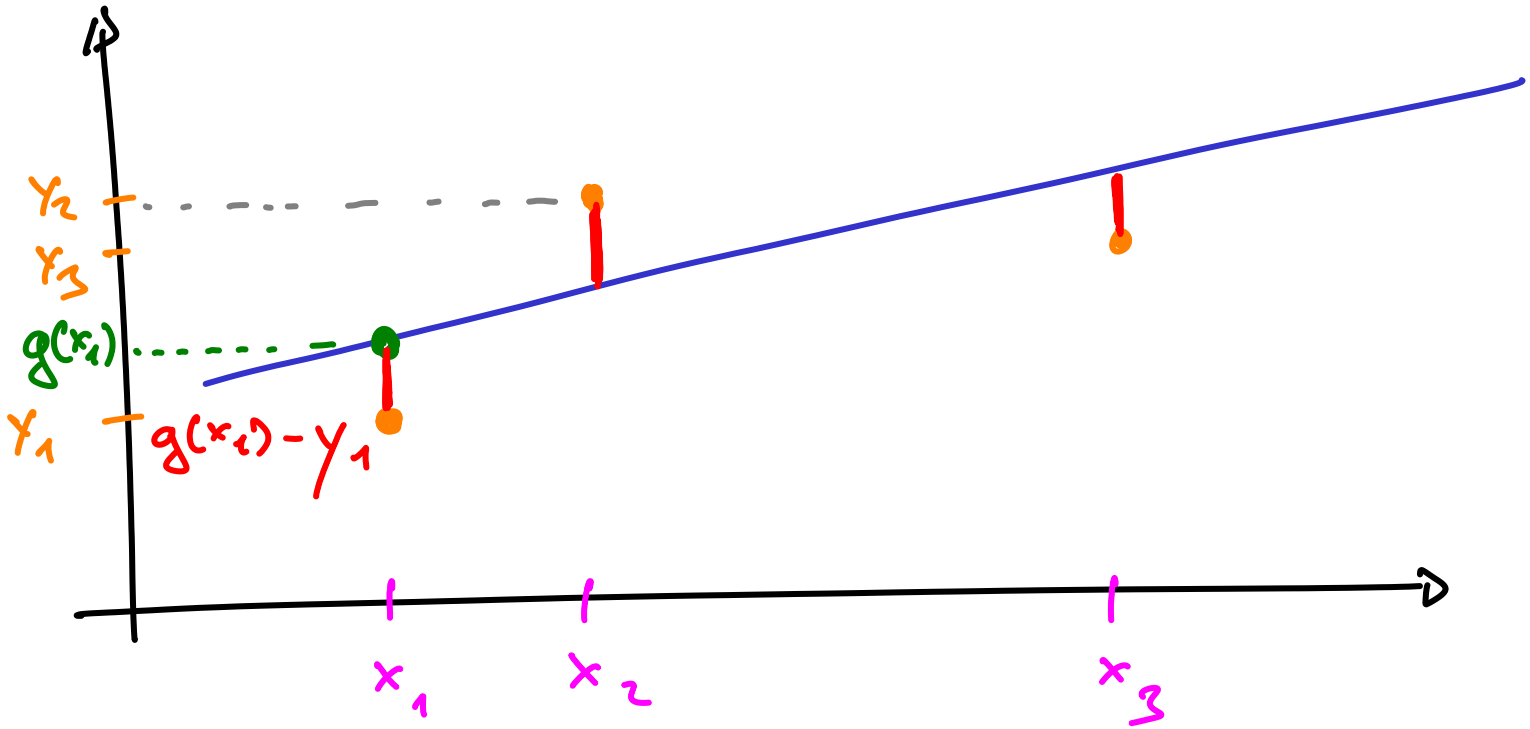


$$D(m,b) = \sqrt{\sum_{i=1}^{3} \left(g(x_i) - y_i \right)^2}$$

$mx_i + b$

$$f(m,b) = \left[D(m,b) \right]^2 = \sum_{i=1}^{3} \left(g(x_i) - y_i \right)^2$$

$$f(b,m) = \sum_{i=1}^{n} (m x_i + b - y_i)^2$$

$$\frac{\partial f}{\partial b} = \sum_{i=1}^{n} 2(m x_i + b - y_i) \stackrel{!}{=} 0$$

$$\frac{\partial f}{\partial m} = \sum_{i=1}^{n} 2(m x_i + b - y_i) \cdot x_i \stackrel{!}{=} 0$$

$$\frac{\partial f}{\partial b} = \sum_{i=1}^{n} 2(m x_i + b - y_i) \stackrel{!}{=} 0 \qquad \Big| \cdot \frac{1}{2}$$

$$\Leftrightarrow \underbrace{\sum_{i=1}^{n} m x_i}_{} + \underbrace{\sum_{i=1}^{n} b}_{= n b} - \underbrace{\sum_{i=1}^{n} y_i}_{= n \bar{y}} = 0 \qquad \Big| \cdot \frac{1}{n}$$

$$, \quad \bar{y} = \frac{1}{n} \sum_{i=1}^{n} y_i$$

$$= m \sum_{i=1}^{n} x_i$$

$$= m\, n\, \bar{x} \quad \text{mit} \quad \bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i$$

$$\Leftrightarrow \quad m\bar{x} + b - \bar{y} = 0$$

$$\sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})$$

$$= \sum_{i=1}^{n} \left(x_i y_i - x_i \bar{y} - \bar{x} y_i + \bar{x}\bar{y} \right)$$

$$= \sum_{i=1}^{n} x_i y_i - \bar{y} \underbrace{\sum_{i=1}^{n} x_i}_{= n\bar{x}} - \bar{x} \underbrace{\sum_{i=1}^{n} y_i}_{= n\bar{y}} + n\bar{x}\bar{y}$$

$$= \sum_{i=1}^{n} x_i y_i - n\bar{x}\bar{y}$$

Dürfte wir durch

$$\sum_{i=1}^{n} \underbrace{(x_i - \bar{x})^2}_{\geq 0}$$ teilen?

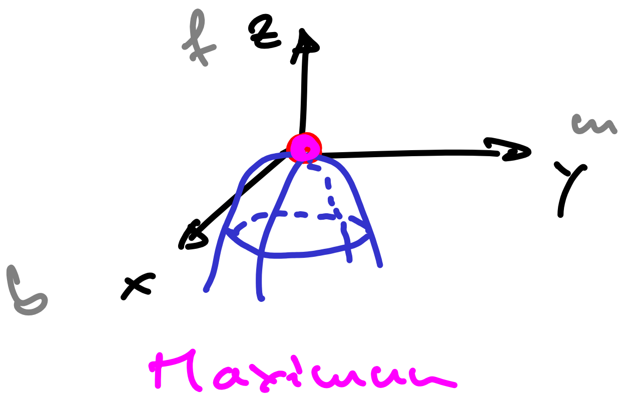
ganze Summe $= 0$ nur dann wenn

$x_i - \bar{x} = 0$ für <u>alle</u> i

d.h. wenn alle x_i gleich sind $\leftarrow$ ausgeschlossen

also O.K.

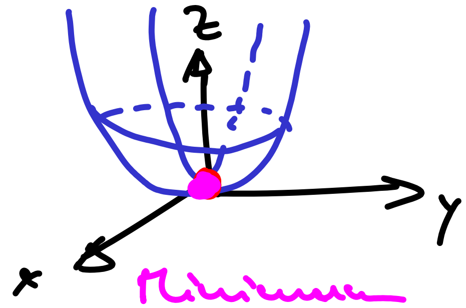
Definitheit



z.B. $z = -x^2 - y^2$

$$z'' = \begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix}$$

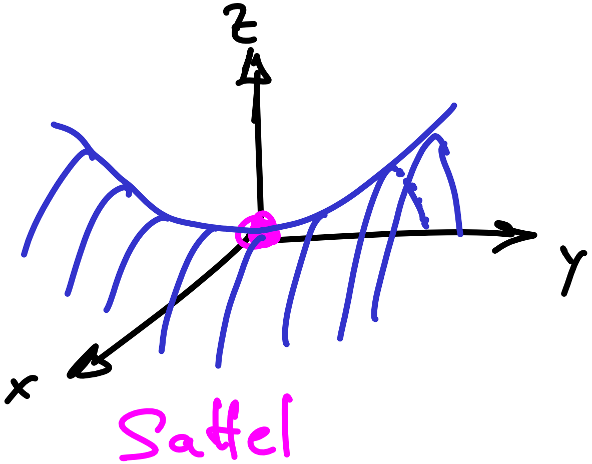
negativ definit



z.B. $z = x^2 + y^2$

$$z'' = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$$

positiv definit



z.B. $z = y^2 - x^2$

$$z = \begin{pmatrix} -2 & 0 \\ 0 & 2 \end{pmatrix}$$

indefinit

Definitheit von 2x2-Matrizen

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}, \quad \vec{x} = \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\vec{x}^T A \vec{x} = (x \ \ y) \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = (x \ \ y) \begin{pmatrix} ax + by \\ cx + dy \end{pmatrix}$$

$$= a x^2 + \underbrace{b xy + c yx}_{= 2bxy \text{ falls } c = b} + d y^2$$

also A symmetrisch

quadrat. Ergänzung

$$= a \left(x^2 + 2 \frac{b}{a} xy \right) + d y^2$$

$$= a \left(\left(x + \frac{b}{a} y \right)^2 - \frac{b^2}{a^2} y^2 \right) + d y^2$$

$$= a \underbrace{\left(x + \frac{b}{a} y \right)^2}_{\geq 0} + \left(d - \frac{b^2}{a} \right) \underbrace{y^2}_{\geq 0}$$

Falls $a > 0$ und $da - b^2 > 0$

dann ist A pos. definit

$$A = \begin{pmatrix} a & b \\ b & d \end{pmatrix}$$

---

für $$H = \begin{pmatrix} 2n & 2n\bar{x} \\ 2n\bar{x} & 2\sum_{i=1}^{n} x_i^2 \end{pmatrix}$$

① $2n > 0$ ✓

② $$4n \sum_{i=1}^{n} x_i^2 - 4n^2\bar{x}^2 = 4n\left(\sum_{i=1}^{n} x_i^2 - n\bar{x}^2\right)$$

Nenner von m also > 0

$\Rightarrow f''$ pos. definit

$\Rightarrow$ Minimum