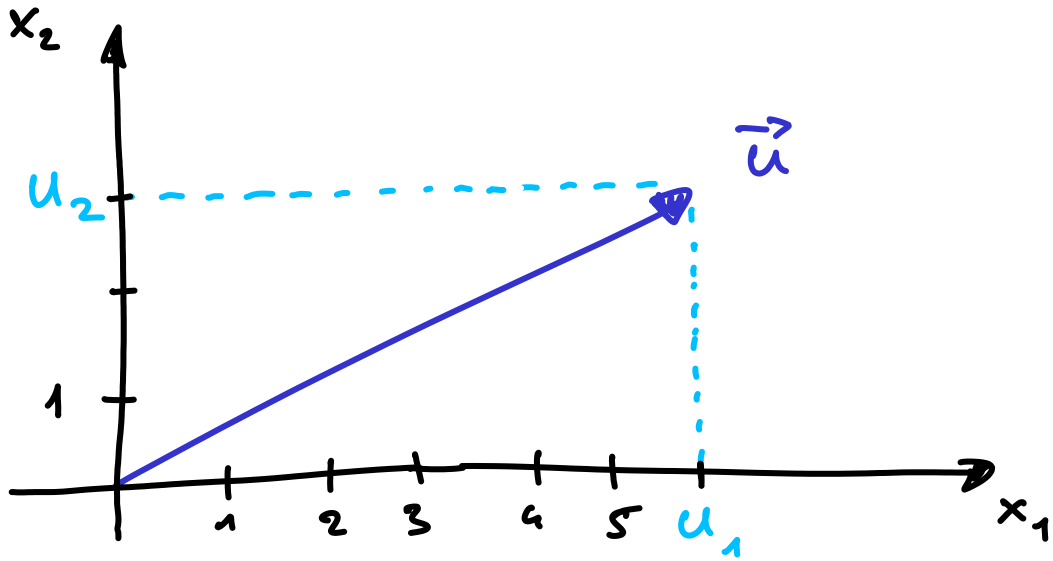
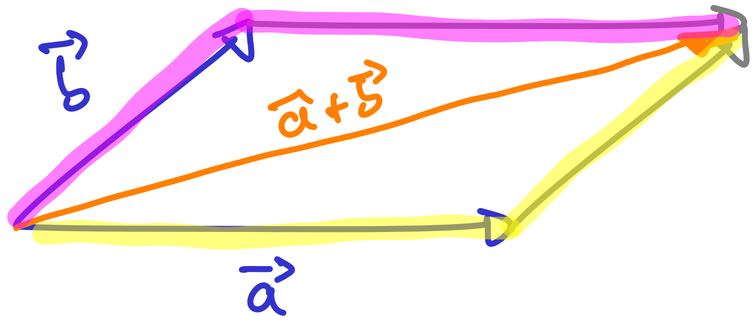


$$\vec{u} = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}, \quad |\vec{u}| = \sqrt{u_1^2 + u_2^2}$$

Länge des Pfeils

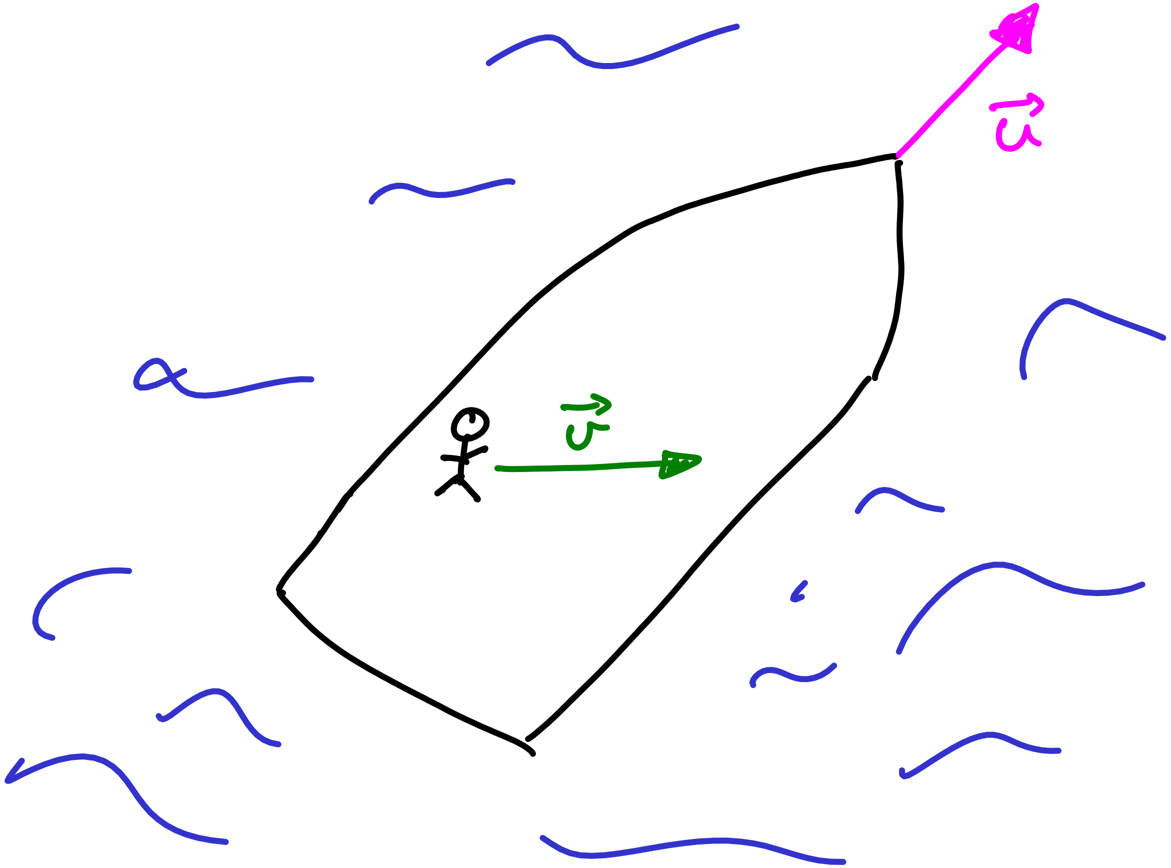
$$\vec{u} = \begin{pmatrix} 6 \\ 3 \end{pmatrix}, \quad |\vec{u}| = \sqrt{36 + 9} = \sqrt{45} = 3\sqrt{5}$$



$$\vec{a}+\vec{b} = \vec{b}+\vec{a}$$

Bsp:

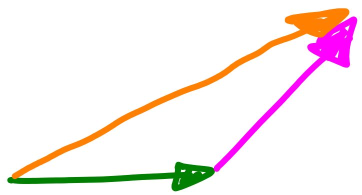
$$\vec{a} = \begin{pmatrix} 3 \\ 0 \end{pmatrix}, \quad \vec{b} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}, \quad \vec{a}+\vec{b} = \begin{pmatrix} 3+2 \\ 0+2 \end{pmatrix} = \begin{pmatrix} 5 \\ 2 \end{pmatrix}$$

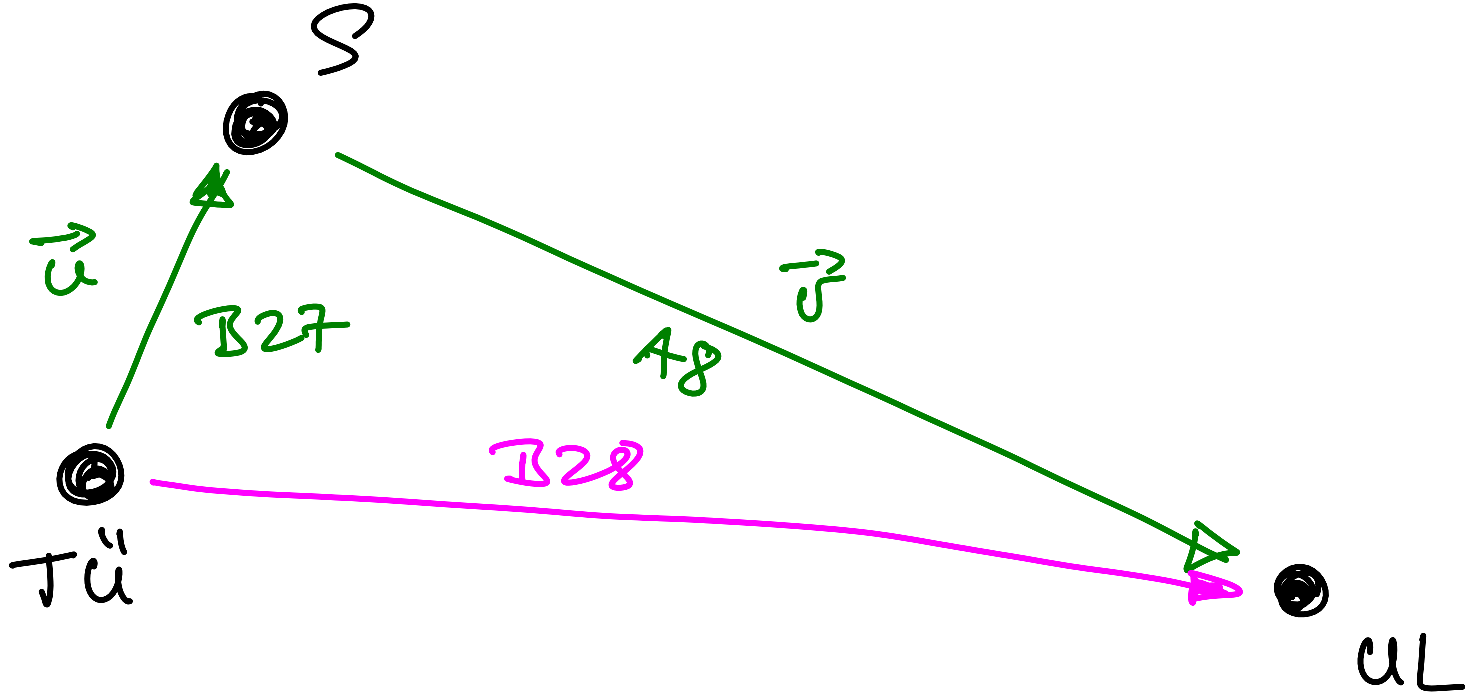


$\vec{u}$: Geschw. Boot gegenüber Wasser

$\vec{v}$: Geschw. Person gegenüber Boot

$\vec{u} + \vec{v}$: Geschw. von gegenüber Wasser

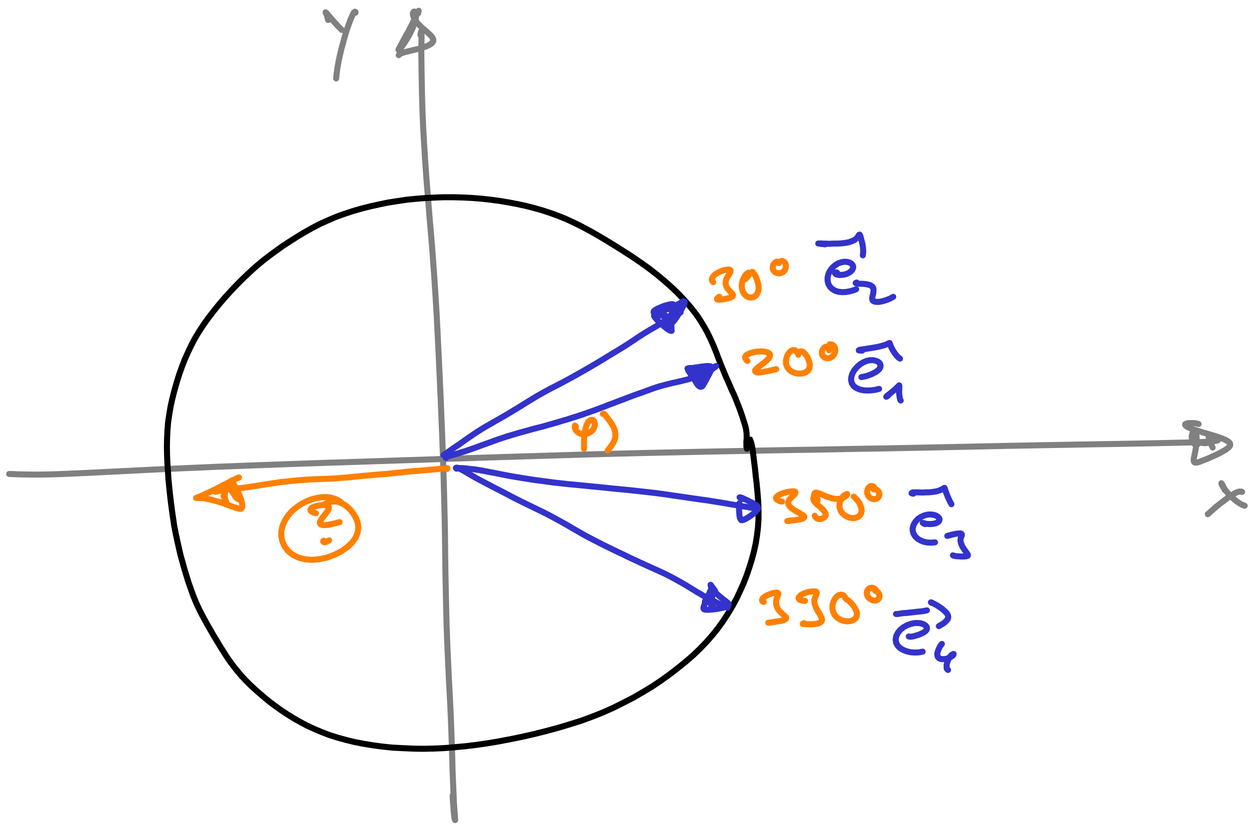




$$\vec{u} = \frac{50\,\text{km}}{\sqrt{5}} \begin{pmatrix} 1 \\ 2 \end{pmatrix} \qquad \sqrt{5} = \sqrt{1^2 + 2^2}$$

$$\vec{v} = \frac{100\,\text{km}}{\sqrt{58}} \begin{pmatrix} 7 \\ -3 \end{pmatrix} \qquad \sqrt{58} = \sqrt{7^2 + (-3)^2}$$

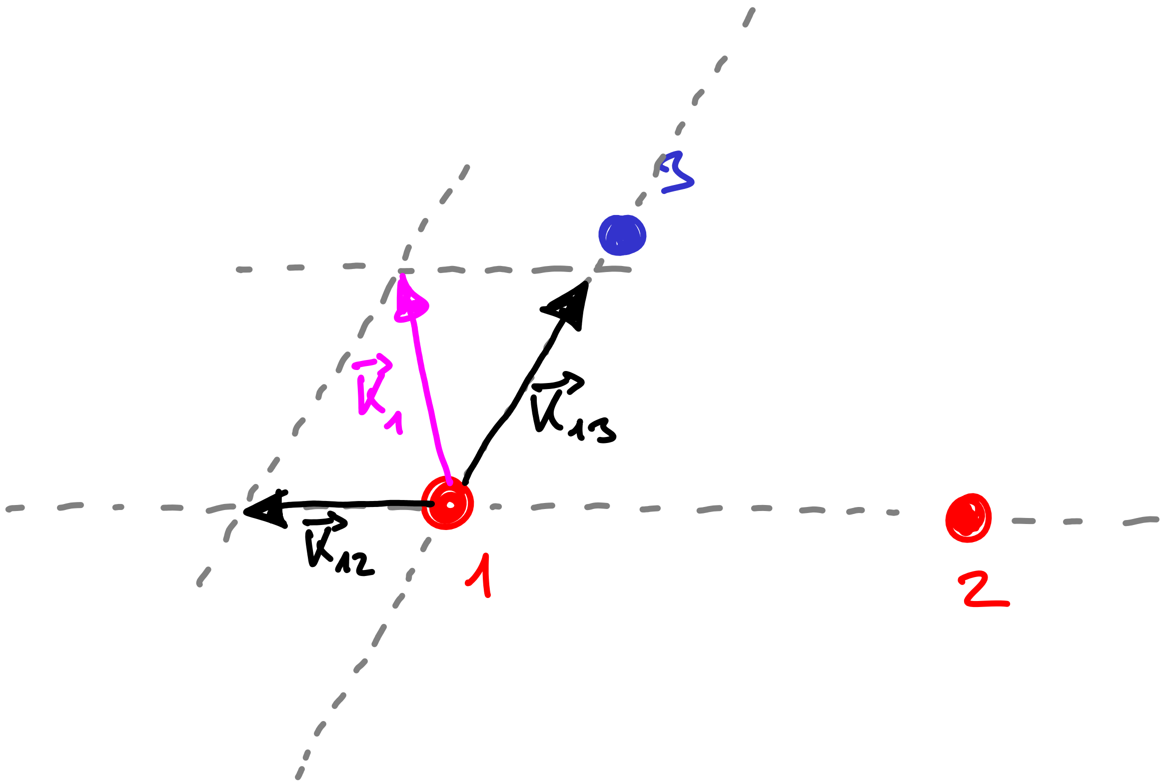
$$|\vec{u} + \vec{v}| = \left| \frac{50\,\text{km}}{\sqrt{5}} \begin{pmatrix} 1 \\ 2 \end{pmatrix} + \frac{100\,\text{km}}{\sqrt{58}} \begin{pmatrix} 7 \\ -3 \end{pmatrix} \right|$$



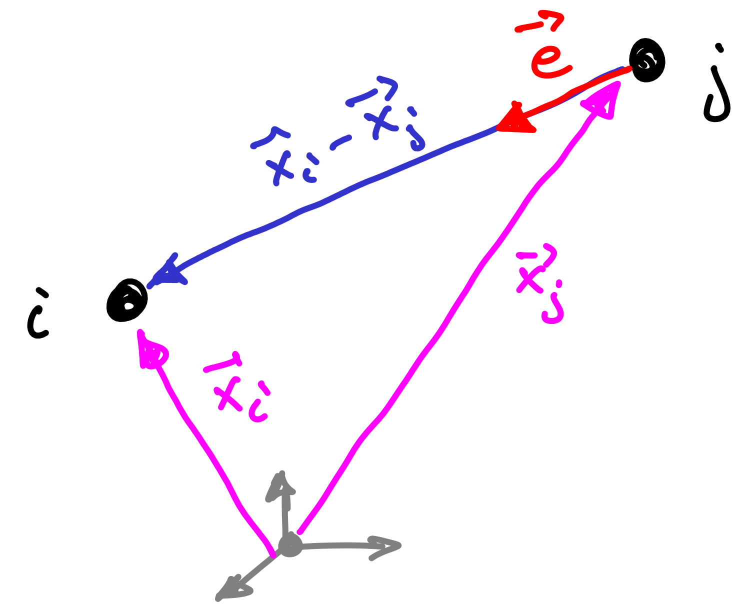
$$\bar{\varphi} = \frac{20° + 30° + 350° + 330°}{4} = 182{,}5°$$

$$\vec{e} = \frac{\vec{e}_1 + \vec{e}_2 + \vec{e}_3 + \vec{e}_4}{4}$$ ← zeigt nach rechts ☺

ersetze evtl. $\vec{e}$ am Ende durch $\frac{\vec{e}}{|\vec{e}|}$

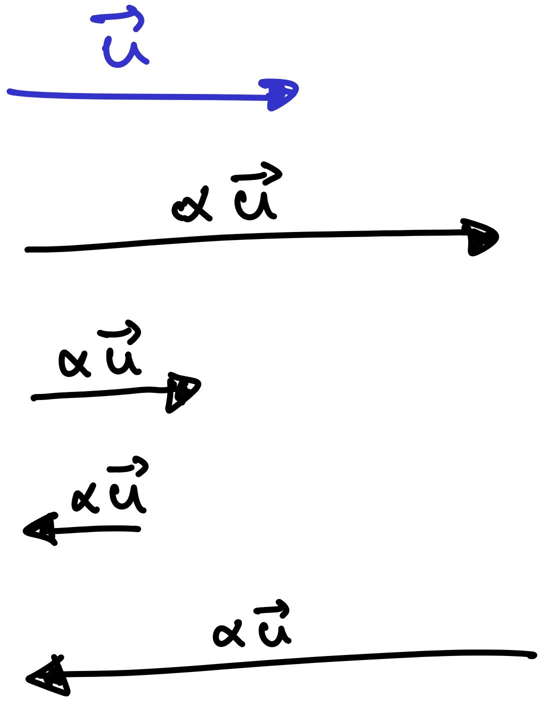


$$\vec{K}_1 = \vec{K}_{12} + \vec{K}_{13}$$



$$K_{ij} = \frac{q_i q_j}{|\vec{x}_i - \vec{x}_j|^2}\, \vec{e} = \frac{q_i q_j}{|\vec{x}_i - \vec{x}_j|^2} \frac{\vec{x}_i - \vec{x}_j}{|\vec{x}_i - \vec{x}_j|} = \frac{q_i q_j}{|\vec{x}_i - \vec{x}_j|^3} (\vec{x}_i - \vec{x}_j)$$

Vektor d. Länge 1



$\alpha > 1$:

$0 < \alpha < 1$:

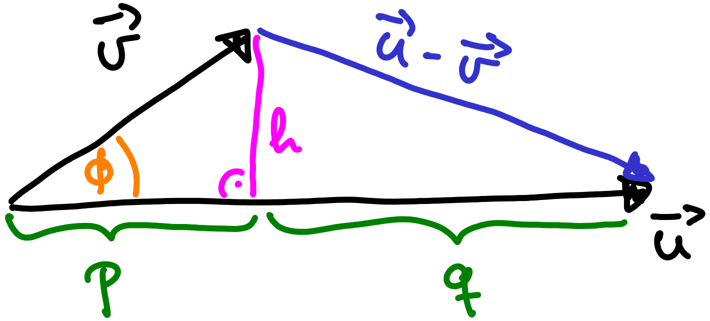
$-1 < \alpha < 0$:

$\alpha < -1$:

alle parallel zu $\vec{u}$
(bzw. antiparallel)

$\vec{u} = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}, \quad \vec{v} = \begin{pmatrix} 3 \\ 4 \\ 7 \end{pmatrix}$

$\vec{u} \cdot \vec{v} = 1 \cdot 3 + 2 \cdot 4 + 0 \cdot 7 = 11$



$\vec{u}^2 = |\vec{u}|^2$

$$\frac{p}{|\vec{v}|} = \cos\phi$$

Pythagoras: $p^2 + h^2 = |\vec{v}|^2$, $q^2 + h^2 = |\vec{u}-\vec{v}|^2$

$|\vec{u}| = p + q \Rightarrow q = |\vec{u}| - p$

$\Rightarrow q^2 = |\vec{u}|^2 + p^2 - 2p|\vec{u}|$

$$\Rightarrow |\vec{v}|^2 - p^2 = |\vec{u}-\vec{v}|^2 - q^2$$

$$\Leftrightarrow |\vec{v}|^2 - p^2 = \underbrace{|\vec{u}-\vec{v}|^2}_{} - |\vec{u}|^2 - p^2 + 2p|\vec{u}|$$

$$= (\vec{u}-\vec{v})\cdot(\vec{u}-\vec{v}) = \vec{u}^2 + \vec{v}^2 - 2\vec{u}\cdot\vec{v}$$

$$\Leftrightarrow \vec{v}^2 = \vec{u}^2 + \vec{v}^2 - 2\vec{u}\cdot\vec{v} - \vec{u}^2 + 2|\vec{u}|p$$

$$\Leftrightarrow \vec{u}\cdot\vec{v} = |\vec{u}|p = |\vec{u}||\vec{v}|\cos\phi$$

Ausblick: Matrizen

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1m} \\ a_{21} & a_{22} & \cdots & a_{2m} \\ \vdots & & & \\ a_{n1} & a_{n2} & \cdots & a_{nm} \end{pmatrix}$$

$n \times m$

Zeilenindex

Spaltenindex

z.B.

$$A = \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ -1 & 1 \end{pmatrix}$$

2×2 $\qquad$ 3×2

$\cancel{A \cdot B}$ geht nicht

$B \cdot A$ ist definiert

$$\begin{pmatrix}1 & 3\\ 2 & 4\end{pmatrix} = A$$

$$B = \begin{pmatrix}1 & 0\\ 0 & 1\\ -1 & 1\end{pmatrix} \qquad \begin{pmatrix}1 & 3\\ 2 & 4\\ 1 & 1\end{pmatrix} = B \cdot A$$

$$1 \cdot 1 + 0 \cdot 2 = 1$$

$$-1 \cdot 3 + 1 \cdot 4 = 1$$