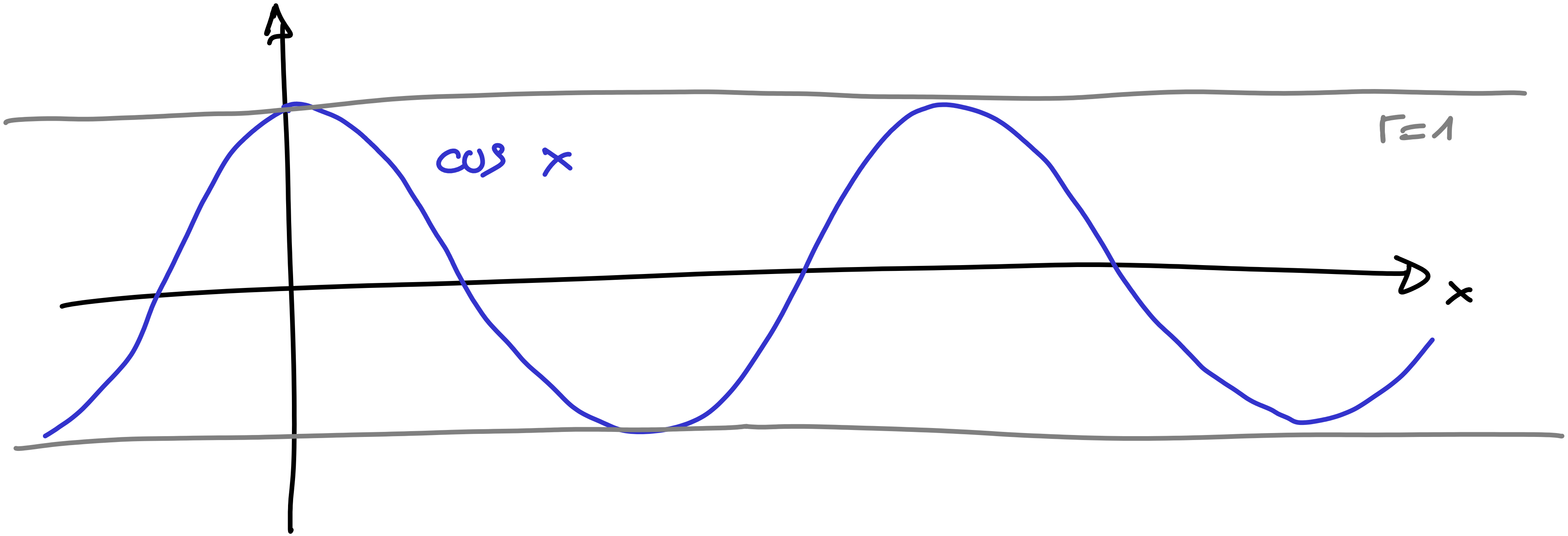
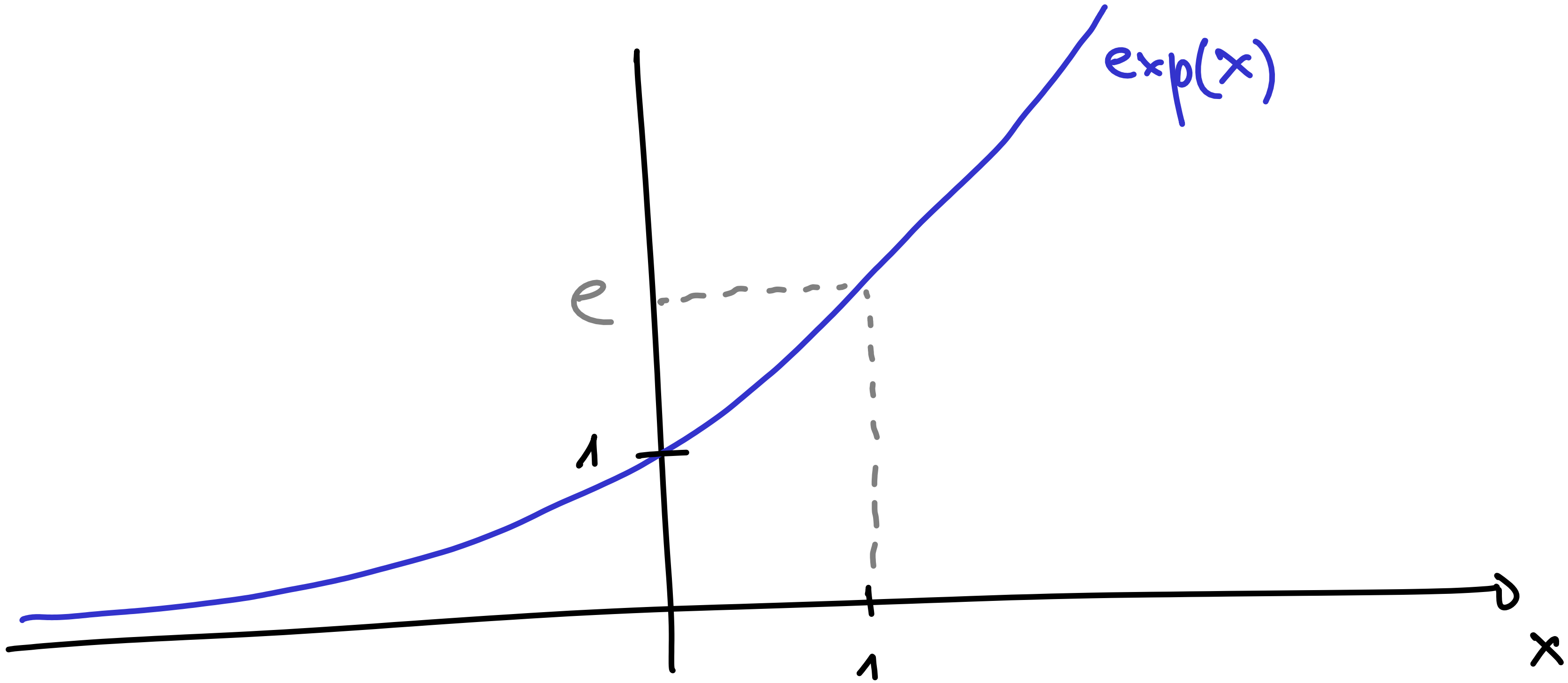


anderes Bsp:

$$f(x) = 3 + \cos x$$

$$\Rightarrow \quad 2 \leq f(x) \leq 4$$

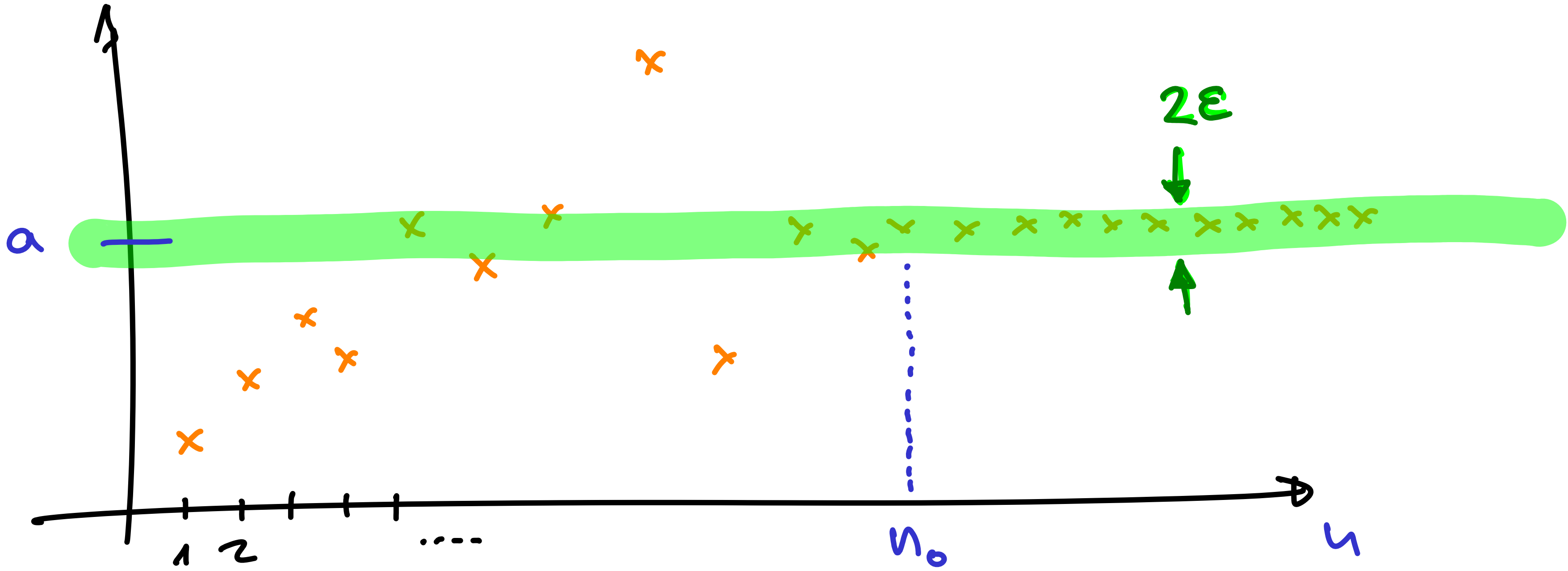
beschränkt mit $r = 4$ (z.B.), denn $|f(x)| \leq 4$



$$\exp: (-\infty, 1] \longrightarrow (0, e]$$

beschränkt mit $r = e$, denn

$$|\exp(x)| \leq e \quad \forall\, x \in (-\infty, 1]$$



2ε
a
1 2 ....
n₀
n

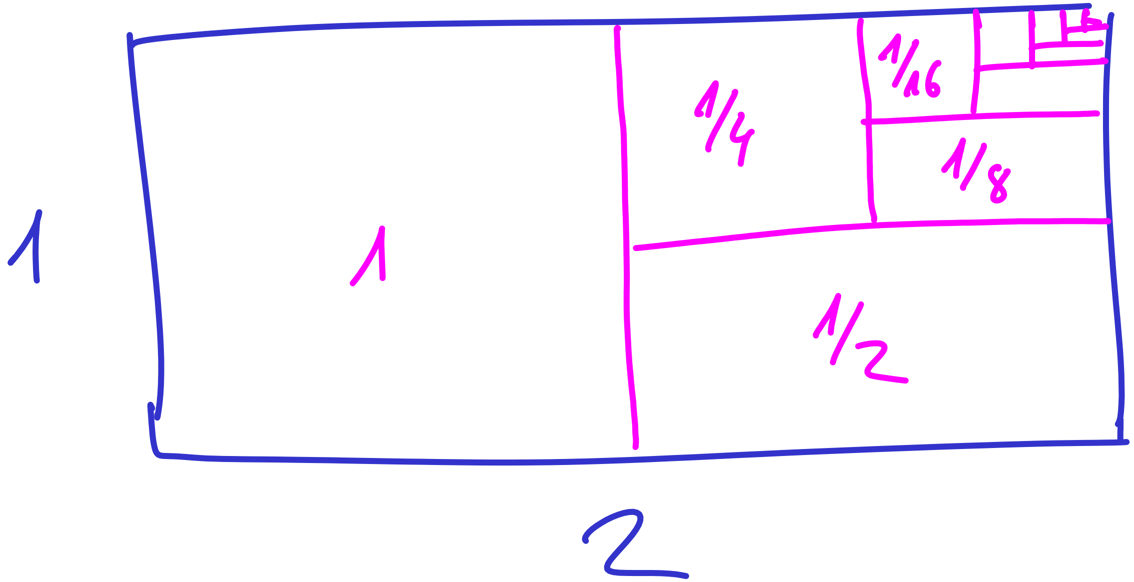
$a_n = \frac{1}{n} \quad , \quad n \in \mathbb{N}$

Behauptung: $\lim_{n \to \infty} a_n = 0$

$$|a_n - 0| = \left|\frac{1}{n} - 0\right| = \left|\frac{1}{n}\right| = \underbrace{\frac{1}{n} \overset{!}{<} \varepsilon}_{\text{hätten wir gerne}}$$

$$\Leftrightarrow \quad n > \frac{1}{\varepsilon}$$

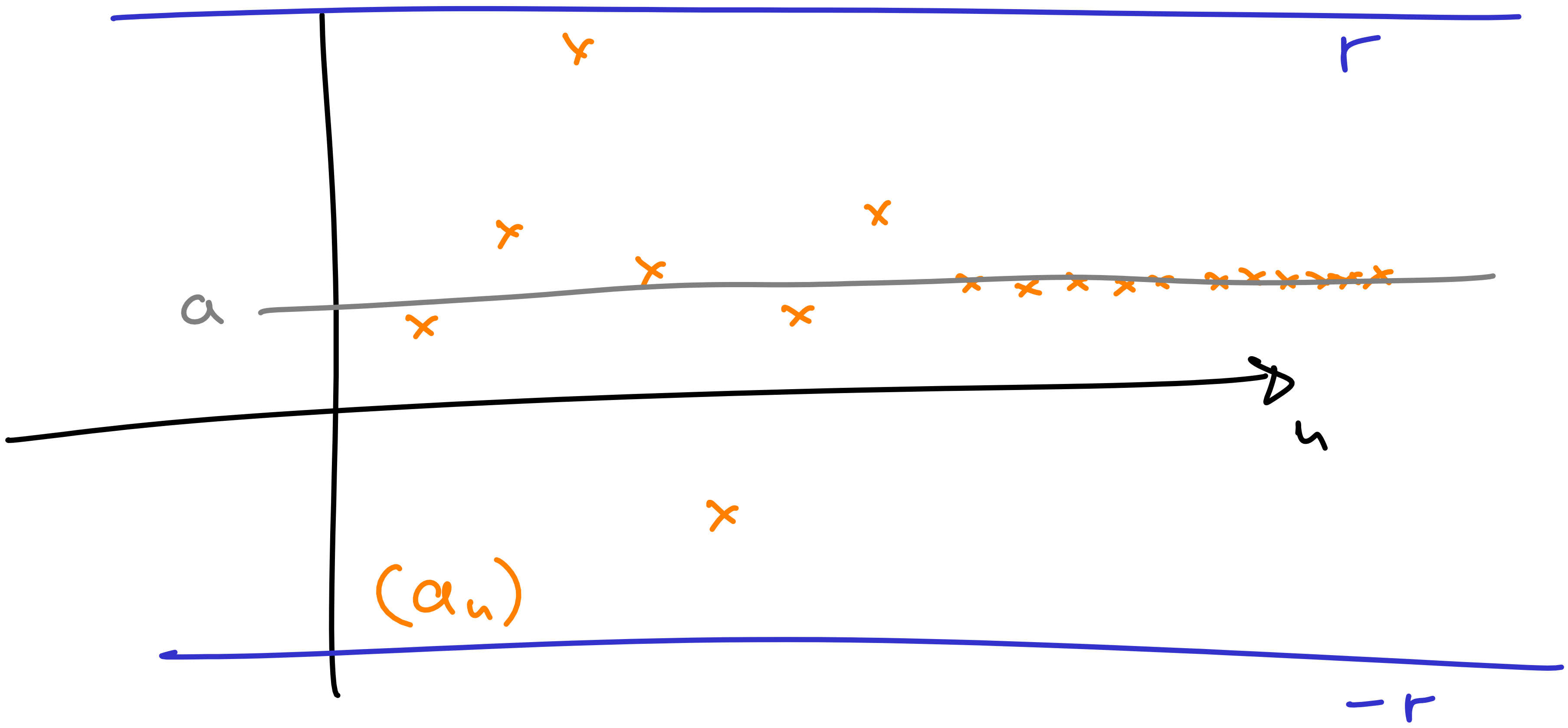
Wähle $n_0 \geq \frac{1}{\varepsilon}$, damit klappt's ☺



Fläche: $2 = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \dots$

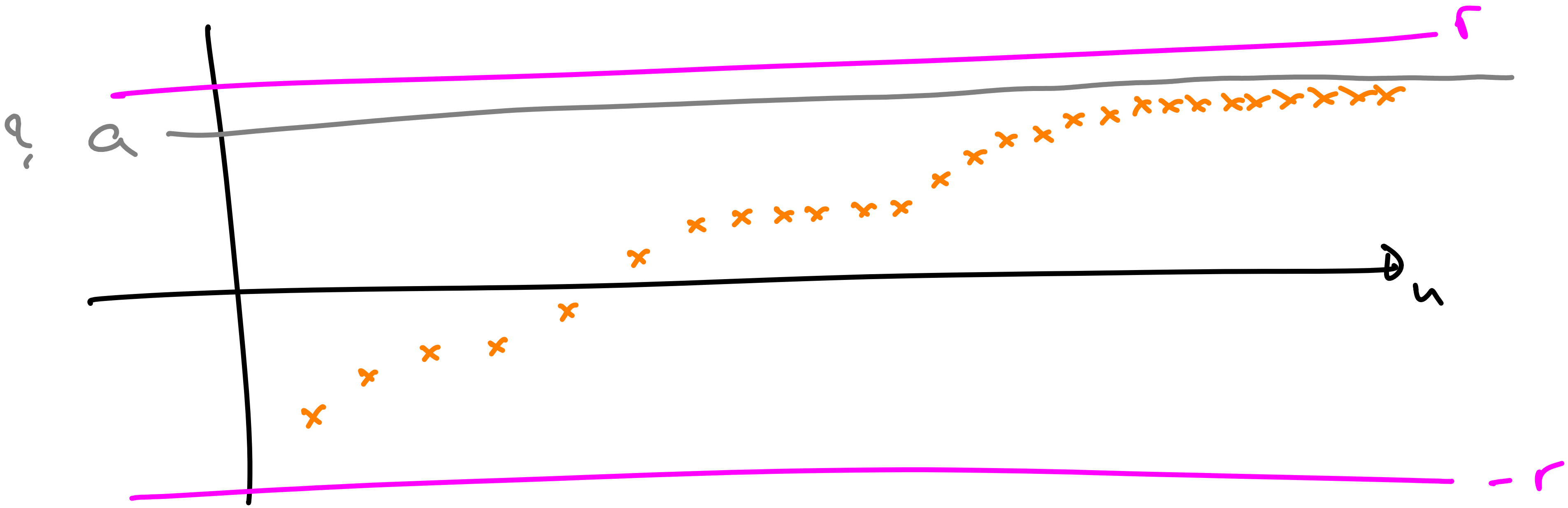
$$\vec{a}_n = \begin{pmatrix} 1/n \\ \sum_{k=0}^{n} \left(\frac{1}{2}\right)^k \end{pmatrix} \xrightarrow[n \to \infty]{} \begin{pmatrix} 0 \\ 2 \end{pmatrix}$$

$$\vec{b}_n = \begin{pmatrix} 1/n \\ (-1)^n \end{pmatrix}$$ konvergiert nicht

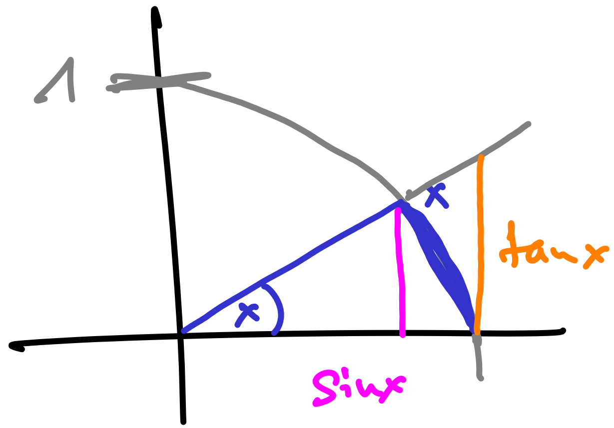


r
a
n
(a_n)
-r

Folge a_n sei monoton & beschränkt



$$\frac{\sin\left(\frac{1}{n}\right)}{\frac{1}{n}} = n \sin\left(\frac{1}{n}\right) \xrightarrow[n\to\infty]{} \ ?$$



for $0 < x < \frac{\pi}{2}$

$$\sin x < x < \tan x$$

$$\Leftrightarrow \quad \frac{1}{\sin x} > \frac{1}{x} > \frac{1}{\tan x} = \frac{\cos x}{\sin x} \quad \Big| \cdot \sin x$$

$$\Leftrightarrow \quad 1 > \frac{\sin x}{x} > \cos x$$

d.h.

$$1 > \frac{\sin\left(\frac{1}{n}\right)}{\frac{1}{n}} > \cos\left(\frac{1}{n}\right) \xrightarrow[n\to\infty]{} 1$$

$$\Rightarrow \quad \lim_{n \to \infty} \frac{\sin\left(\frac{1}{n}\right)}{\frac{1}{n}} = 1$$

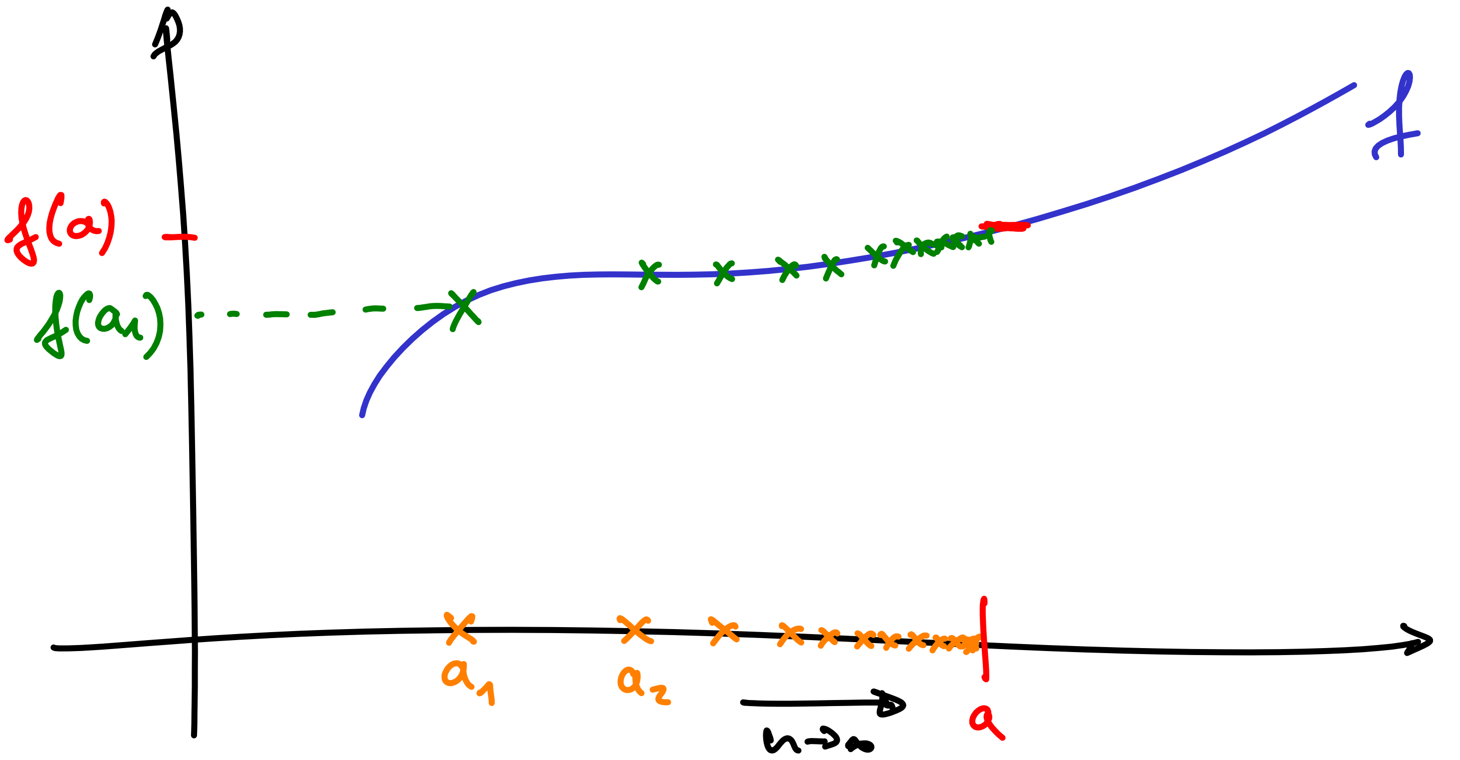
$$\lim_{n\to\infty} \frac{n^{27} - 3n^2}{n^5 - 2n^{27}} = \lim_{n\to\infty} \frac{1 - \frac{3}{n^{25}}}{\frac{1}{n^{22}} - 2}$$

Produkt

$$= \frac{\lim_{n\to\infty}\left(1 - \frac{3}{n^{25}}\right)}{\lim_{n\to\infty}\left(\frac{1}{n^{22}} - 2\right)} = \frac{\lim_{n\to\infty} 1 - \lim_{n\to\infty} \frac{3}{n^{25}}}{\lim_{n\to\infty} \frac{1}{n^{22}} - \lim_{n\to\infty} 2}$$

Summe

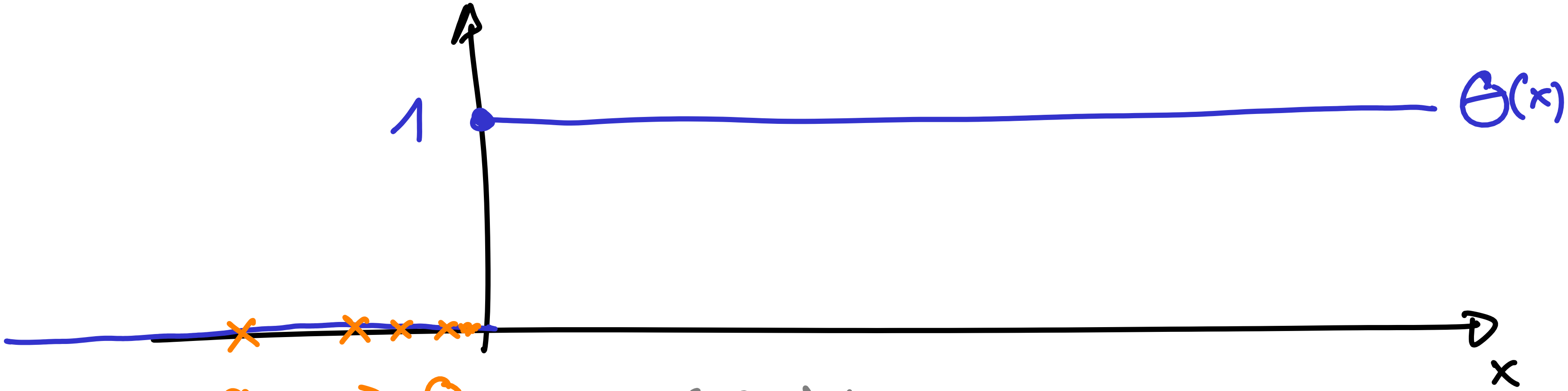
$$= \frac{1 - 0}{0 - 2} = -\frac{1}{2}$$



$$\lim_{n\to\infty} f(a_n) = f\left(\lim_{n\to\infty} a_n\right) = f(a)$$

Stetigkeit von f

$$\Theta(x) = \begin{cases} 0, & x < 0 \\ 1, & x \geq 0 \end{cases}$$



$a_n \to 0$ $\qquad$ $a_n < 0 \ \forall n$

$$a_n = -\frac{1}{n}, \quad n \in \mathbb{N}$$

$$\lim_{n\to\infty} \Theta(a_n) = \lim_{n\to\infty} 0 = 0$$

$$\neq \Theta\left(\lim_{n\to\infty} a_n\right) = \Theta(0) = 1$$

übrigens: $\lim\limits_{x\to 0} \Theta(x)$ existiert nicht

$a_n = \frac{1}{n}$ $\qquad$ $b_n = -\frac{1}{n}$

$$\lim_{n\to\infty} \Theta(a_n) = 1$$

$$\lim_{n\to\infty} \Theta(b_n) = 0$$

$$\sin(n\omega(t+T)) = \sin\left(n\frac{2\pi}{T}(t+T)\right)$$

$$= \sin\left(n\frac{2\pi}{T}t + n2\pi\right) = \sin(n\omega t)$$

$\frac{1}{2}$, $\frac{1}{2}\cdot\frac{1}{2} = \frac{1}{4}$, $\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2} = \frac{1}{8}$ ... $\longrightarrow 0$

$-\frac{1}{3}$, $\left(-\frac{1}{3}\right)^2 = \frac{1}{9}$, $\left(-\frac{1}{3}\right)^3 = -\frac{1}{27}$ $\longrightarrow 0$

geom. Summe

$$S_n = \sum_{k=0}^{n} q^k = 1 + q + q^2 + \ldots + q^n$$

$$q \cdot S_n = q + q^2 + \ldots + q^n + q^{n+1}$$

Differenz

$$S_n - q \cdot S_n = 1 - q^{n+1} \quad \Big| \cdot \frac{1}{1-q}, \quad q \neq 1$$

$$\Rightarrow \boxed{\sum_{k=0}^{n} q^k = \frac{1-q^{n+1}}{1-q} \quad \forall\, q \neq 1}$$

also für $|q| < 1$:

$$\lim_{n\to\infty} \sum_{k=0}^{n} q^k = \underbrace{\sum_{k=0}^{\infty} q^k}_{\text{geom. Reihe}} = \frac{1 - \overbrace{\lim q^{n+1}}^{=0,\ \text{da } |q|<1}}{1-q} = \frac{1}{1-q}$$