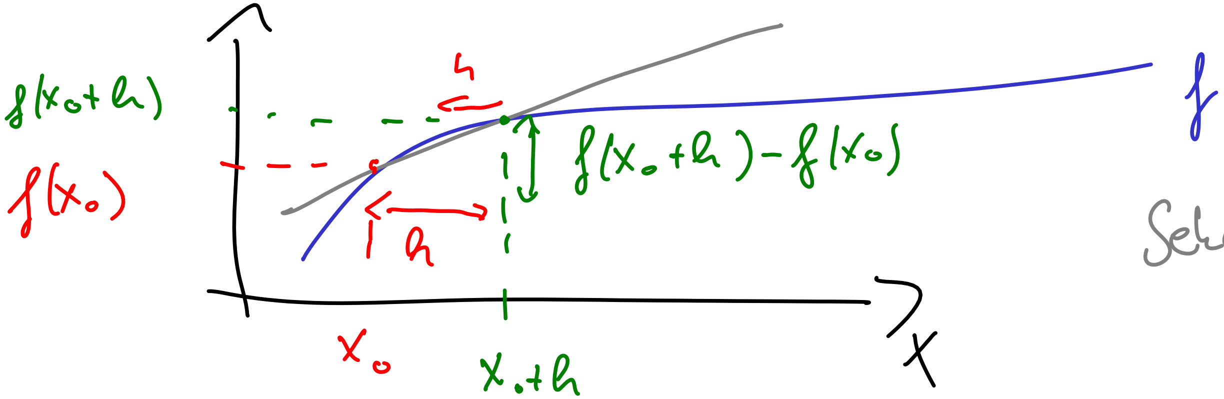


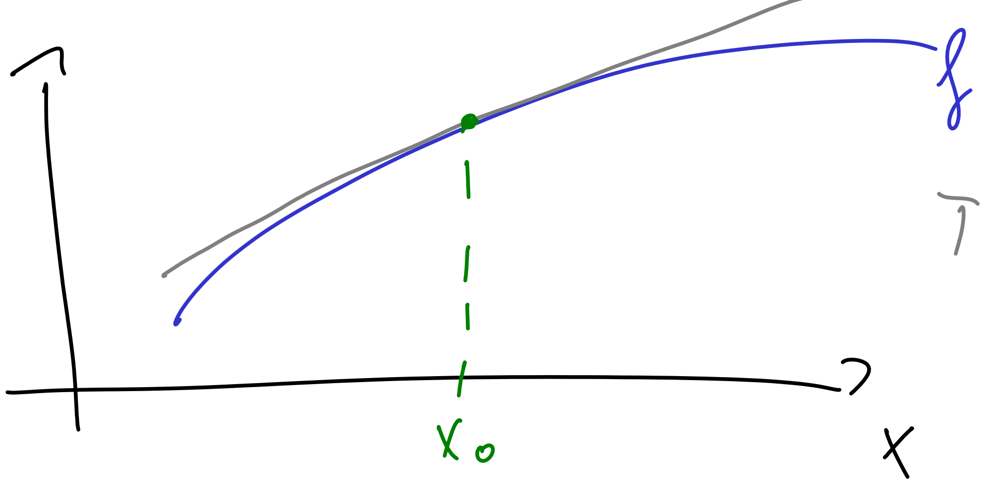
Sekante $\xrightarrow{h\to 0}$ Tangente

Notation:

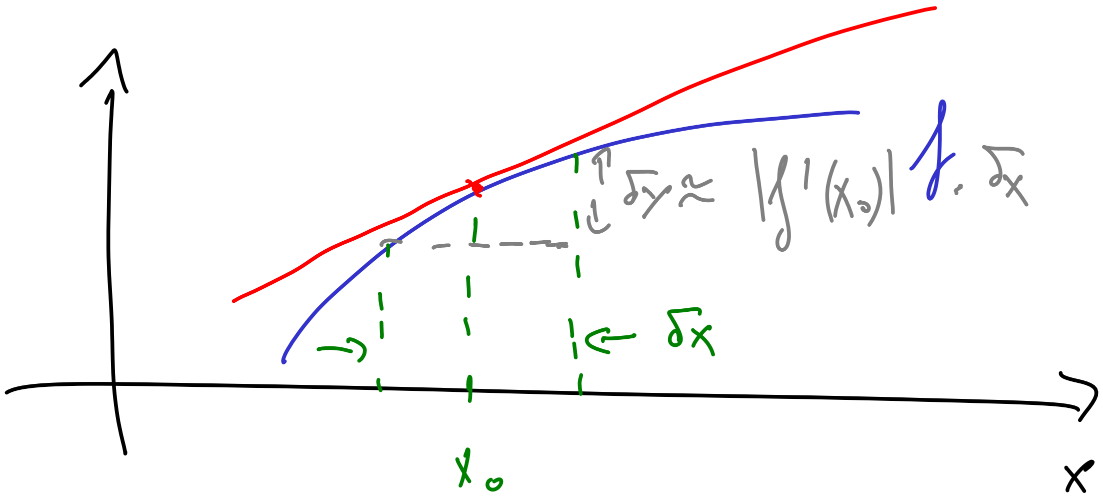
$$f' = \frac{df}{dx} = \left(\frac{d}{dx}\right) f$$

$$f'' = f^{(2)} = \frac{d^2 f}{dx^2} = \left(\frac{d}{dx}\right)^2 f$$

$$f'''''(x) = f^{(5)} \quad - \quad \ldots$$

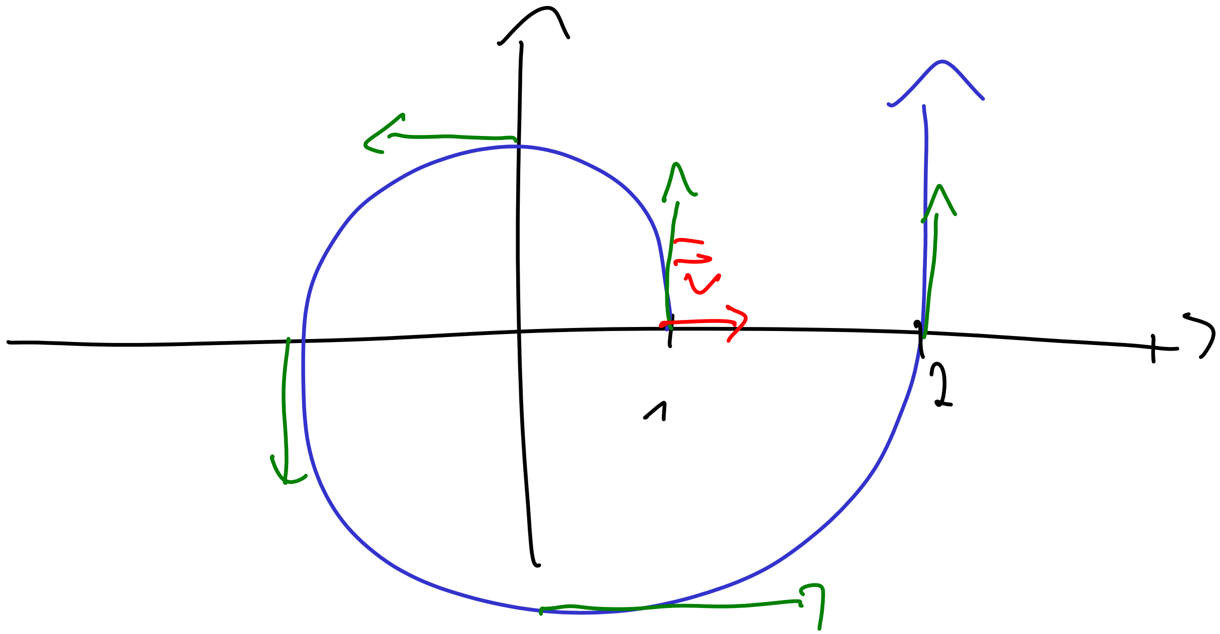


$$T(x) = f(x_0) + f'(x_0)(x - x_0)$$



$$x \in \left[x - \frac{\delta x}{2},\ x + \frac{\delta x}{2}\right]$$

Kurve $\vec{x}: [0, 2\pi] \to \mathbb{R}^2$



in Formeln:

zunächst

$$\vec{x}(t) = \begin{pmatrix} r\cos t \\ r \sin t \end{pmatrix}, \quad 0 \le t \le 2\pi$$

Radius

Spirale:

$$\vec{x}(t) = \begin{pmatrix} \left(1 + \frac{t}{2\pi}\right) \cos t \\ \left(1 + \frac{t}{2\pi}\right) \sin t \end{pmatrix}, \quad 0 \le t \le 2\pi$$

$$\dot{\vec{x}} = \begin{pmatrix} \frac{1}{2\pi}\cos t + \left(1+\frac{t}{2\pi}\right)(-\sin t) \\ \frac{1}{2\pi}\sin t + \left(1+\frac{t}{2\pi}\right)\cos t \end{pmatrix}$$

$$\vec{\bar{v}} = \frac{\vec{x}(2\pi) - \vec{x}(0)}{2\pi} = \frac{\begin{pmatrix} 2 \\ 0 \end{pmatrix} - \begin{pmatrix} 1 \\ 0 \end{pmatrix}}{2\pi} = \frac{1}{2\pi}\begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

Bsp. für Ableitungen

$$f(x) = \frac{1}{x}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{1}{h}\left(\frac{1}{x+h} - \frac{1}{x}\right)$$

$$= \lim_{h\to 0} \frac{1}{h} \frac{x-(x+h)}{x(x+h)} = \lim_{h\to 0} \frac{-\cancel{h}}{\cancel{h}\,x(x+h)} = -\frac{1}{x^2}$$

$$f(x) = \frac{1}{x} = x^{-1} \; ; \; f'(x) = -x^{-2} = -\frac{1}{x^2}$$

$$f(x) = a^x$$

$$f'(x) = \lim_{h\to 0} \frac{a^{x+h} - a^x}{h} = \lim_{h\to 0} \frac{a^x a^h - a^x}{h}$$

$$= a^x \underbrace{\lim_{h\to 0} \frac{a^h - 1}{h}}_{\text{hängt nicht von } x \text{ ab !!}}$$

Für $a = e = 2{,}718\ldots$ gilt $\lim\limits_{h\to 0} \frac{e^h - 1}{h} = 1$

$$f(x) = \sin x$$

$$f'(x) = \lim_{h\to 0} \frac{\sin(x+h) - \sin(x)}{h}$$ Add.-Th.

$$= \lim_{h\to 0} \frac{\sin x \cos h + \cos x \sin h - \sin x}{h}$$

$$= \lim_{h\to 0}\left(\sin x \, \frac{\cos h - 1}{h}\right) + \lim_{h\to 0}\left(\cos x \, \frac{\sin h}{h}\right)$$

$$= \sin x \lim_{h\to 0}\left(\frac{\cos h - 1}{h}\right) + \cos x \underbrace{\lim_{h\to 0}\frac{\sin h}{h}}_{=1}$$

letzte Woche

bleibt noch:

$$\lim_{h\to 0}\frac{\cos h - 1}{h} = \lim_{h\to 0}\frac{\sqrt{1-\sin^2 h}-1}{h}\cdot\frac{\sqrt{1-\sin^2 h}+1}{\sqrt{1-\sin^2 h}+1}$$

$$= \lim_{h\to 0}\frac{(1-\sin^2 h)-1}{h\underbrace{\left(\sqrt{1-\sin^2 h}+1\right)}_{h\to 0\ =2}} = \lim_{h\to 0}\left(-\underbrace{\frac{\sin h}{h}}_{1}\frac{\overset{0}{\sin h}}{\underbrace{\left(\sqrt{}+1\right)}_{2}}\right)$$

$$= 0$$

Produktregel:

$$(f \cdot g)'(x) = \lim_{h \to 0} \frac{f(x+h)\,g(x+h) - f(x)\,g(x)}{h}$$

$$= \lim_{h \to 0} \frac{[\,f(x+h) - f(x)\,]\,g(x+h) + f(x)\,g(x+h) - f(x)\,g(x)}{h}$$

$$= \lim_{h \to 0} \underbrace{\frac{f(x+h) - f(x)}{h}}_{\to f'(x)} \underbrace{g(x+h)}_{g(x)} + \lim_{x+h} f(x) \underbrace{\frac{g(x+h) - g(x)}{h}}_{g'(x)}$$

Idee Kettenregel:

$$[f(g(x))]' = \lim_{h \to 0} \frac{f(g(x+h)) - f(g(x))}{h}$$

nähere g

durch Tangente

$$= \lim_{h \to 0} \frac{f(g(x) + g'(x)h) - f(g(x))}{h\, g'(x)} g'(x)$$

$g'(x)h \to 0$

$$= f'(g(x)) \cdot g'(x)$$

Abl. der Umkehrfkt.

$$f^{-1}(f(x)) = x \quad \Rightarrow \text{ ableiten}$$

$$f^{-1\,\prime}(\underbrace{f(x)}_{=:y}) \cdot f'(\underbrace{x}_{=f^{-1}(y)}) = 1$$

$$\Leftrightarrow \quad f^{-1\,\prime}(y) = \frac{1}{f'(f^{-1}(y))}$$

$$f'(f^{-1}(f(x))) \neq 0$$

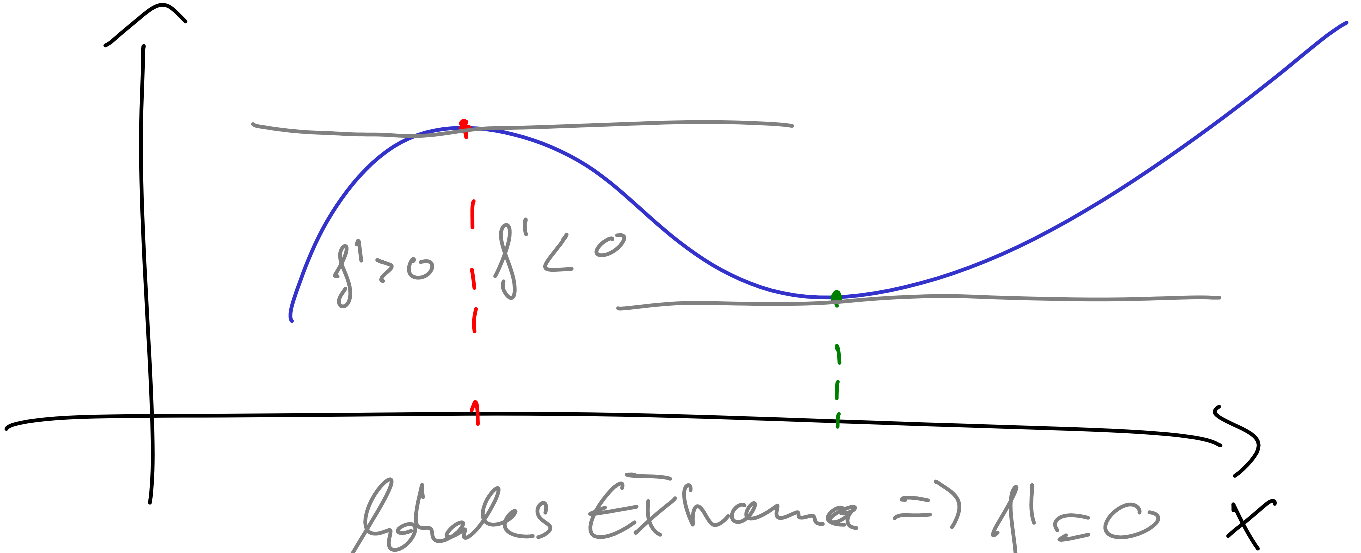
Nenne y in x um

$$\boxed{f^{-1\,\prime}(x) = \frac{1}{f'(f^{-1}(x))}}$$

Bsp:

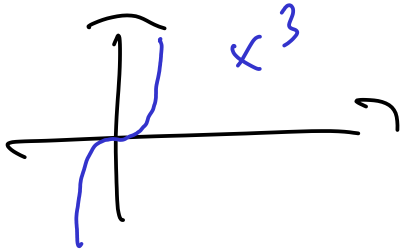
$$f^{-1}(x) = \log x \qquad \left(f(x) = e^x = \exp x \right)$$

$$(\log x)' = \frac{1}{\exp'(\log(x))} = \frac{1}{\exp(\log(x))} = \frac{1}{x}$$



lokales Extrema $\Rightarrow f' = 0$

Umkehrung gilt nicht



$f'(0) = 0$

kein Extrempunkt !!