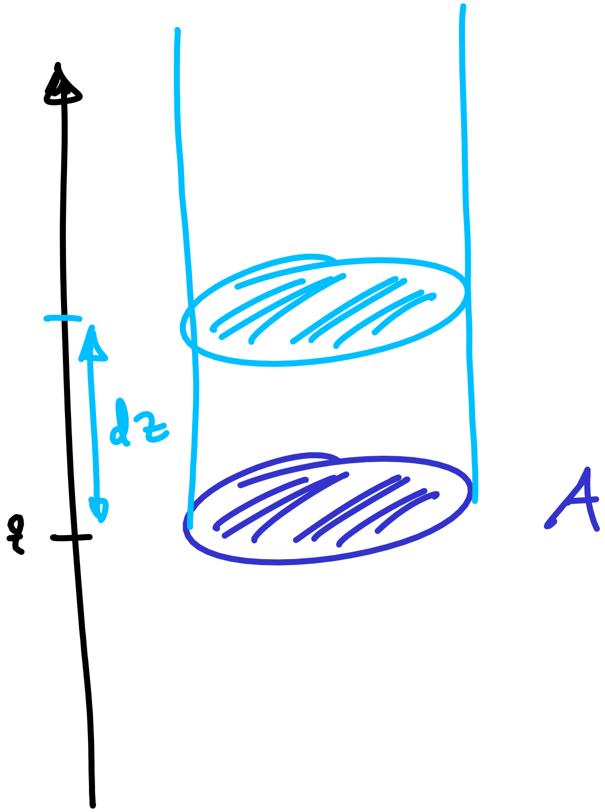


Druck = Kraft pro Fläche

Gewichtskraft der Luftsäule

Druckänderung: $z \to z + dz$

Masse Luftsäule zw. z & $z+dz$

$$dp = -g \cdot m_{LS}/A$$

Erdbeschl.

Grundfläche

Volumen

Dichte (abh. v. z)

Masse der Luftsäule: $m_{LS} = V \cdot \rho = A \cdot dz \cdot \rho$

d.h. $$p'(z) = \frac{dp}{dz} = -g\,\rho(z)$$

ideales Gas $$pV = nRT$$

Molzahl

Gaskonst.

Temp.

$$V = \frac{m}{\rho}$$

$$\Longleftrightarrow \quad \rho = \frac{p\,m}{n R T} \qquad \frac{m}{n} = M \text{ (molare Masse)}$$

$$\boxed{p'(z) = -\underbrace{\frac{gM}{R}}_{\text{Konstante}} \frac{1}{T(z)}\, p(z)}$$

Wie findet man Lösungen?

① $T(z) = T$ konstant

$$\frac{dp}{dz} = p'(z) = -\frac{gM}{RT}\, p(z) \qquad \Big| \cdot \frac{1}{p(z)} \cdot dz$$

$$\int \frac{dp}{p} = -\frac{gM}{RT} \int dz$$

$$\log p = -\frac{gM}{RT} z + C \qquad \text{mit } C \in \mathbb{R} \text{ beliebig}$$

exp(...)

$$p = e^{-\frac{gM}{RT}z} \cdot \underbrace{e^{C}}_{=:\,\tilde{C}}$$

neue Konstante

Vergleiche mit

$$p = p_0\, e^{-\frac{gM}{RT}(z-z_0)} = e^{-\frac{gM}{RT}z}\, \underbrace{p_0\, e^{+\frac{gM}{RT}z_0}}_{=\tilde{C}}$$

② $T(z) = T_0 + \gamma\,(z-z_0)$ $\quad (T(z_0) = T_0)$

$$\int \frac{dp}{p} = -\frac{gM}{R}\int \frac{1}{T_0+\gamma(z-z_0)}\,dz$$

$$\log p = -\frac{gM}{R\gamma}\log\left(T_0+\gamma(z-z_0)\right) + C \quad \Big|\ \exp(\ldots)$$

$$p = \left(T_0 + \gamma(z - z_0)\right)^{-\frac{gM}{R\gamma}} \cdot \underbrace{e^{C}}_{= \tilde{C}\ \text{(bel. Konstante)}}$$

Klammern T_0 aus

$$= p_0 \left(1 + \frac{\gamma}{T_0}(z - z_0)\right)^{-\frac{gM}{R\gamma}}$$

$$p_0 = \underbrace{T_0^{-\frac{gM}{R\gamma}} \cdot e^{C}}_{\text{beliebige Konstante}}$$

Bspe für DGLn

① $\dot{x} + x = 0$ autonome DGL 1. Ordnung

$d=1,\ n=1$

$$\dot{x} = f(x, \cancel{\dot{x}}) = -x$$

② $\dot{x} + x = \sin t$ zeitabhängige DGL 1. Ord.

$d=1,\ n=1$

$$\dot{x} = f(x,t) = -x + \sin t$$

③ $\ddot{x} + x = 0$ autonome DGL 2. Ord.

$d=1,\ n=2$

$$\ddot{x} = f(x, \cancel{\dot{x}}, \cancel{\ddot{x}}) = -x$$

④ $\left.\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -x_1 \end{aligned}\right\}$ autonomes System 1. Ord.

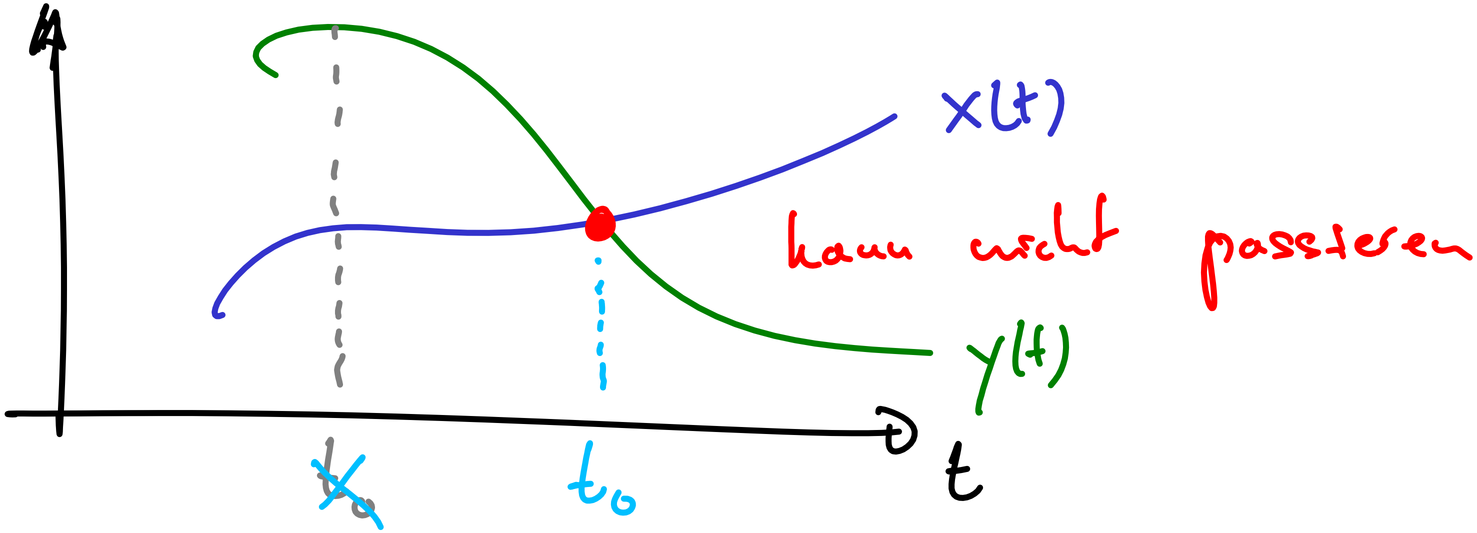
$d=2,\ n=1$

In Vektorschreibweise

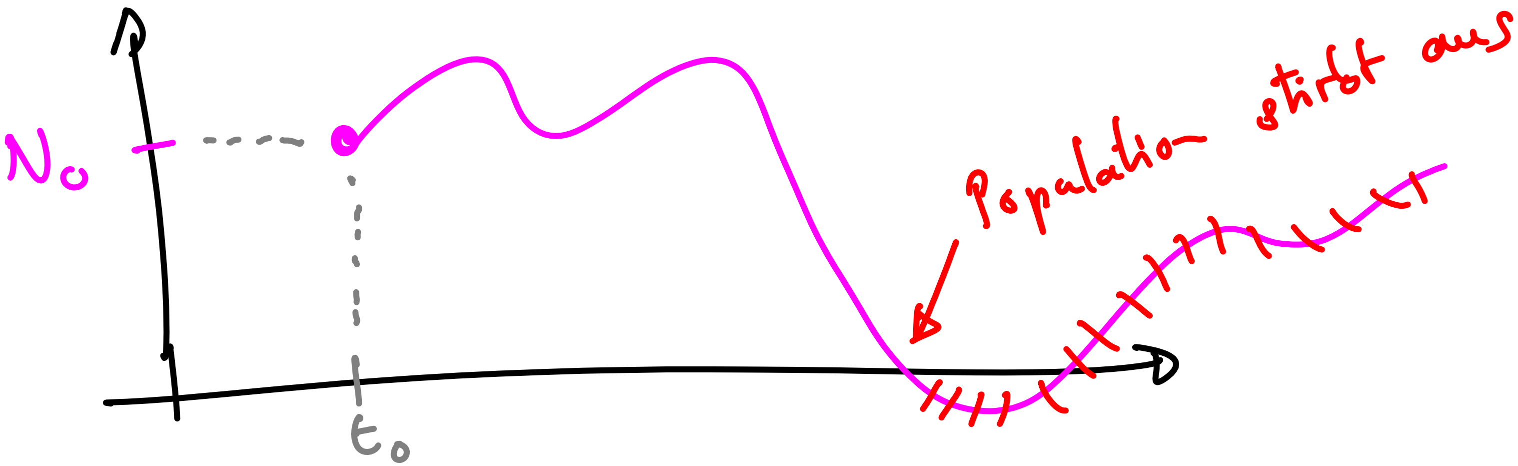
$$\vec{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

$$\dot{\vec{x}} = \begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \underbrace{\begin{pmatrix} x_2 \\ -x_1 \end{pmatrix}}_{= \underbrace{\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}}_{=: A} \underbrace{\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}}_{= \vec{x}}} = \vec{f}(x_1, x_2, \cancel{t}) = \vec{f}(\vec{x}, \cancel{t})$$

$$\dot{\vec{x}} = \underbrace{A \vec{x}}_{= \vec{f}(\vec{x}, \cancel{t})}$$



Populationsgröße $N(t)$



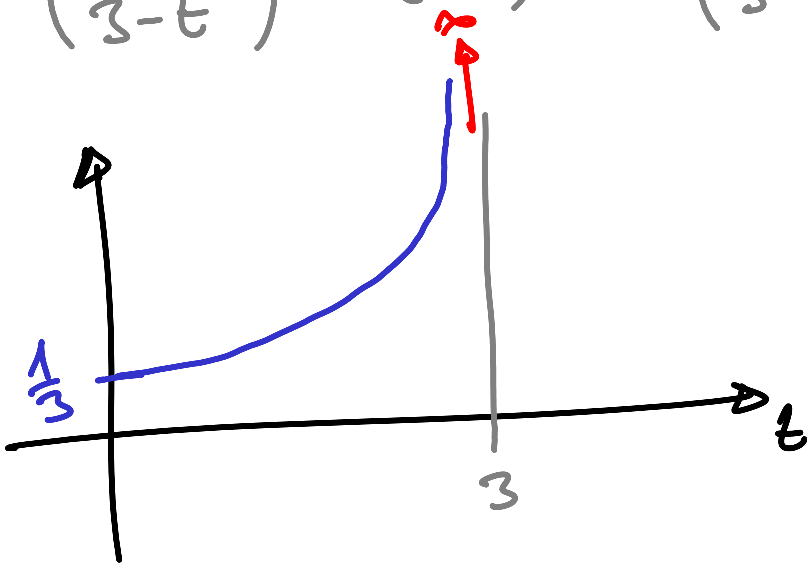
anderes Bsp

AWP: $\dot{x} = x^2$, $x(0) = \frac{1}{3}$

Lösung $x(t) = \dfrac{1}{3-t}$

denn

$$\dot{x} = -\left(\frac{1}{3-t}\right)^2 \cdot (-1) = \left(\frac{1}{3-t}\right)^2 = x^2$$



Reduktion

$$\ddot{x} + \omega^2 x = 0 \qquad (n=2,\ d=1)$$

$$\left.\begin{aligned} x_1 &= x \\ x_2 &= \dot{x} \end{aligned}\right. \quad \Longrightarrow \quad \left.\begin{aligned} \dot{x}_1 &= \dot{x} = x_2 \\ \dot{x}_2 &= \ddot{x} = -\omega^2 x = -\omega^2 x_1 \end{aligned}\right\} \begin{pmatrix} n=1 \\ d=2 \end{pmatrix}$$

DGL

folglich:

AWP: $\ddot{x} + \omega^2 x = 0, \quad x(0) = x_0, \quad \dot{x}(0) = v_0$

hat eindeutige Lösung

$$S + E \underset{k_{-1}}{\overset{k_1}{\rightleftharpoons}} SE$$

s, e, c – Konzentration von S, E, SE

$$SE \xrightarrow{k_2} P + E$$

$$\dot{s} = -k_1 \cdot s e + k_{-1} c$$

$$\dot{c} = k_1 \cdot s e - k_{-1} c - k_2 c$$