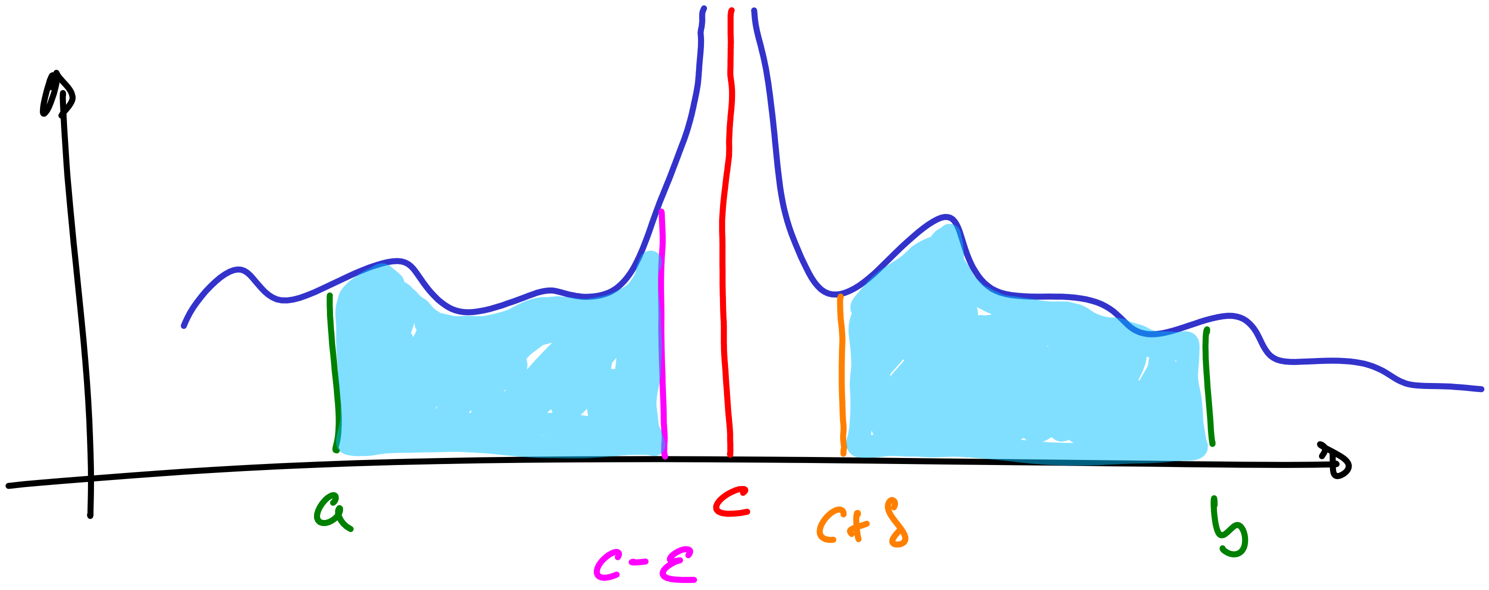


dann $\varepsilon \to 0+$, $\delta \to 0+$

Beispiel

$$\int_{-1}^{2} \frac{dx}{\sqrt{|x|}} = \int_{-1}^{0} \frac{dx}{\sqrt{|x|}} + \int_{0}^{2} \frac{dx}{\sqrt{|x|}}$$

$$= \lim_{\varepsilon \to 0+} \int_{-1}^{-\varepsilon} \frac{dx}{\sqrt{-x}} + \lim_{\delta \to 0+} \int_{\delta}^{2} \frac{dx}{\sqrt{x}}$$

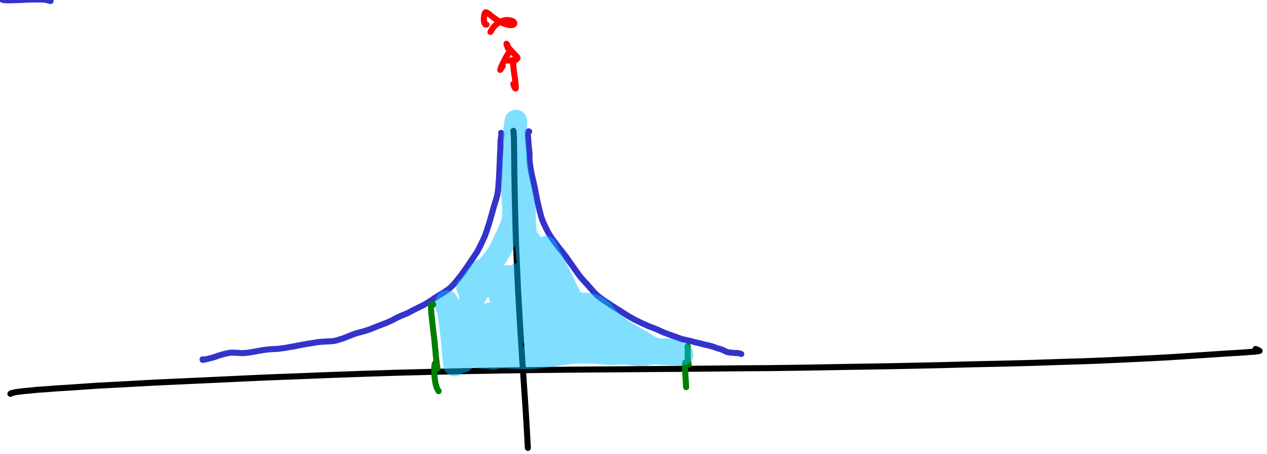
$x < 0$: $\frac{1}{\sqrt{|x|}} = \frac{1}{\sqrt{-x}}$

$x > 0$: $\frac{1}{\sqrt{|x|}} = \frac{1}{\sqrt{x}}$

$$= \lim_{\varepsilon \to 0+} \left[-2(-x)^{1/2}\right]_{-1}^{-\varepsilon} + \lim_{\delta \to 0+} \left[2x^{1/2}\right]_{\delta}^{2}$$

$$= \lim_{\varepsilon \to 0+} \left(-2\sqrt{\varepsilon} - (-2\sqrt{1})\right) + \lim_{\delta \to 0+} \left(2\sqrt{2} - 2\sqrt{\delta}\right)$$

$$= 2 + 2\sqrt{2} = 2(1+\sqrt{2})$$

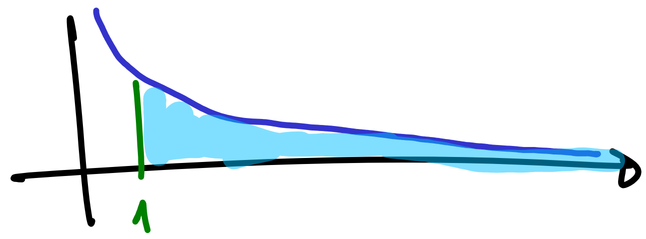


oder kurz:

$$\int_0^2 \frac{1}{\sqrt{x}}\,dx = \left[2\sqrt{x}\right]_0^2 = 2\sqrt{2}$$

versteckter Grenzwert

$$\int_1^\infty \frac{dx}{x^2}$$



$$= \lim_{b\to\infty} \int_1^b \frac{dx}{x^2} = \lim_{b\to\infty} \left[-\frac{1}{x}\right]_1^b = \lim_{b\to\infty} \left(-\frac{1}{b} - \left(-\frac{1}{1}\right)\right)$$

$$= 0 + 1 = 1$$

kurz: $\displaystyle\int_1^\infty \frac{dx}{x^2} = \left[-\frac{1}{x}\right]_1^\infty = 0 - (-1) = 1$

anderes Bsp:

$$\int_1^\infty \frac{dx}{x} = \lim_{b\to\infty} [\log x]_1^b = \lim_{b\to\infty} \log b = \infty$$

Calc-zilla

$$f(x,y) = y^x = e^{\log(y^x)} = e^{x \log y}$$

$$\frac{\partial f}{\partial x}(x,y) = e^{x \log y} \cdot \log y = y^x \cdot \log y$$

... Ableitung nach y?

$$\frac{\partial f}{\partial y} = x\, y^{x-1}$$

$$f : \mathbb{R}^2 \to \mathbb{R}$$

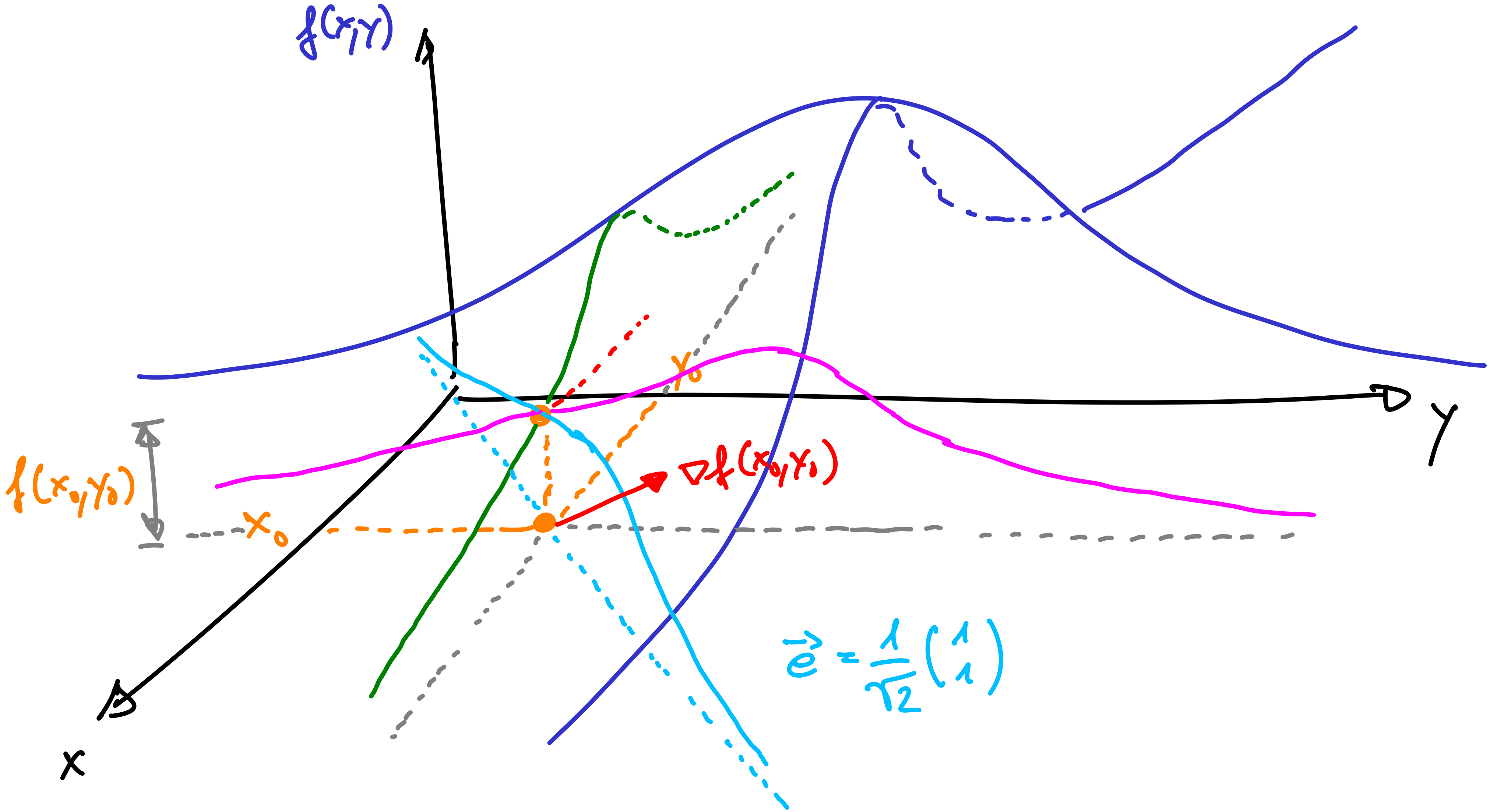
$$\vec{e}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \vec{e}_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\vec{x} = \begin{pmatrix} x \\ y \end{pmatrix} \longmapsto f(x,y)$$

$$\frac{\partial f}{\partial x}(x,y) = \lim_{h \to 0} \frac{f(\vec{x} + h\vec{e}_1) - f(\vec{x})}{h}$$

$$\vec{x} + h\,\vec{e}_1 = \begin{pmatrix} x \\ y \end{pmatrix} + h \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} x+h \\ y \end{pmatrix}$$

$$= \lim_{h \to 0} \frac{f(x+h,y) - f(x,y)}{h}$$



$\frac{\partial f}{\partial x}(x_0,y_0)$ = Steigung des grünen Weges an der Stelle (x_0,y_0) < 0

$\frac{\partial f}{\partial y}(x_0,y_0)$ = Steigung des rosa Weges an der Stelle (x_0,y_0) > 0

$\frac{\partial f}{\partial \vec{e}}(x_0, y_0)$ = Steigung des Weges an der Stelle (x_0, y_0)

Bsp: $f(x,y) = x e^y$

Stelle $(1,0)$

Richtung $\vec{e} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$

$\frac{\partial f}{\partial x} = e^y$

$\frac{\partial f}{\partial y} = x e^y$

$$\frac{\partial f}{\partial \vec{e}}(1,0) = \left(\frac{\partial f}{\partial x}(1,0), \frac{\partial f}{\partial y}(1,0)\right) \cdot \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

Matrix-Produkt

$$= (1, 1) \cdot \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix} = 0$$

$$= \begin{pmatrix} 1 \\ 1 \end{pmatrix} \cdot \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

Skalarprodukt in $\mathbb{R}^2$

1D : Tangente

Fkt. f, Stelle x_0

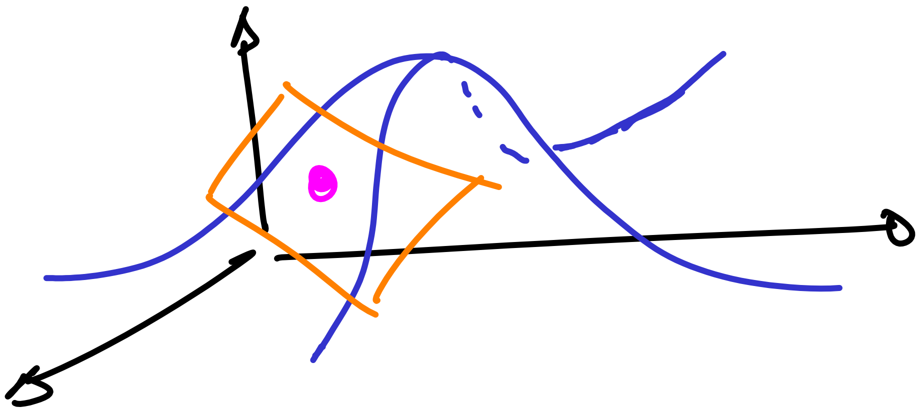
Tangente $t(x) = f(x_0) + f'(x_0)\,(x - x_0)$

analog mehrdim.

Fkt. f, Stelle $\vec{x}_0$

Tangentialebene

$$T(\vec{x}) = f(\vec{x}_0) + \nabla f(\vec{x}_0) \cdot (\vec{x} - \vec{x}_0)$$



zweite Ableitungen

$f(x,y) = x \cdot \sin y$

$$\frac{\partial f}{\partial x} = \sin y \quad , \quad \frac{\partial f}{\partial y} = x \cdot \cos y$$

$$\frac{\partial^2 f}{\partial y \partial x} = \frac{\partial}{\partial y}(\sin y) = \cos y$$

$$\frac{\partial^2 f}{\partial x \partial x} = \frac{\partial}{\partial x}(\sin y) = 0$$

$$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial}{\partial x}(x \cdot \cos y) = \cos y$$

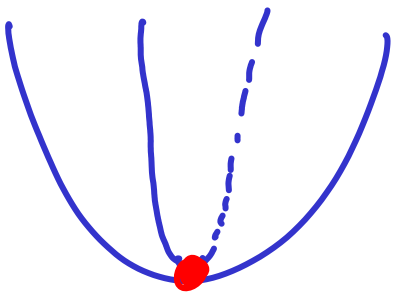
$$\frac{\partial^2 f}{\partial y \partial y} = \frac{\partial^2 f}{\partial y^2} = \frac{\partial}{\partial y}(x \cdot \cos y) = -x \sin y$$

Sind die immer gleich ?!?

Ja, falls f lieb ist ☺

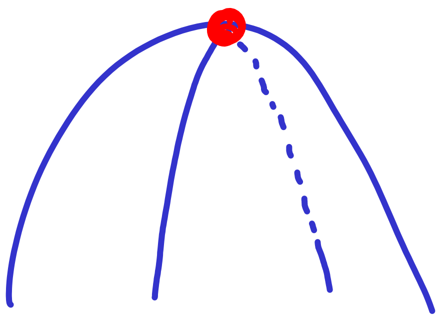
genauer: wenn $\frac{\partial^2 f}{\partial x \partial y}$ stetig

$$H = \begin{pmatrix} 0 & \cos y \\ \cos y & -x \cdot \sin y \end{pmatrix}$$



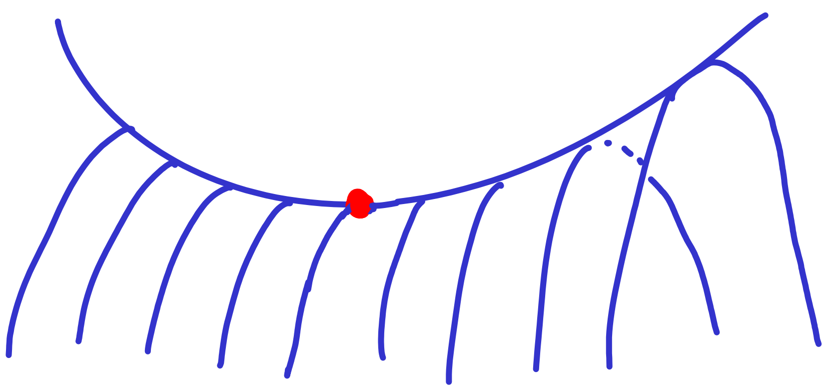
Minimum

H pos. def.



Maximum

H neg. def.



Sattel

H indefinit