$$f : [0, 2\pi] \longrightarrow \mathbb{R}^2$$
$$x \longmapsto \begin{pmatrix} \sin x \\ e^x - x \end{pmatrix}$$

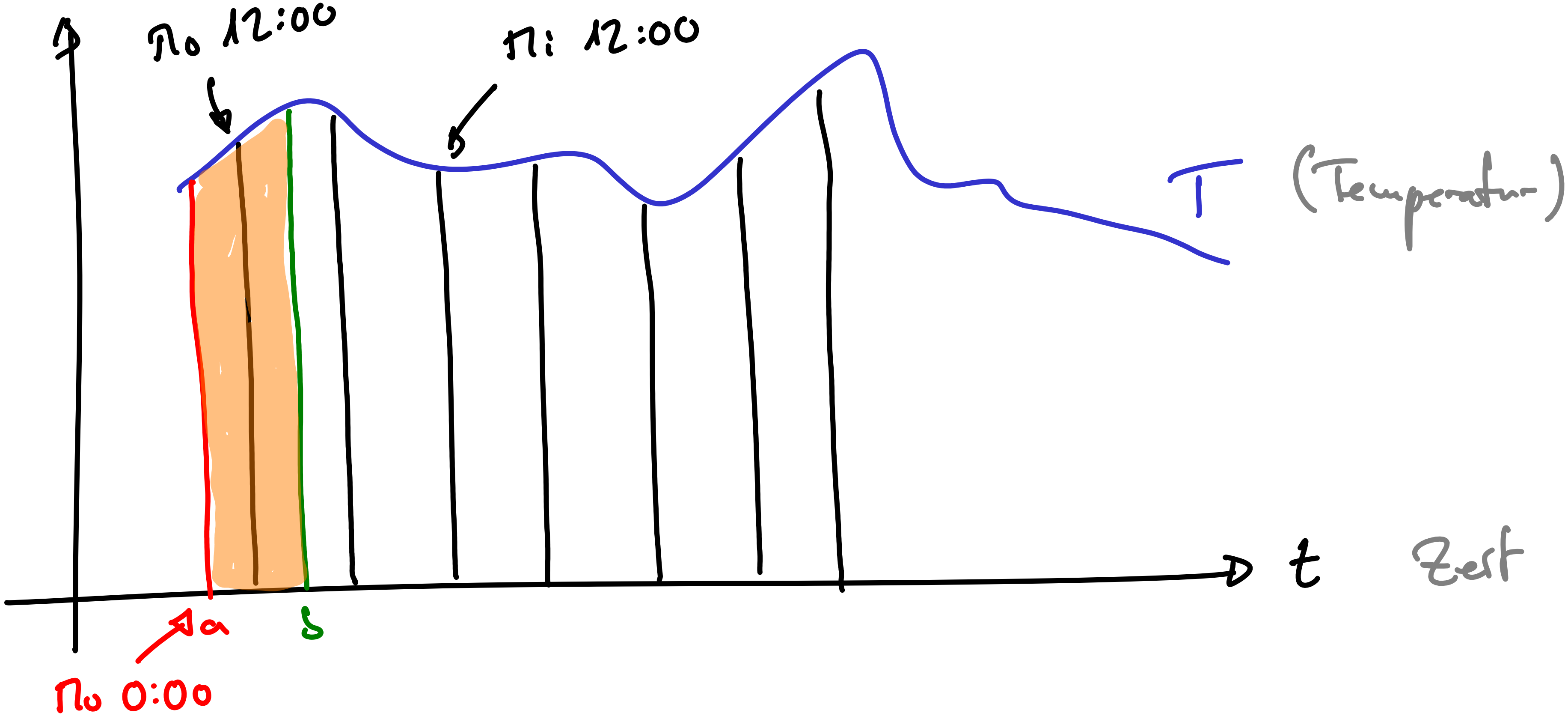
ges.: F mit $F' = f$

$$F : [0, 2\pi] \to \mathbb{R}^2$$
$$x \longmapsto \begin{pmatrix} -\cos x \\ e^x - \frac{1}{2}x^2 \end{pmatrix}$$

auch Stammfkt.

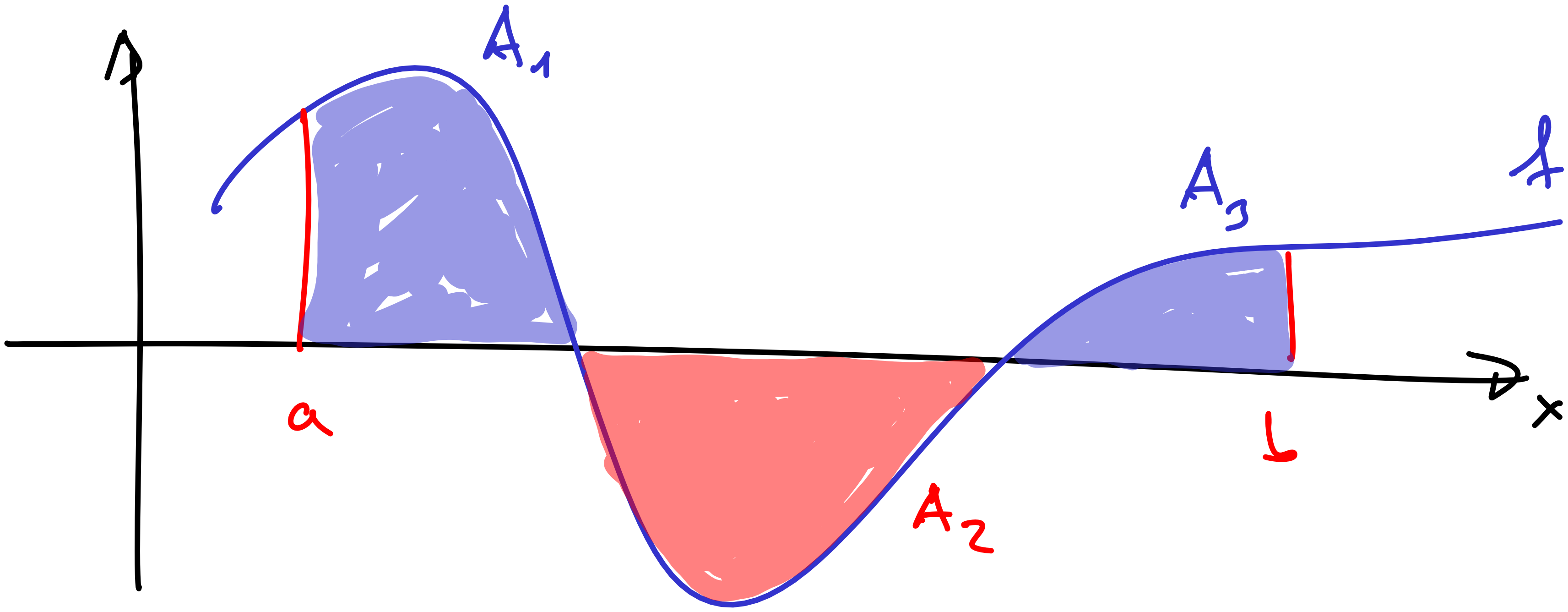
$$\tilde{F}(x) = \begin{pmatrix} 23 - \cos x \\ e^x + 42 - \frac{x^2}{2} \end{pmatrix}$$ erfüllt auch $\tilde{F}' = f$

hier $C = \begin{pmatrix} 23 \\ 42 \end{pmatrix}$



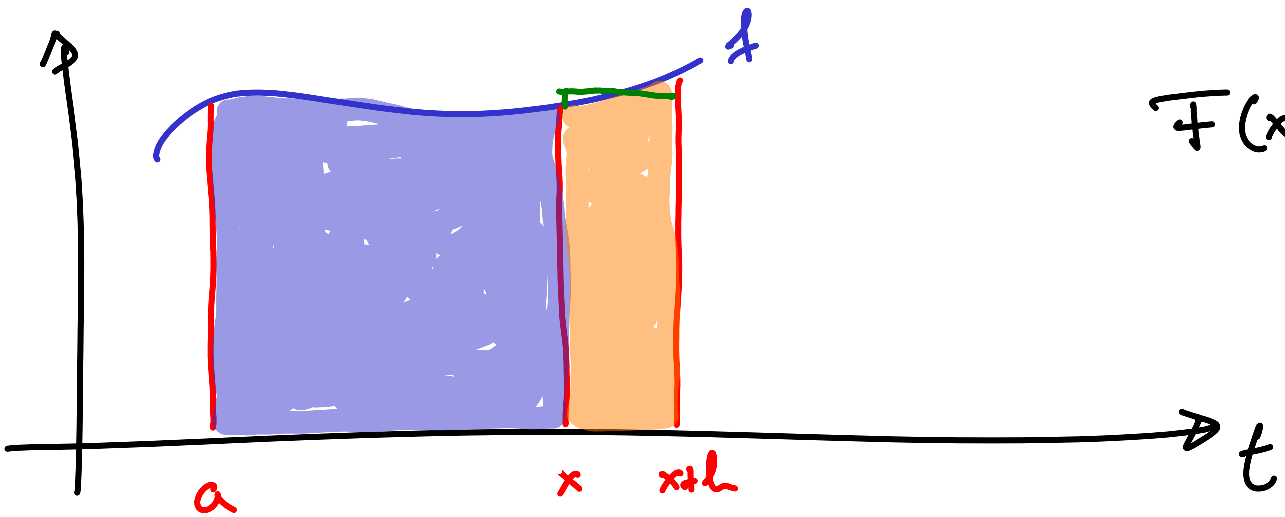
$$24h\text{-Mittel} = \frac{\text{Fläche}}{24\,h}$$

$$\text{Fläche} = \int_a^b f(x)\,dx$$



$$\int_a^b f(x)\,dx = A_1 - A_2 + A_3$$

Beweisidee zu HS



$$F(x) = \int_a^x f(t)\,dt$$

z.Z.: $F' = f$

es gibt ein $\xi \in [x, x+h]$ so dass

$$F'(x) = \lim_{h \to 0} \frac{F(x+h) - F(x)}{h} = \lim_{h \to 0} \frac{f(\xi) \cdot \cancel{h}}{\cancel{h}}$$

$\xi \xrightarrow[h \to 0]{} x$

$$= f(x)$$

Beispiele

$$\int_0^1 (3x^5 - 11x^2 + x)\,dx$$

$$= \left[\frac{1}{2}x^6 - \frac{11}{3}x^3 + \frac{1}{2}x^2\right]_0^1$$

Schreibweise für

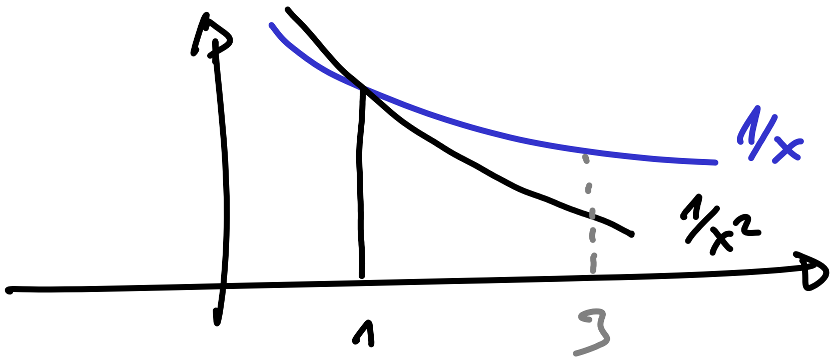
$$= \left(\frac{1}{2} - \frac{11}{3} + \frac{1}{2}\right) - (0 - 0 + 0)$$

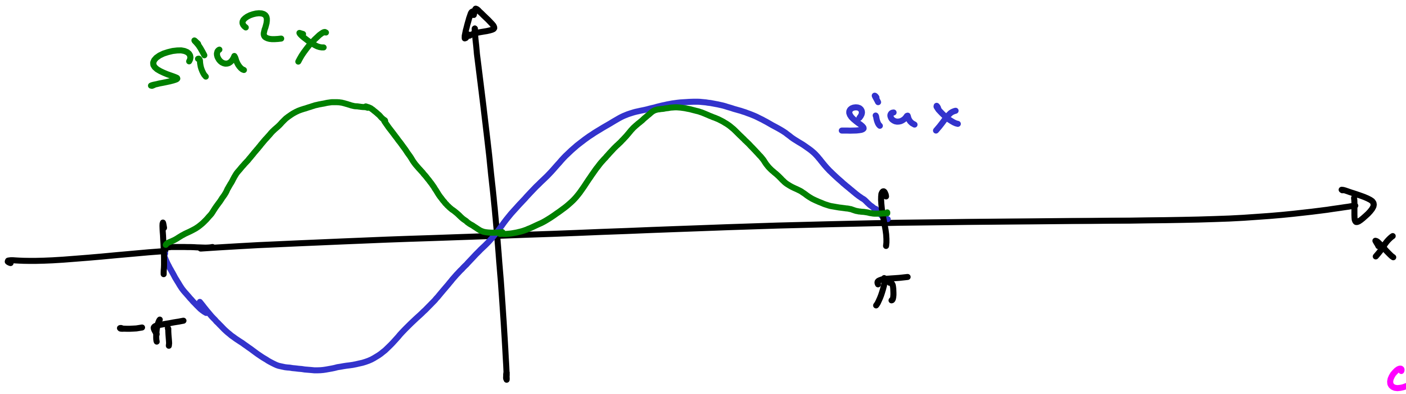
$$= -\frac{8}{3}$$

$$\int_1^3 \frac{dx}{x^2} = \int_1^3 \frac{1}{x^2}\,dx = \int_1^3 x^{-2}\,dx = \left[-x^{-1}\right]_1^3$$

$$= \left[-\frac{1}{x}\right]_1^3 = -\frac{1}{3} - (-1) = \frac{2}{3}$$

$$\int_1^3 \frac{dx}{x} = [\log x]_1^3 = \log 3 - \underbrace{\log 1}_{=0} = \log 3$$





cos(nπ)

$$\int_{-\pi}^{\pi} \sin(nx)\,dx = \left[-\frac{1}{n}\cos(nx)\right]_{-\pi}^{\pi} = -\frac{1}{n}\left((-1)^n - (-1)^n\right) = 0$$

$= 0$ da $\cos(-y) = \cos y$

$$\cos(2x) = \cos^2 x - \sin^2 x$$ Add.-Th. des cos

$$= 1 - 2\sin^2 x$$

Pythagoras $\sin^2 x + \cos^2 x = 1$

$$\Rightarrow \sin^2 x = \frac{1}{2} - \frac{1}{2}\cos(2x)$$

$$\int_{-\pi}^{\pi} \sin^2(nx)\,dx = \int_{-\pi}^{\pi} \left(\frac{1}{2} - \frac{1}{2}\cos(2nx)\right)dx$$

$$= \left[\frac{1}{2}x - \frac{1}{4n}\sin(2nx)\right]_{-\pi}^{\pi}$$

$$= \left(\frac{\pi}{2} - 0\right) - \left(-\frac{\pi}{2} - 0\right) = \pi$$

$\sin(k\pi) = 0 \quad \forall k \in \mathbb{Z}$

Produktregel: $(f(x)\,g(x))' = f'(x)\,g(x) + f(x)\,g'(x)$

$$\int_a^b \ldots\ldots \, dx \quad \Longrightarrow$$

$$[f(x)\,g(x)]_a^b = \underbrace{\int_a^b f'(x)\,g(x)\,dx} + \int_a^b f(x)\,g'(x)\,dx$$

$$\Longleftrightarrow \quad \int_a^b f'(x)\,g(x)\,dx = [f(x)\,g(x)]_a^b - \int_a^b f(x)\,g'(x)\,dx$$

Beispiel:

$$\int_0^{\pi/2} \underset{g}{x}\ \underset{f'}{\cos x}\ dx = [x \sin x]_0^{\pi/2} - \int_0^{\pi/2} 1 \cdot \sin x \ dx$$

$$= \frac{\pi}{2} \cdot 1 - 0 \cdot 0 + [\cos x]_0^{\pi/2}$$

$$= \frac{\pi}{2} + 0 - 1 = \frac{\pi}{2} - 1$$

$$\int \log x \, dx = \int \underset{f'}{1} \cdot \underset{g}{\log x} \, dx$$

$$= x \log x - \int x \cdot \frac{1}{x} \, dx = x \log x - \int dx$$

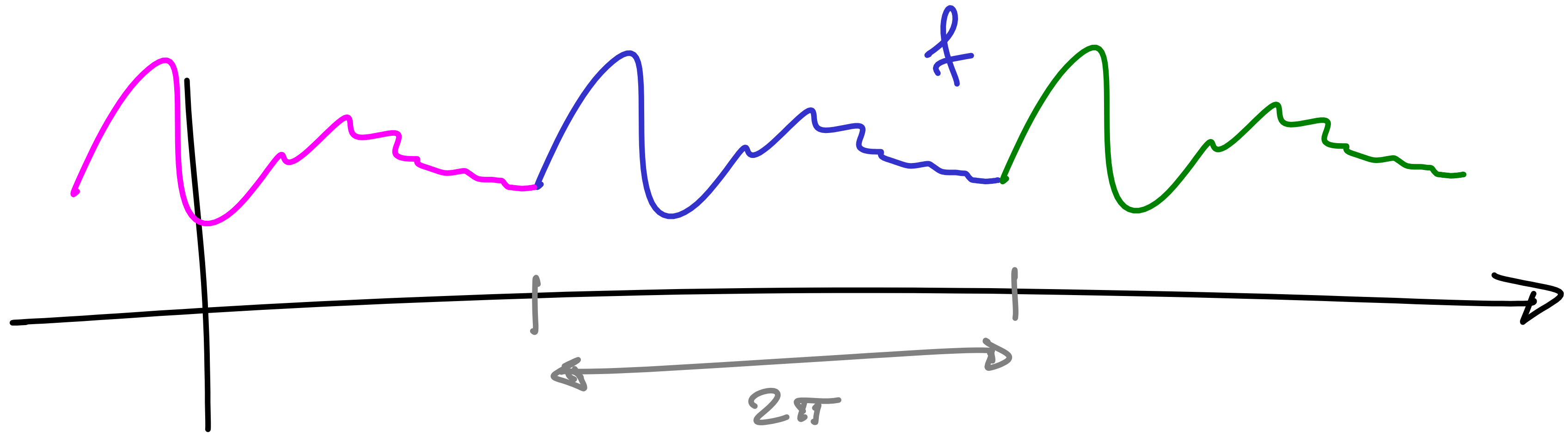
$$= x \log x - x = x \left(\log x - 1 \right)$$

$$\int_{-\pi}^{\pi} \underbrace{\sin(nx)}_{f'} \, \underbrace{\sin(mx)}_{g} \, dx = \underbrace{\left[-\frac{\cos(nx)}{n} \sin(mx) \right]_{-\pi}^{\pi}}_{=0} + \int_{-\pi}^{\pi} \underbrace{\frac{\cos(nx)}{n}}_{f'} \, \underbrace{\cos(mx) \cdot m}_{g} \, dx$$

$$= \frac{m}{n} \left(\underbrace{\left[\frac{\sin(nx)}{n} \cos(mx) \right]_{-\pi}^{\pi}}_{=0} - \int_{-\pi}^{\pi} \frac{\sin(nx)}{n} \left(-\sin(mx) \cdot m \right) dx \right)$$

$$= \frac{m^2}{n^2} \int_{-\pi}^{\pi} \sin(nx) \sin(mx) \, dx$$

$$\Rightarrow \underbrace{\left(1 - \frac{m^2}{n^2} \right)}_{\neq 0} \underbrace{\int_{-\pi}^{\pi} \sin(nx) \sin(mx) \, dx}_{=0} = 0$$



ist Summe von sin- & cos-Termen

$$\int_{-\pi}^{\pi} f(t)\,dt = \underbrace{\int_{-\pi}^{\pi} a_0\,dt}_{=2\pi a_0} + \sum_{n=1}^{\infty}\left(\underbrace{\int_{-\pi}^{\pi} a_n\cos(nt)\,dt}_{=0} + \underbrace{\int_{-\pi}^{\pi} b_n\sin(nt)\,dt}_{=0}\right)$$

$$\Rightarrow \quad a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(t)\,dt$$

$$\int_{-\pi}^{\pi} f(t)\sin(mt)\,dt = \underbrace{\int_{-\pi}^{\pi} a_0 \sin(mt)\,dt}_{=0}$$

$$+ \sum_n \left(\underbrace{\int_{-\pi}^{\pi} a_n \cos(nt)\sin(mt)\,dt}_{=0 \ (\text{ÜA})} + \underbrace{\int_{-\pi}^{\pi} b_n \sin(nt)\sin(mt)\,dt}_{= \begin{cases} \pi b_m, & n=m \\ 0, & n\neq m \end{cases}} \right)$$

$$\Rightarrow b_m = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t)\sin(mt)\,dt$$