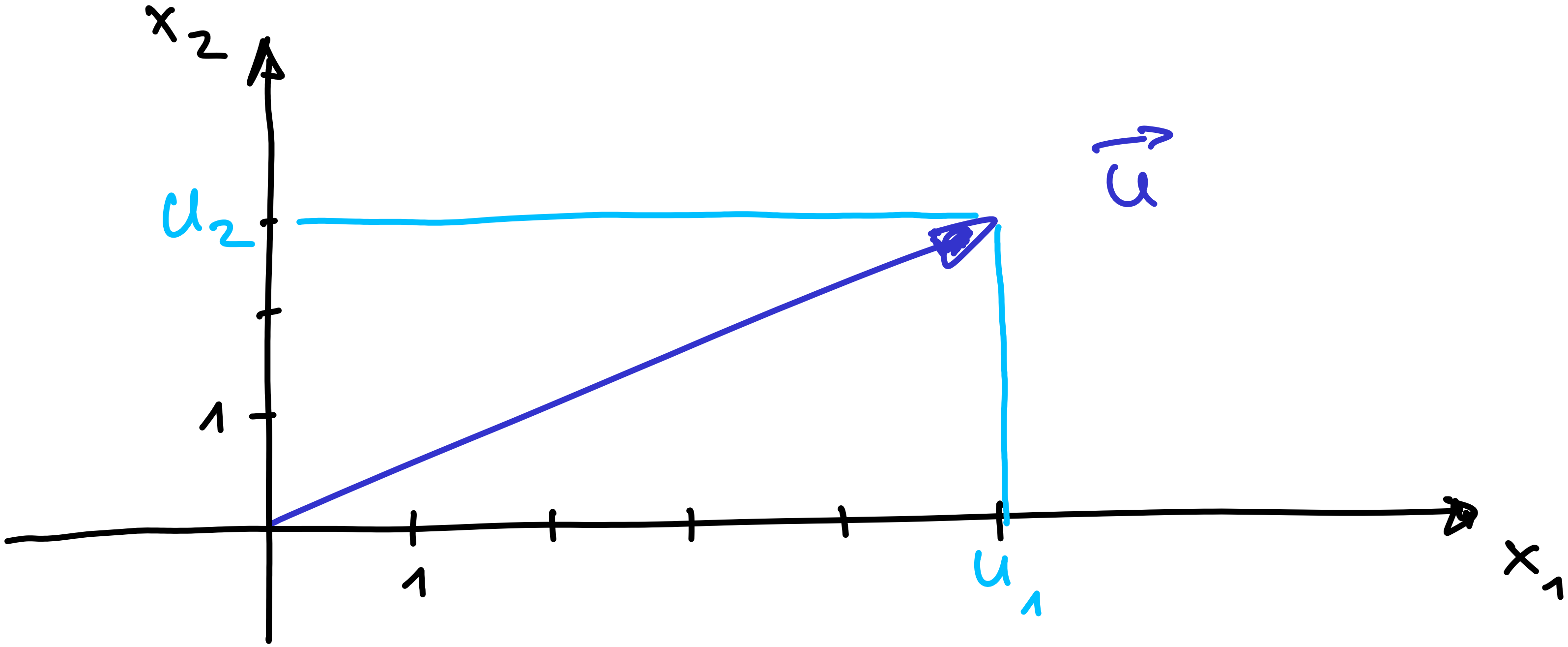
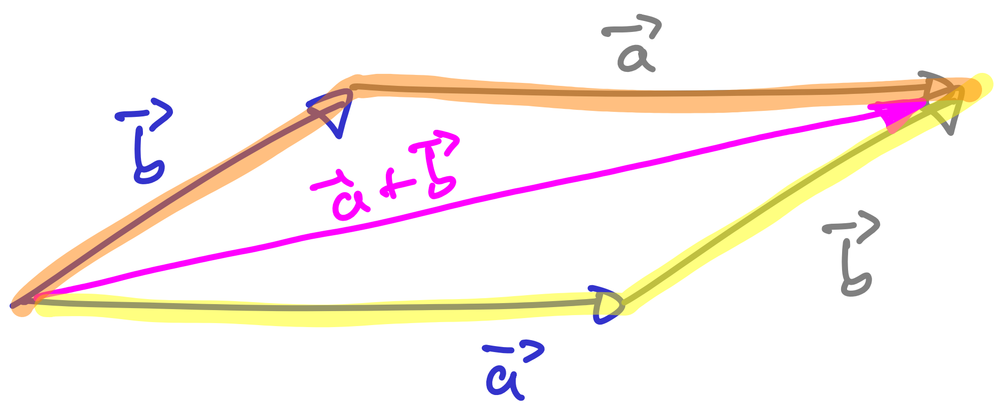


$$\vec{u} = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}, \quad |\vec{u}| = \sqrt{u_1^2 + u_2^2}$$

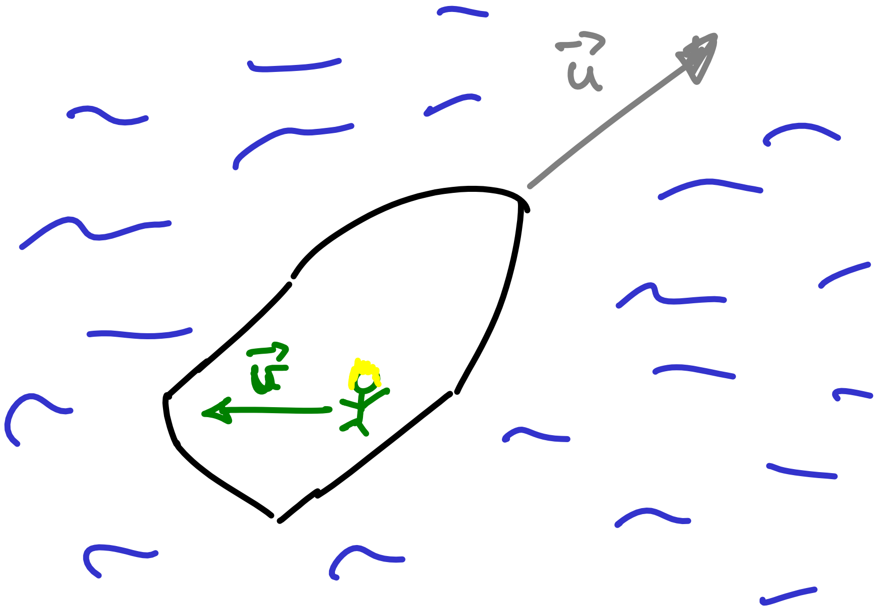
Betrag: Länge des Pfeils

z.B. $\left|\begin{pmatrix} 5 \\ 3 \end{pmatrix}\right| = \sqrt{25+9} = \sqrt{34}$



$$\vec{a}+\vec{b} = \vec{b}+\vec{a}$$

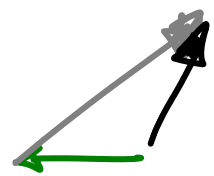
$$\vec{a} = \begin{pmatrix} 4 \\ 0 \end{pmatrix}, \quad \vec{b} = \begin{pmatrix} 3 \\ 2 \end{pmatrix} : \quad \vec{a}+\vec{b} = \begin{pmatrix} 4+3 \\ 0+2 \end{pmatrix} = \begin{pmatrix} 7 \\ 2 \end{pmatrix}$$

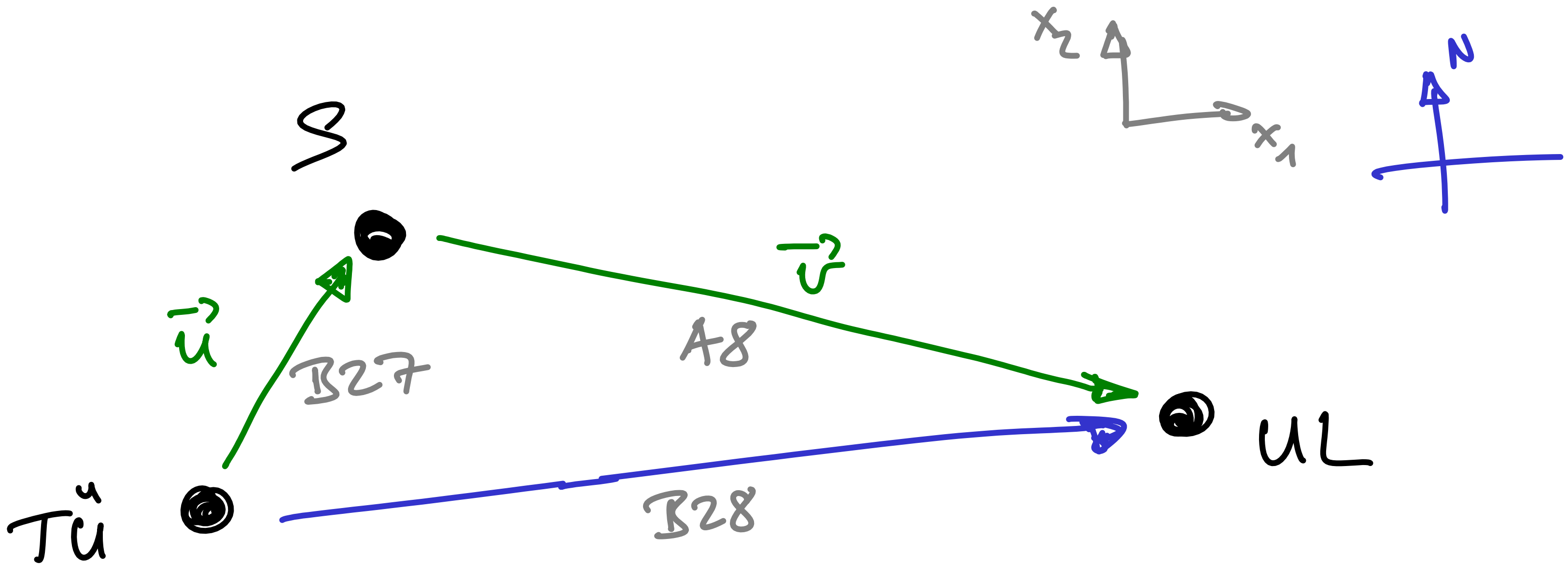


$\vec{v}$: Geschw. der Pers. auf Schiff

$\vec{u}$: Geschw. des Schiffs

$\vec{u}+\vec{v}$: Geschw. der Pers. gegenüber dem Wasser

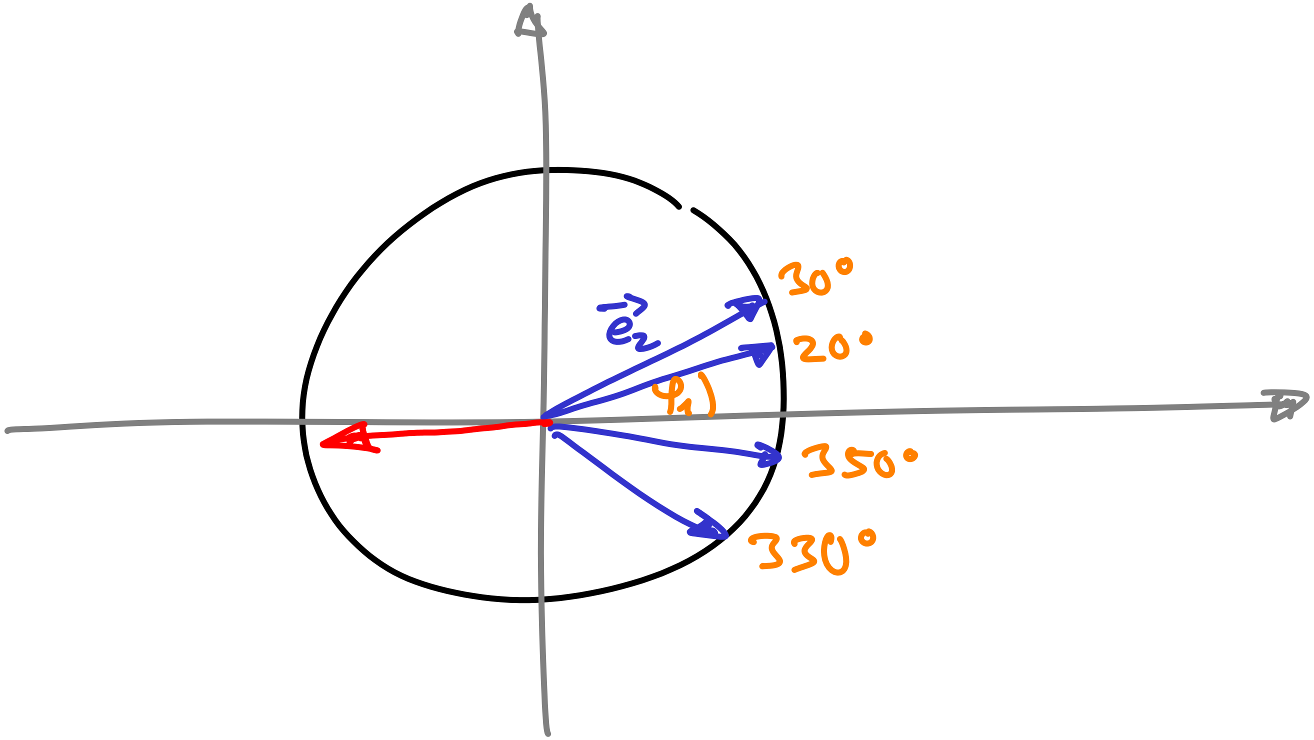




$$\vec{u} = \frac{50\,\text{km}}{\sqrt{5}} \begin{pmatrix} 1 \\ 2 \end{pmatrix} \qquad \sqrt{5} = \sqrt{2^2 + 1^2}$$

$$\vec{v} = \frac{100\,\text{km}}{\sqrt{10}} \begin{pmatrix} 3 \\ -1 \end{pmatrix}$$

$$|\vec{u} + \vec{v}| = \left| \frac{50 \cdot \sqrt{2}}{\sqrt{10}} \begin{pmatrix} 1 \\ 2 \end{pmatrix} + \frac{100}{\sqrt{10}} \begin{pmatrix} 3 \\ -1 \end{pmatrix} \right| \text{km}$$

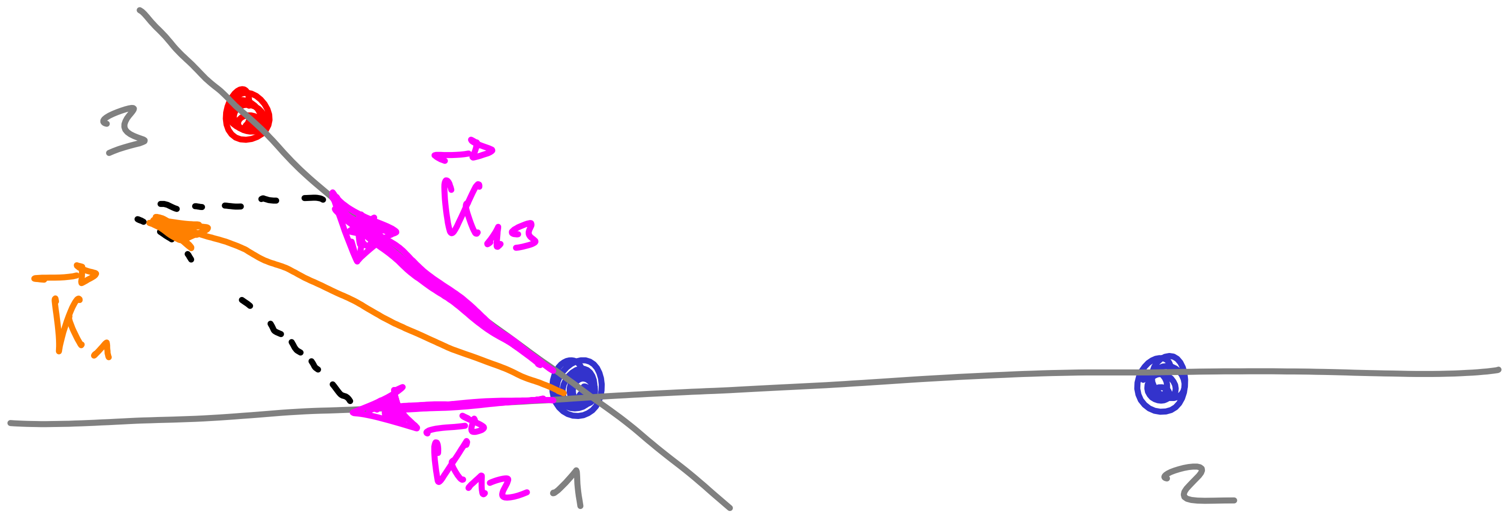


$$\overline{\varphi} = \frac{20° + 30° + 350° + 330°}{4} = 182{,}5$$

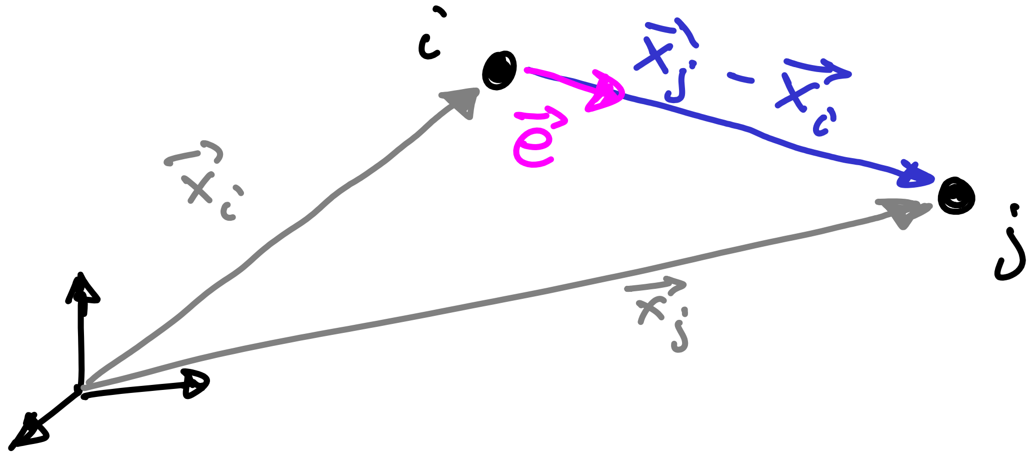
$$\overline{\vec{e}} = \frac{\vec{e}_1 + \vec{e}_2 + \vec{e}_3 + \vec{e}_4}{4}$$

← zeigt nach rechts

besser ☺



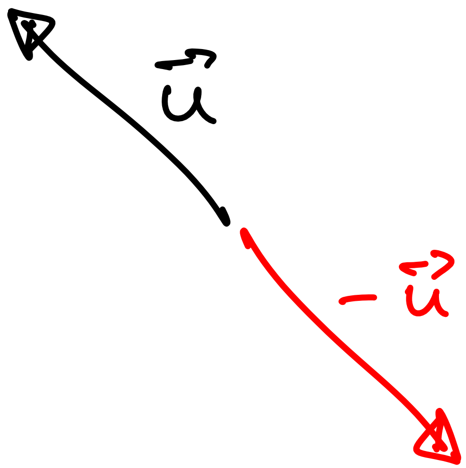
$$\vec{K}_i = \sum_{\substack{j=1 \\ j \neq i}}^{N} \vec{K}_{ij} = \vec{K}_{12} + \vec{K}_{13}$$



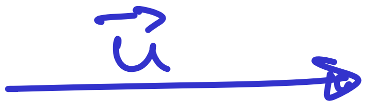
$$|\vec{e}| = 1$$

$$\vec{K}_{ij} = \frac{q_i q_j}{|\vec{x}_j - \vec{x}_i|^2} \vec{e} = \frac{q_i q_j}{|\vec{x}_j - \vec{x}_i|^3} (\vec{x}_j - \vec{x}_i)$$

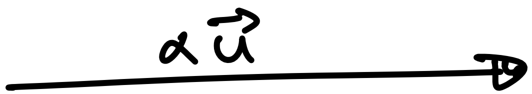
$$\vec{e} = \frac{\vec{x}_j - \vec{x}_i}{|\vec{x}_j - \vec{x}_i|}$$



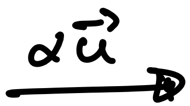
$$\vec{u} = \begin{pmatrix} -2 \\ 1 \end{pmatrix} \Rightarrow -\vec{u} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$$



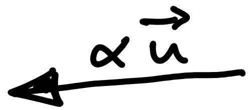
$\alpha > 1$:



$0 < \alpha < 1$:



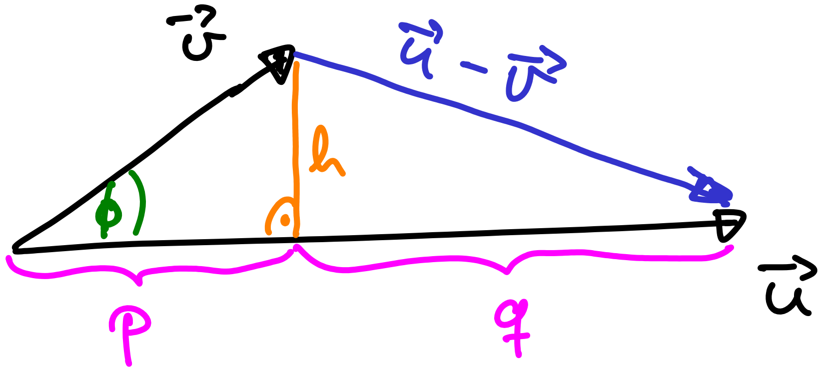
$-1 < \alpha < 0$:



alle parallel zu $\vec{u}$
(bzw. antiparallel)

$$\vec{u} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}, \quad \vec{v} = \begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix}$$

$$\vec{u} \cdot \vec{v} = 1 \cdot 4 + 0 \cdot 5 + (-1) \cdot 6 = -2$$



$\cos\phi = \dfrac{p}{|\vec{v}|}$

Pythagoras: $\quad p^2 + h^2 = |\vec{v}|^2 \quad , \quad q^2 + h^2 = |\vec{u}-\vec{v}|^2$

$\Rightarrow \quad |\vec{v}|^2 - p^2 = |\vec{u}-\vec{v}|^2 - q^2 \quad , \quad q = |\vec{u}| - p$

$q^2 = |\vec{u}|^2 + p^2 - 2|\vec{u}|p$

$$\Leftrightarrow \quad |\vec{v}|^2 - p^2 = \underbrace{|\vec{u}-\vec{v}|^2}_{=(\vec{u}-\vec{v})\cdot(\vec{u}-\vec{v})} - |\vec{u}|^2 - p^2 + 2|\vec{u}|p$$

$$\Leftrightarrow \quad |\vec{v}|^2 = \underline{|\vec{u}|^2 + |\vec{v}|^2 - 2\,\vec{u}\cdot\vec{v}} - |\vec{u}|^2 + 2|\vec{u}|p$$

$$\Leftrightarrow \quad \vec{u}\cdot\vec{v} = |\vec{u}|\cdot p = |\vec{u}|\cdot|\vec{v}|\cos\phi$$

Ausblick: Matrizen

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}, \quad \begin{pmatrix} 0 & 1 \\ 1 & -1 \\ 1 & 0 \end{pmatrix} = B$$

$$\cancel{A \cdot B} = \cancel{?}$$

$$\begin{array}{c|c} & \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \\ \hline \begin{pmatrix} 0 & 1 \\ 1 & -1 \\ 1 & 0 \end{pmatrix} & \begin{pmatrix} 3 & 4 \\ -2 & -2 \\ 1 & 2 \end{pmatrix} \end{array} = B \cdot A$$

$$-2 = 1 \cdot 1 + (-1) \cdot 3$$