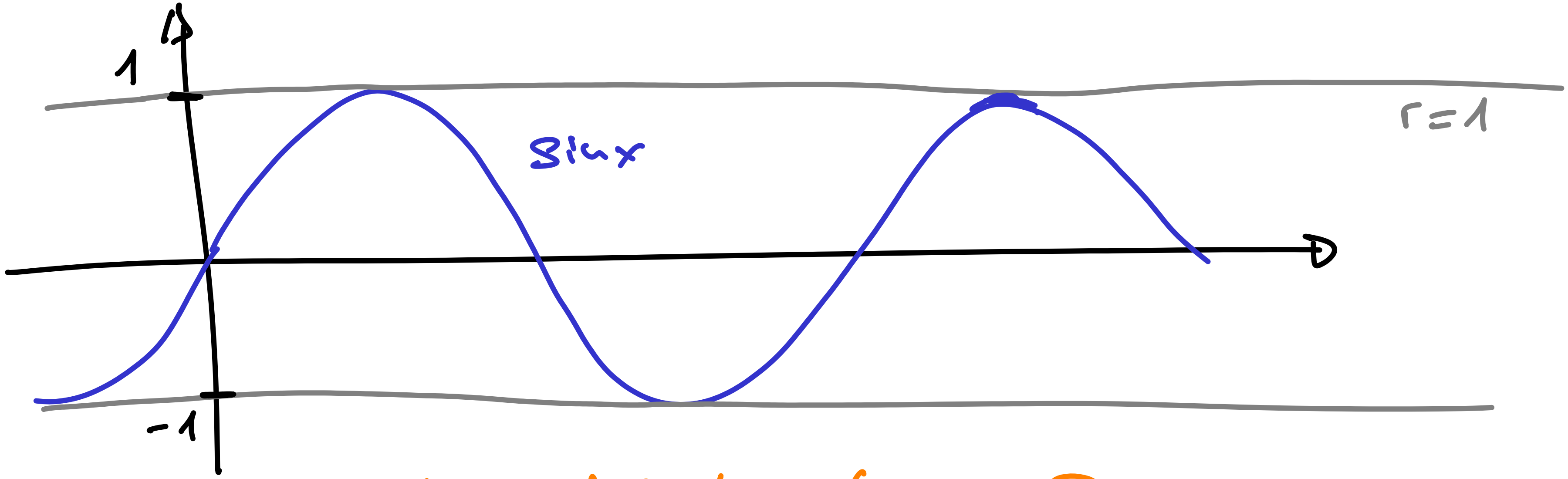


$|\sin x| \leq 1 \quad \forall\, x \in \mathbb{R}$

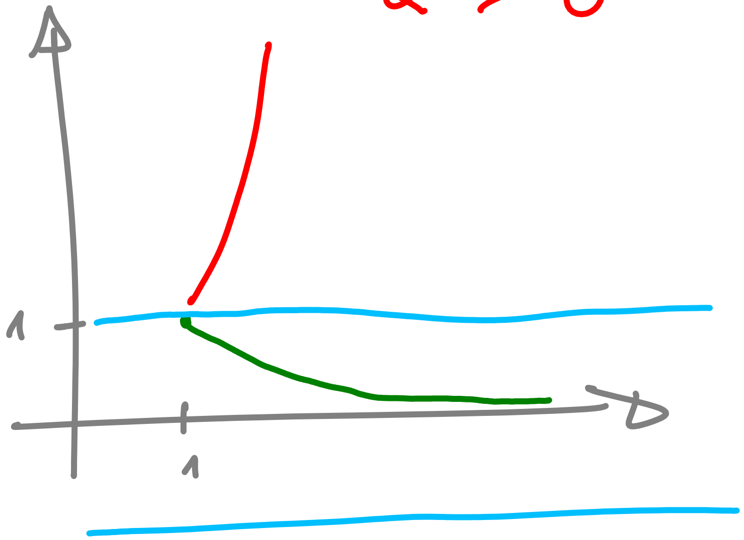
anderes Bsp:

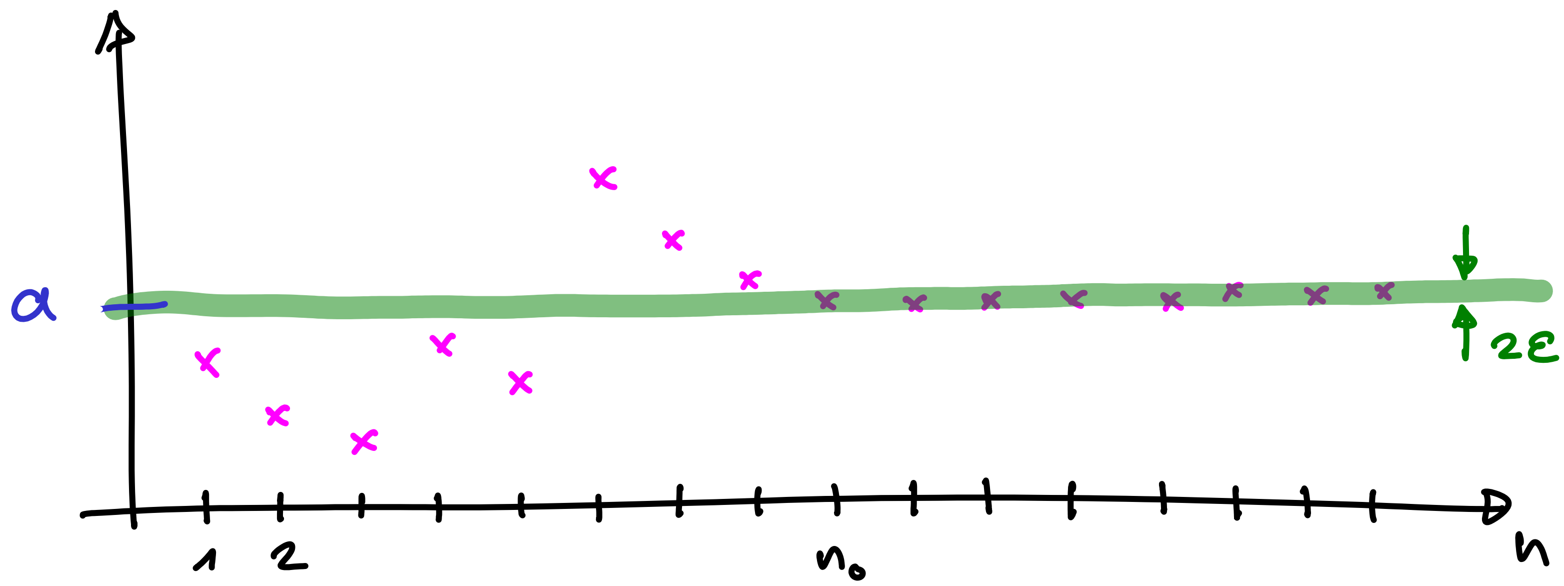
$$5 \sin x - 2 =: f(x)$$

$$\Rightarrow \quad -7 \leq f(x) \leq 3 \quad \forall\, x \in \mathbb{R}$$

d.h. beschränkt mit $r=7$, denn $|f(x)| \leq 7 \;\forall\, x \in \mathbb{R}$.

$x \mapsto x^{\alpha}$, $\alpha < 0$ auf $[1, \infty)$
$\alpha > 0$



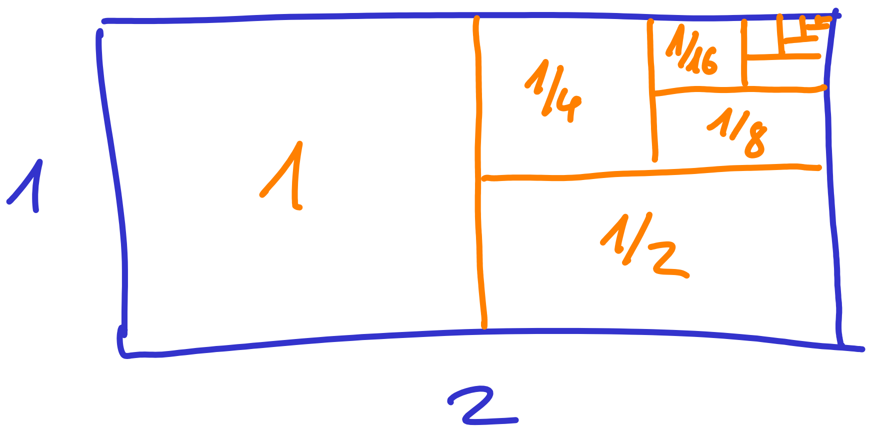


Beh: $a_n = \frac{1}{n} \xrightarrow[n\to\infty]{} 0$

Warum?

$$|a_n - 0| = \left|\frac{1}{n}\right| = \underbrace{\frac{1}{n} \overset{!}{<} \varepsilon}_{\text{hätten wir gerne}}$$

$\Longleftrightarrow \; n > \frac{1}{\varepsilon}$ d.h. wähle $n_0 \geq \frac{1}{\varepsilon}$, damit klappt's ☺



Fläche: $2 = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \dots = \sum_{k=0}^{\infty} \left(\frac{1}{2}\right)^k$

$$\vec{a}_n = \underbrace{\begin{pmatrix} \sum_{k=0}^{n} \left(\frac{1}{2}\right)^k \\ 1/n \end{pmatrix}}_{\in \mathbb{R}^2} \xrightarrow[n\to\infty]{} \begin{pmatrix} 2 \\ 0 \end{pmatrix}$$

$$\vec{b}_n = \underbrace{\begin{pmatrix} \sum_{k=0}^{\infty} \left(\frac{1}{2}\right)^k \\ (-1)^n \\ 1/n \end{pmatrix}}_{\in \mathbb{R}^3}$$

konvergiert <u>nicht</u>, da $(-1)^n$ <u>nicht</u> konvergiert

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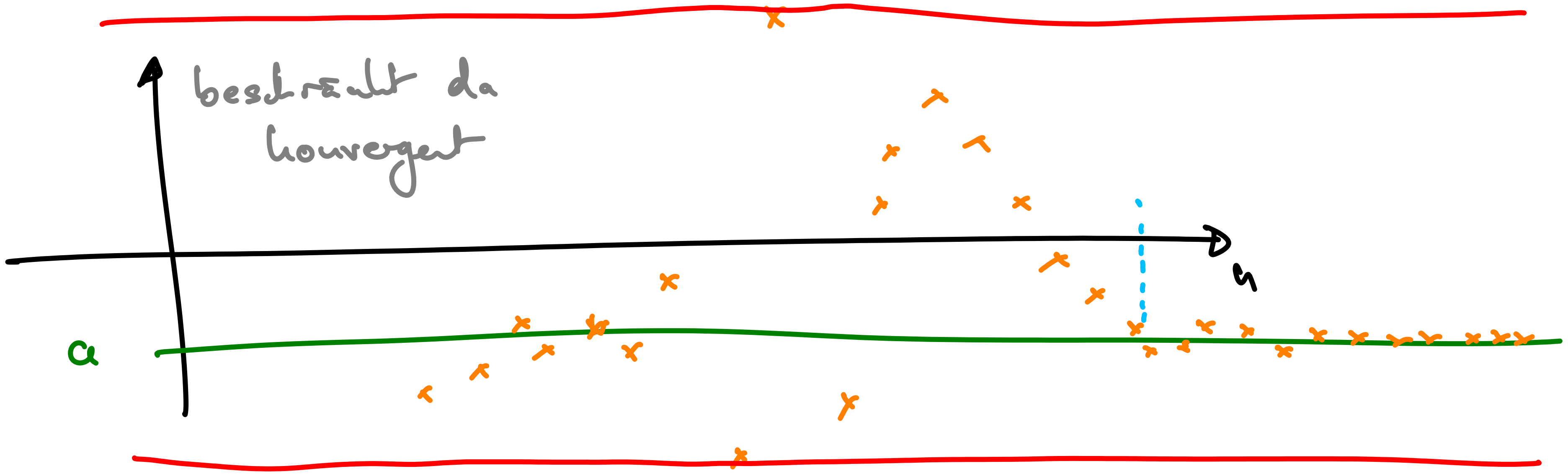
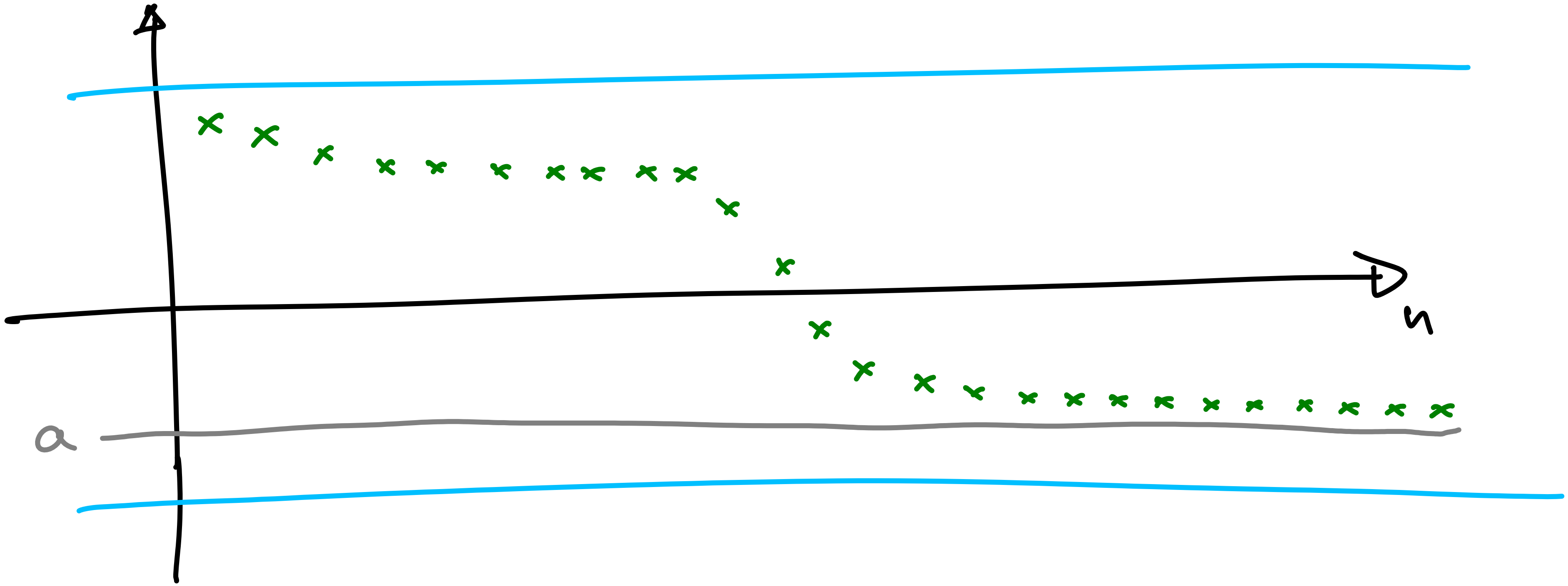
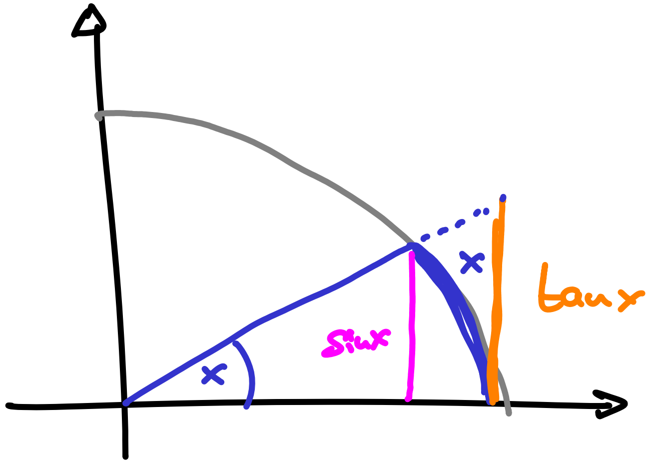


Folge a_n sei monoton fallend & beschränkt



für $0 \leq x \leq \frac{\pi}{2}$:

$$\sin x \leq x \leq \tan x$$



uns interessiert $\frac{\sin\left(\frac{1}{n}\right)}{\frac{1}{n}}$ für $n \to \infty$

d.h. $\frac{\sin x}{x}$ für kleine positive x

$$\sin x \leq x \leq \tan x$$

$$\Longleftrightarrow \quad \frac{1}{\sin x} \geq \frac{1}{x} \geq \frac{1}{\tan x} \quad \Big| \cdot \sin x$$

$$\Longleftrightarrow \quad 1 \geq \frac{\sin x}{x} \geq \cos x$$

d.h.

$$1 \geq \frac{\sin\left(\frac{1}{n}\right)}{\frac{1}{n}} \geq \cos\left(\frac{1}{n}\right) \xrightarrow[n \to \infty]{} 1$$

$$\Longrightarrow \quad \lim_{n \to \infty} \frac{\sin\left(\frac{1}{n}\right)}{\frac{1}{n}} = 1$$

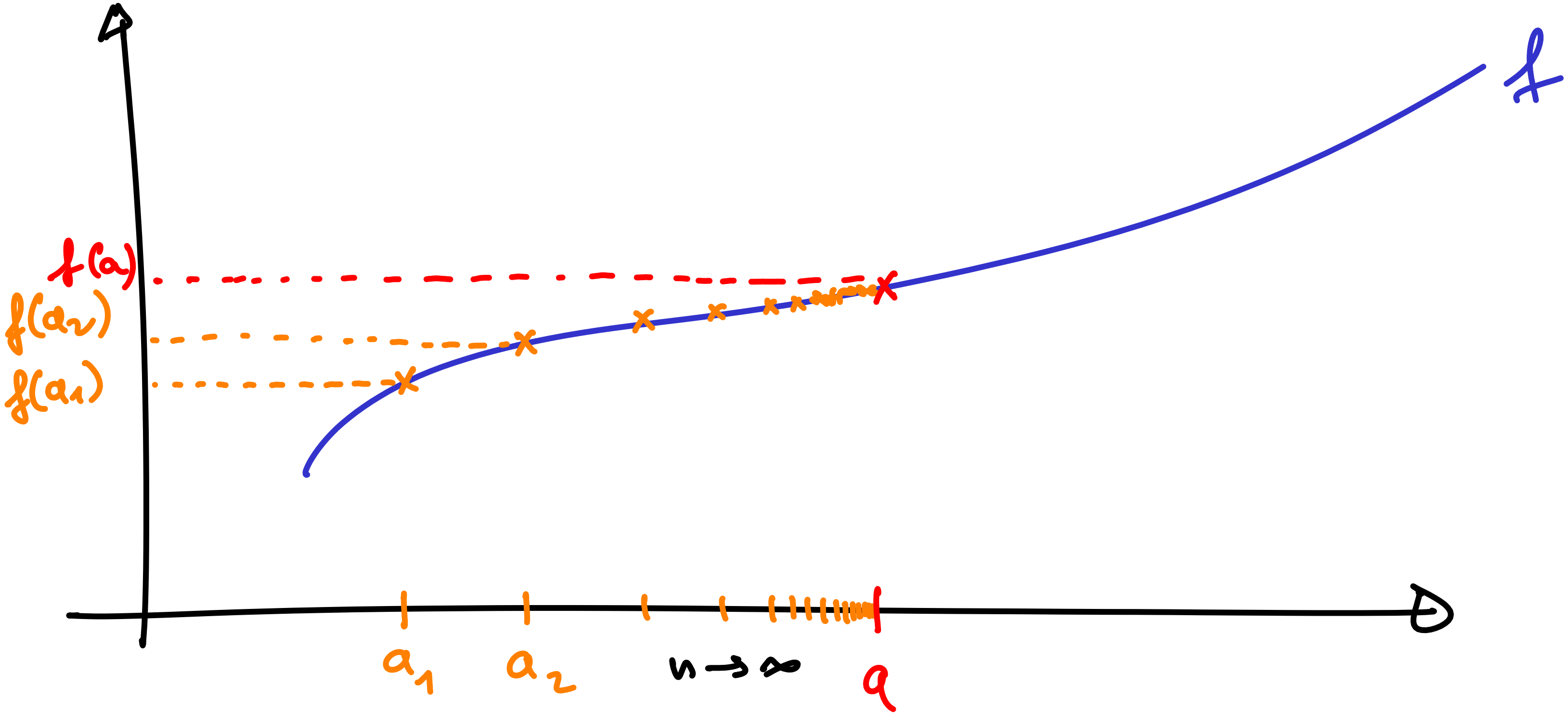
$$\lim_{n\to\infty} \frac{n^{2013} - 2n^2}{n^{12} + 3n^{2013}} = \lim_{n\to\infty} \frac{1 - 2\frac{1}{n^{2011}}}{\frac{1}{n^{2001}} + 3}$$

$$\overset{\text{Produkt}}{=} \frac{\lim\limits_{n\to\infty}\left(1 - \frac{2}{n^{2011}}\right)}{\lim\limits_{n\to\infty}\left(\frac{1}{n^{2001}} + 3\right)} \underset{\text{Summe}}{=} \frac{\lim\limits_{n\to\infty} 1 - \lim\limits_{n\to\infty} \frac{2}{n^{2011}}}{\lim\limits_{n\to\infty} \frac{1}{n^{2001}} + \lim\limits_{n\to\infty} 3}$$

$$= \frac{1-0}{0+3} = \frac{1}{3}$$

aber nicht

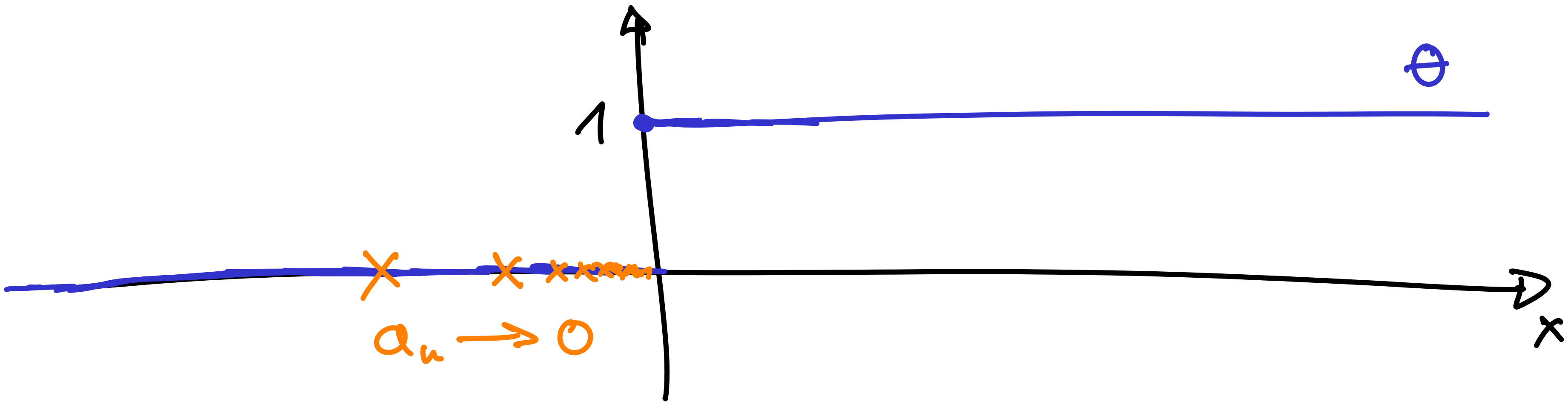
$$\lim_{n\to\infty} n \cdot \sin\left(\frac{1}{n}\right) \neq \left(\lim_{n\to\infty} n\right) \cdot \left(\lim_{n\to\infty} \sin\frac{1}{n}\right)$$



$$\lim_{n\to\infty} f(a_n) = f\left(\lim_{n\to\infty} a_n\right) = f(a)$$

Stetigkeit von f

$$\Theta(x) = \begin{cases} 1, & x \geq 0 \\ 0, & x < 0 \end{cases}$$



z.B. $a_n = -\frac{1}{n}$

$$\lim_{n\to\infty} \Theta(a_n) \underset{a_n < 0 \ \forall n}{=} \lim_{n\to\infty} 0 = 0$$

$$\neq$$

$$\Theta\left(\lim_{n\to\infty} a_n\right) = \Theta(0) = 1$$

$q^0 = 1, \quad q^1 = q, \quad q^2, \quad q^3, \quad q^4, \; \ldots$

z.B.

$1, \quad \frac{1}{2}, \quad \frac{1}{4}, \quad \frac{1}{8}, \quad \frac{1}{16}, \; \ldots$

$1, \quad \frac{2}{3}, \quad \frac{4}{9}, \quad \frac{8}{27}, \; \ldots$

kleiner

da Multiplikation mit Faktor $|\ldots| < 1$.

geom. Summe

$$S_n = \sum_{k=0}^{n} q^k = \underline{\underline{1}} + q + q^2 + q^3 + \dots + q^n$$

$$q \cdot S_n = q + q^2 + q^3 + q^4 + \dots + \underline{\underline{q^{n+1}}}$$

Differenz

$$S_n - q \cdot S_n = 1 - q^{n+1} \quad | \; 1-q$$

$$\underset{q \neq 1}{\Leftrightarrow} \quad S_n = \frac{1-q^{n+1}}{1-q}$$

also

$$\boxed{\sum_{k=0}^{n} q^k = \frac{1-q^{n+1}}{1-q} \quad \forall \; q \neq 1}$$

nun für $|q| < 1$:

$$\lim_{n\to\infty} \sum_{k=0}^{n} q^k = \underline{\sum_{k=0}^{\infty} q^k} = \frac{1 - \lim q^{n+1}}{1-q} = \frac{1}{1-q}$$

$= 0$, da $|q| < 1$