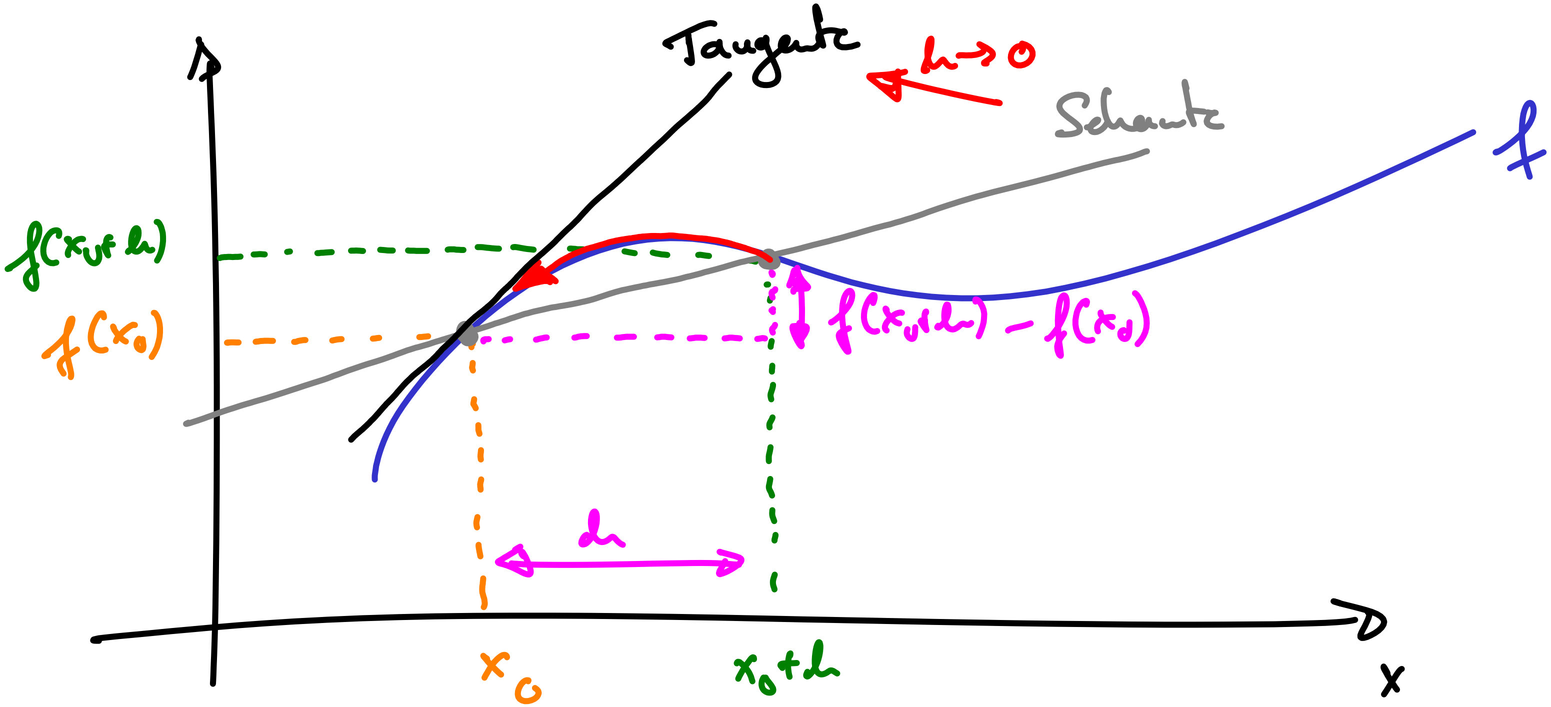


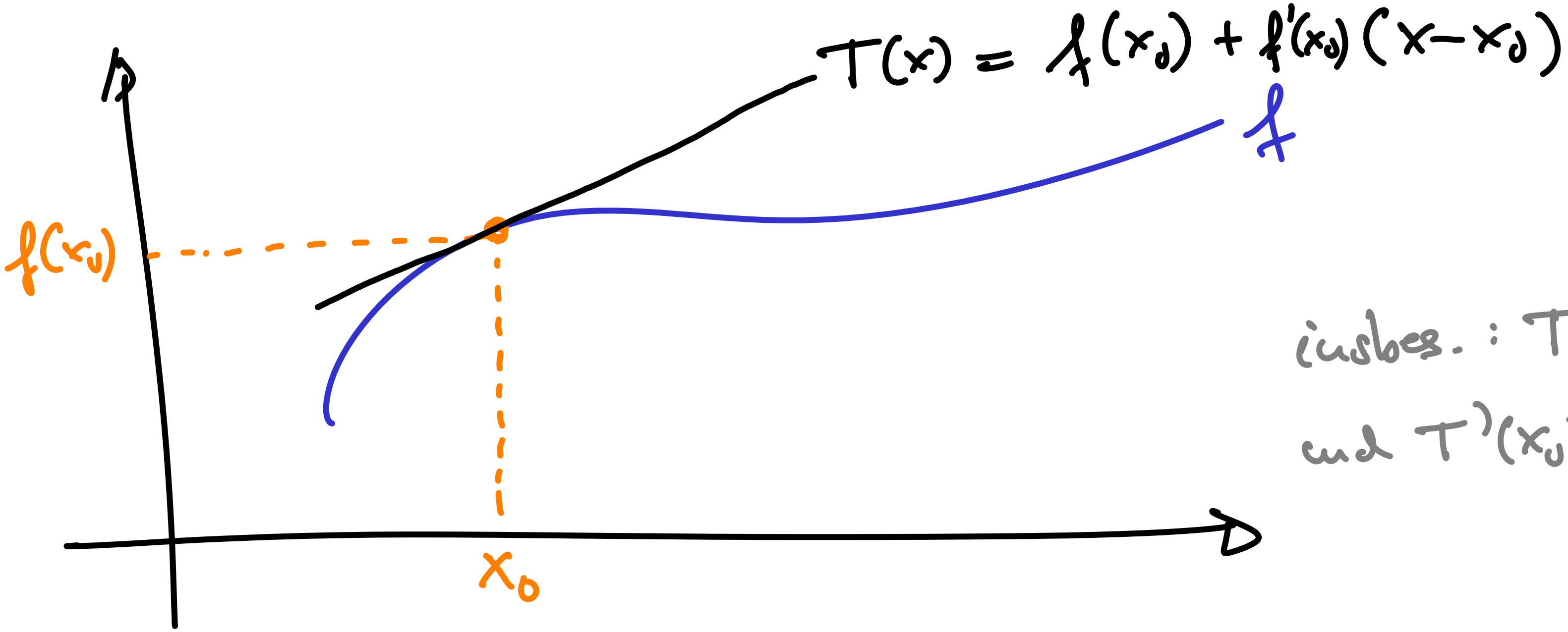
Notation

$$f'(x) = \frac{df}{dx}(x) = \frac{d}{dx} f(x)$$

$$f' = \frac{df}{dx} = \frac{d}{dx} f$$

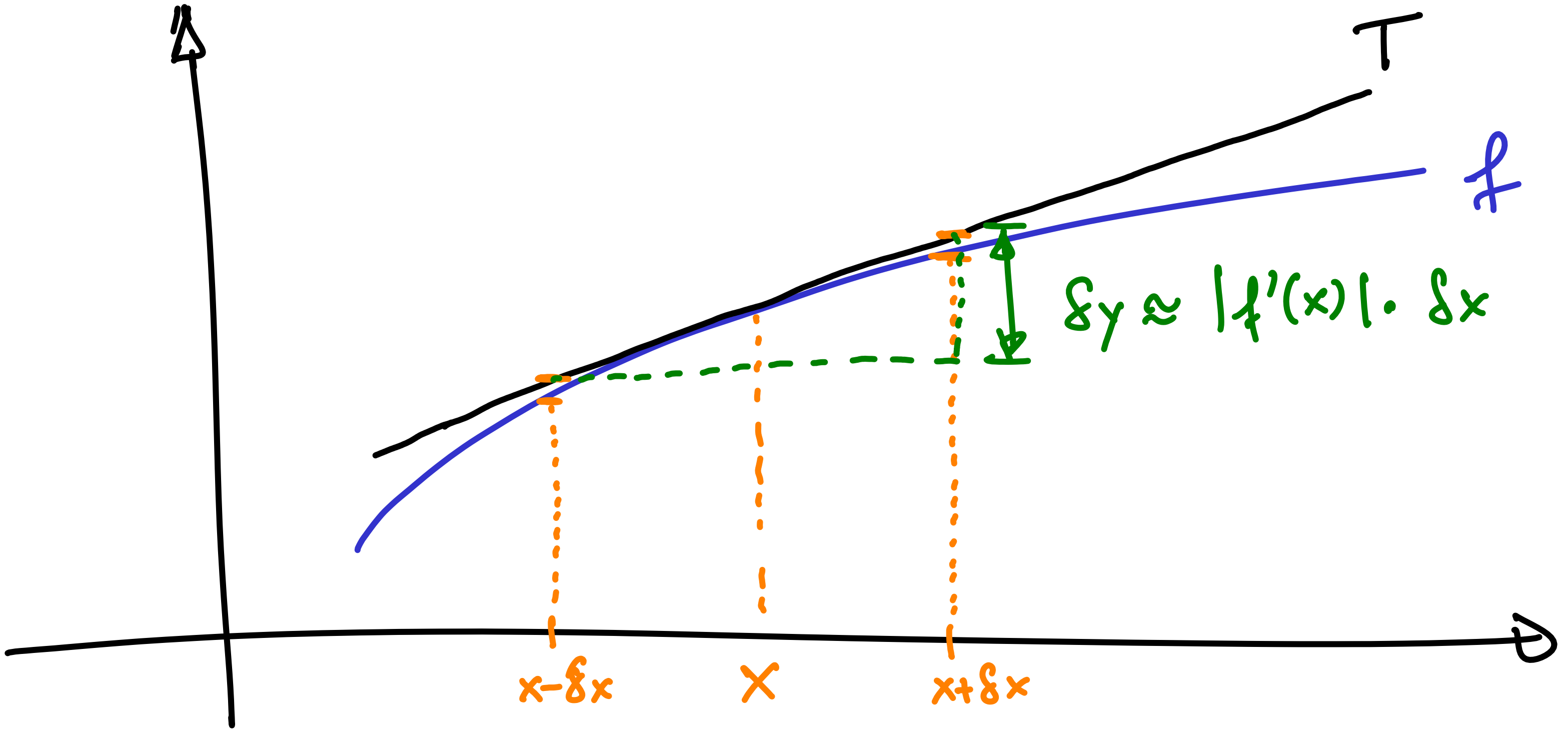
$$f'' = f^{(2)} = \frac{d^2 f}{dx^2} = \left(\frac{d}{dx}\right)^2 f$$

$$f''''''' = f^{(7)}$$



insbes.: $T(x_0) = f(x_0)$

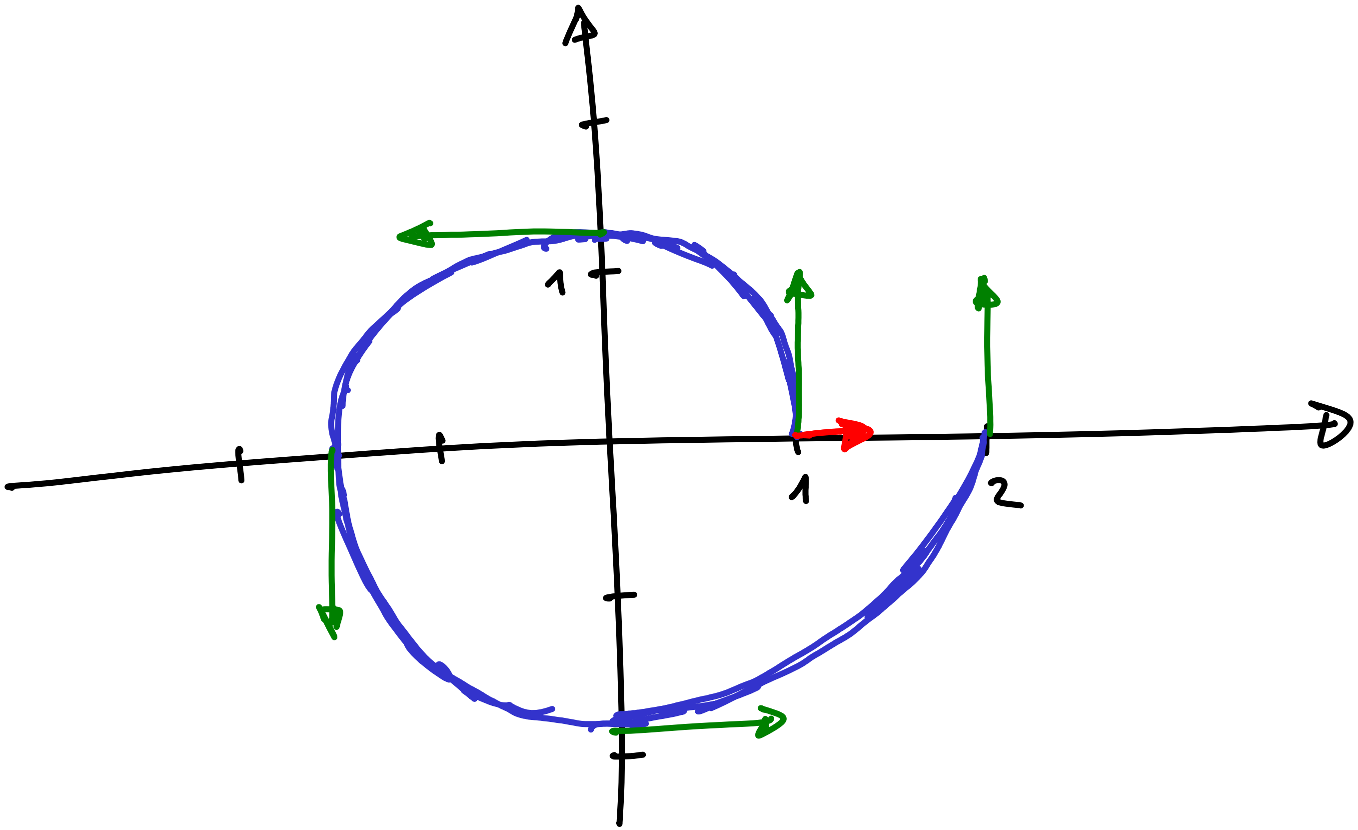
und $T'(x_0) = f'(x_0)$



$x \in [x-\delta x, \; x+\delta x]$

dann ist ungefähr $y \in [f(x) - |f'(x)| \cdot \delta x, \; f(x) + |f'(x)| \delta x]$

Kurve $\vec{x}: [0, 2\pi] \to \mathbb{R}^2$



zunächst Kreis

$$\vec{x}(t) = \begin{pmatrix} \overset{\text{Radius}}{r} \cos t \\ r \sin t \end{pmatrix}, \quad t \in [0, 2\pi]$$

Spirale

$$\vec{x}(t) = \begin{pmatrix} \left(1+\frac{t}{2\pi}\right)\cos t \\ \left(1+\frac{t}{2\pi}\right)\sin t \end{pmatrix}, \quad 0 \le t \le 2\pi$$

Ableitung von $\left(1+\frac{t}{2\pi}\right)\cos t$

$$\dot{\vec{x}}(t) = \begin{pmatrix} \frac{1}{2\pi}\cos t + \left(1+\frac{t}{2\pi}\right)(-\sin t) \\ \frac{1}{2\pi}\sin t + \left(1+\frac{t}{2\pi}\right)\cos t \end{pmatrix}$$

$$\overline{\vec{v}} = \frac{\vec{x}(2\pi) - \vec{x}(0)}{2\pi} = \frac{\begin{pmatrix} 2 \\ 0 \end{pmatrix} - \begin{pmatrix} 1 \\ 0 \end{pmatrix}}{2\pi} = \frac{1}{2\pi}$$

Bsp für Ableitung

$$f(x) = \frac{1}{x}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{1}{h} \left(\frac{1}{x+h} - \frac{1}{x} \right)$$

$$= \lim_{h \to 0} \frac{1}{h} \frac{x - (x+h)}{x(x+h)} = \lim_{h \to 0} \frac{-\cancel{h}}{\cancel{h}\, x(x+h)} = -\frac{1}{x^2}$$

$$f(x) = a^x$$

$$f'(x) = \lim_{h \to 0} \frac{a^{x+h} - a^x}{h} = \lim_{h \to 0} \frac{a^x \cdot a^h - a^x}{h}$$

$$= a^x \underbrace{\lim_{h \to 0} \frac{a^h - 1}{h}}_{\text{hängt nicht von } x \text{ ab}}$$

Für $a = e = 2{,}718\ldots$ gilt $\lim_{h \to 0} \frac{e^h - 1}{h} = 1$

$$g(x) = x^7, \quad f(x) = 5x^3$$

$$(x^7 + 5x^3)' = (x^7)' + 5 \cdot (x^3)'$$

$$= 7 \cdot x^6 + 15 \cdot x^2$$

Produktregel

$$(f \cdot g)'(x) = \lim_{h \to 0} \frac{f(x+h)\,g(x+h) - f(x)\,g(x)}{h}$$

$$= \lim_{h \to 0} \frac{f(x+h)\,g(x+h) - f(x)\,g(x+h) + f(x)\,g(x+h) - f(x)\,g(x)}{h}$$

$$= \lim_{h \to 0} \left(\frac{f(x+h) - f(x)}{h}\,g(x+h) + f(x)\,\frac{g(x+h) - g(x)}{h} \right)$$

$$= f'(x) \quad g(x) \quad + \quad f(x) \quad g'(x)$$

Beispiel: $(e^x)' = e^x$, $x^2 = 2x$

$$(x^2 \cdot e^x)' = (x^2)' \cdot e^x + x^2 \cdot (e^x)'$$
$$= 2x\, e^x + x^2 e^x = (2x + x^2)\, e^x$$

Kettenregel

näheren g durch Tangente

$$[f(g(x))]' = \lim_{h \to 0} \frac{f(g(x+h)) - f(g(x))}{h}$$

$$= \lim_{h \to 0} \frac{f(g(x) + g'(x) \cdot h) - f(g(x))}{h \cdot g'(x)} \cdot g'(x)$$

$h \cdot g'(x) \to 0$

$$= f'(g(x)) \cdot g'(x)$$

Beispiel: $\left(e^{(x^2)}\right)' = e^{(x^2)} \cdot \underline{2x}$ innere Ableitung

Ableitung der Umkehrfkt.

$$f(f^{-1}(x)) = x$$

$\Longrightarrow$ Ableiten

$$f'(f^{-1}(x)) \cdot \underline{(f^{-1})'(x)} = 1$$

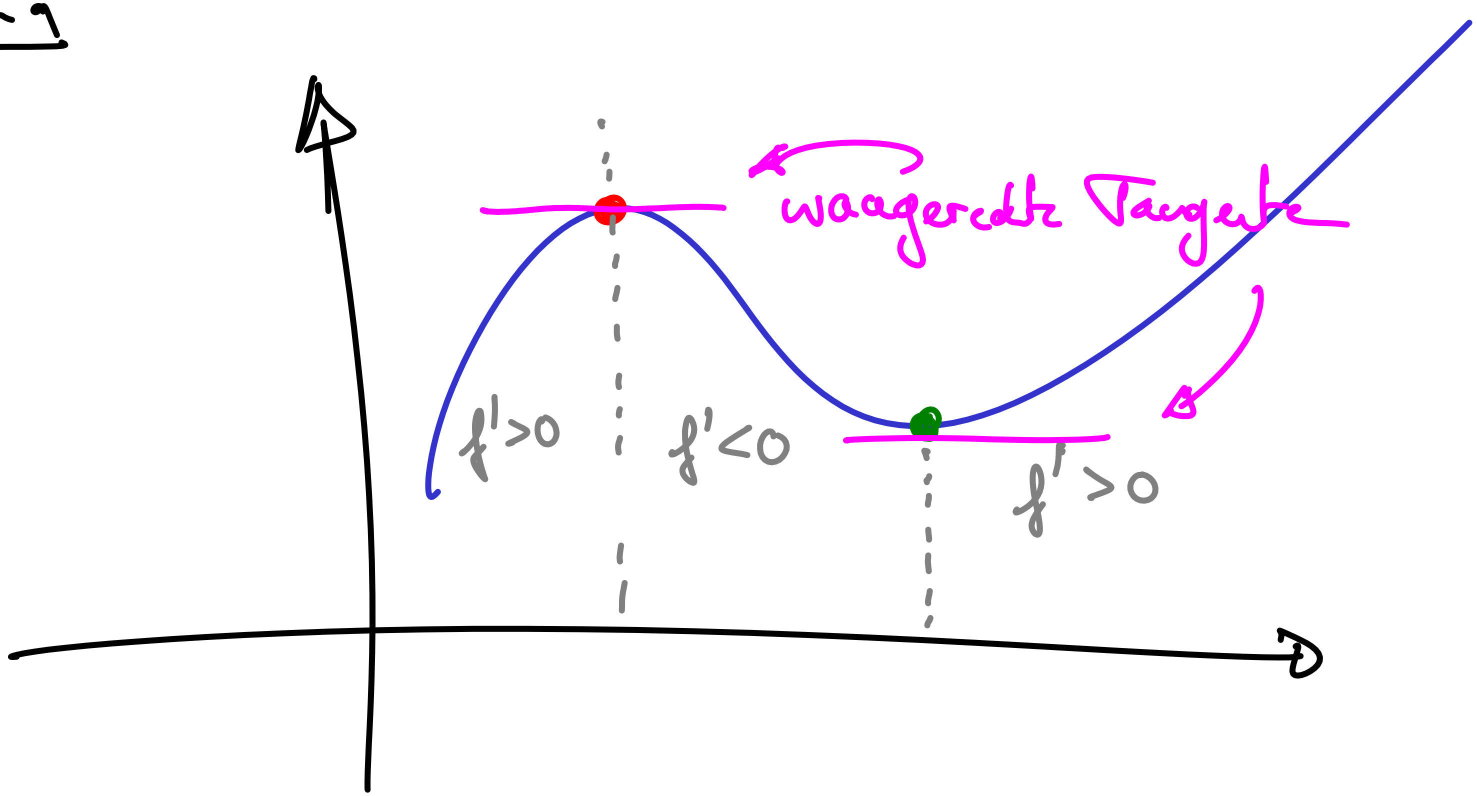
$\Longrightarrow$ $f'(f^{-1}(x)) \neq 0$

$$(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))}$$

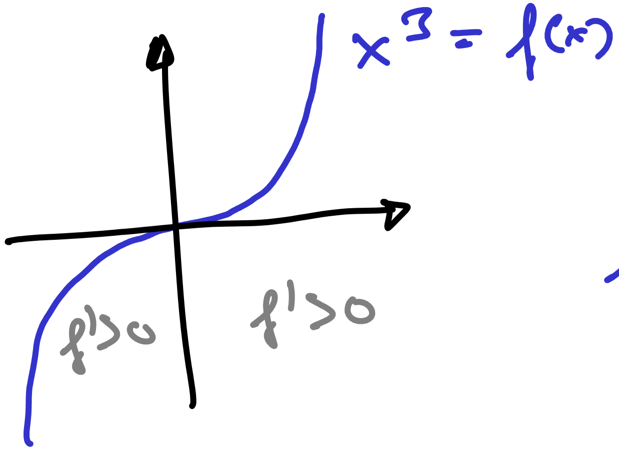
Beispiel: $\log(\exp(x)) = x$

$$(\log x)' = \frac{1}{\exp'(\log(x))} = \frac{1}{\exp(\log x)} = \frac{1}{x}$$

Extrema



Umkehrung gilt nicht



$f'(0) = 0$ aber

kein Extremum